Transmission and Detection of Side Information for Selected Mapping in MIMO OFDM

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***Abstract*— In order to reduce the peak-to-average power ratio (PAR) of multi-antenna orthogonal frequency-division multiplexing (MIMO OFDM) systems selected mapping (SLM) and its MIMO extensions are considered. With SLM the transmission of some side information is necessary. In this paper, we focus on the approach of embedding the side information into the OFDM frame using combinations of unrotated and $\pi/4$ rotated QAM constellations, known for single-antenna systems, and extend it to the MIMO scenario. Moreover, we analyze the detection and estimation of the side information at the receiver. We analyze soft decision for the side information and derive strategies for joint estimation (within one MIMO OFDM frame) of the embedded side information. Numerical results show that the PAR reduction performance behaves conversely to the side information error performance. For instance, directed SLM exhibits the best PAR reduction performance of the SLM variants discussed in this paper but is slightly more prone to transmission errors. However, applying the schemes mentioned above is beneficial to the side information error performance of directed SLM.**

I. Introduction

One essential drawback of *orthogonal frequency-division multiplexing (OFDM)* [4] systems is the high dynamic of the transmit signal. The occurrence of high signal peaks leads to signal clipping at the non-linear power amplifier, which in turn leads to very undesirable out-of-band radiation. In order to avoid violating spectral masks a PAR reduction algorithm should be applied at the transmitter. An overview of different approaches is given in [12]. In this paper, we will concentrate on the *selected mapping (SLM)* [3] family of PAR reduction schemes.

Furthermore, transmitters of modern communication systems are equipped with multiple antennas, resulting in so-called MIMO (multiple-input/multiple-output) systems. In MIMO OFDM the issue of out-of-band radiation when transmitting signals exhibiting high PAR gets even more serious. Recently, the PAR reduction scheme SLM has been extended in many ways in order to utilize the advantages of multi-antenna systems.

In this paper, we focus on the MIMO extensions of SLM, namely ordinary [2], simplified [2], and directed [7], [8], [9], [11] SLM. For this scenario, we will extend the approach of transmitting the side information by combinations of original and $\pi/4$ rotated QAM constellations across the subcarriers [18] and analyze the error performance.

Section II defines the considered system model; Section III reviews the MIMO extensions of SLM. The embedding of the side information at the transmitter is described in Section IV; its decoding at the receiver is studied in Section V. In Section VI numerical results are discussed and conclusions are drawn in Section VII.

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II. System Model

Subsequently, we consider transmission over a frequency-selective MIMO channel. The transmitter is equipped with N_{T} antennas and the receiver with N_{R} antennas. For convenience, we concentrate on the case that both, transmitter and receiver are equipped with the same number of antennas, i.e., $N_{\mathrm{T}} = N_{\mathrm{R}} = K$. Moreover, point-to-point transmission is assumed, i.e., joint signal processing is possible at both the transmitter and the receiver.

In order to equalize the temporal intersymbol interferences, OFDM with D subcarriers is applied. We assume all subcarriers being active.

The complex-valued modulation symbols, drawn from an M-ary QAM constellation, are collected in the $K \times D$ matrix $\boldsymbol{A} = [A_{k,d}]$, the frequency-domain MIMO OFDM frame. The row vectors $\boldsymbol{A}_k = [A_{k,d}]$, $k = 1, \ldots, K$, $d = 0, \ldots, D-1$ of the matrix $\boldsymbol{A}$ are the respective frequency-domain OFDM frames for the K antennas. For convenience, we assume that all elements $A_{k,d}$ are drawn from an $M = 4$-ary QAM constellation with signal points $A_{d,k} \in \{\pm\frac{1}{2} \pm \frac{\mathrm{j}}{2}\}$, which are of zero mean and have the variance $\sigma_a^2 = \mathrm{E}\{|A_{k,d}|^2\} = 0.5$.

The time-domain MIMO OFDM frame $\boldsymbol{a} = [a_{k,\kappa}]$ is now obtained via an inverse discrete Fourier transform (IDFT) [16] along each row of the matrix $\boldsymbol{A}$, i.e., $a_{k,\kappa} = 1/\sqrt{D}\sum_{d=0}^{D-1} A_{k,d} \cdot \mathrm{e}^{\mathrm{j}2\pi\kappa d/D}$, $\kappa = 0, \ldots, D-1$, $k = 1, \ldots, K$.

Subsequently, it is always assumed that the cyclic prefix [4] is large enough; then it is sufficient to consider the transmission over D parallel flat-fading channels. For simplicity, we do not consider appending the cyclic prefix, pulse shaping or modulation to radio frequency but restrict ourselves to the non-oversampled transmit signal in the equivalent complex baseband [19].

The frequency-selective MIMO channel of length l_h is written in the z-domain as a matrix polynomial $\boldsymbol{h}(z) = \sum_{\kappa=0}^{l_h-1} \boldsymbol{h}_\kappa z^{-\kappa}$, where $\boldsymbol{h}_\kappa$ are $K \times K$ matrices containing the (complex valued) fading coefficients. Due to the OFDM technique [4], the flat-fading channels at the D subcarriers are given by $\boldsymbol{H}_d = \boldsymbol{h}(\mathrm{e}^{\mathrm{j}2\pi d/D})$, $d = 0, \ldots, D-1$.

Throughout this paper, we assume that the spatial interferences at each subcarrier are equalized at the receiver. As strategy a simple linear zero-forcing (ZF) equalizer [6], [21], i.e., multiplication of the received signal with the inverse $\boldsymbol{H}_d^{-1}$ of the channel matrix at the d^{th} subcarrier, is considered.

Due to the central limit theorem, the time-domain samples $a_{k,\kappa}$ are approximately Gaussian distributed, which leads to the high peak-to-average power ratio (PAR) of the transmit signal. As common in literature dealing with MIMO OFDM, we consider the "worst-case" PAR, i.e., the maximum PAR over all transmit antennas, which is defined as

$$\mathrm{PAR} = \frac{\max_{\forall\kappa,\forall k} |a_{k,\kappa}|^2}{\sigma_a^2}\,. \tag{1}$$

As performance measure for the different peak-to-average power ratio reduction schemes, we consider the *complementary cumulative distribution function (ccdf)*, i.e., the probability that the PAR of one MIMO OFDM frame exceeds a certain threshold $\mathrm{PAR}_{\mathrm{th}}$

$$\mathrm{ccdf}(\mathrm{PAR}_{\mathrm{th}}) = \Pr\{\mathrm{PAR} > \mathrm{PAR}_{\mathrm{th}}\}\,. \tag{2}$$

III. Extensions of Selected Mapping to MIMO OFDM Systems

A. Review of Ordinary and Simplified Selected Mapping

Originally, selected mapping has been proposed for single antenna schemes [3]. In [2], Baek et al. presented generalizations of SLM to MIMO point-to-point scenarios, namely, *ordinary SLM (oSLM)* and *simplified SLM (sSLM)*.

Ordinary SLM is the simple parallel application of single-antenna SLM to each transmit antenna. Considering the k^{th} transmit antenna, the principal idea is to generate out of the original information carrying OFDM frame $\boldsymbol{A}_k$ multiple, say U, different signal representations $\boldsymbol{A}_k^{(u)}$, $u = 1,\ldots,U$, via some bijective mapping. According to [3], these candidates may be obtained by an element-wise multiplication ($\odot$) with randomly selected phase vectors (candidate generating vectors) $\boldsymbol{P}_{\mathrm{c}}^{(u)} = [P_{\mathrm{c},d}^{(u)}]$, $u = 1,\ldots,U$, $d = 0,\ldots,D-1$, with elements $P_{\mathrm{c},d}^{(u)} \in \{\pm 1, \pm \mathrm{j}\}$. This choice is favourable as it does not affect the receiver sided synchronisation but gives the same results as when choosing from a larger set.

Next, the U different signal representations at the K transmit antennas are transformed into time domain and at each antenna that one exhibiting the lowest PAR is selected for transmission. Thus, a total computational complexity of KU calculations of the IDFT and KU metric calculations (PAR) is necessary.

In order to decode the signal correctly, the receiver must be informed which one of the U possible signal representations has been transmitted at the respective antenna. Assuming both transmitter and receiver are aware of a codebook of possible phase vectors it is sufficient to transmit the index $\check{u}_k \in \{1,\ldots,U\}$, $k = 1,\ldots,K$, of the actual used phase vector for the signal at the k^{th} antenna. Hence, in total[1] $\mu = \lceil \log_2(U) \rceil$ bits per transmit antenna are necessary to code these indices. An overview of the required bits of side information for the different variants of SLM is shown in Tab. I.

This side information is especially prone to transmission errors. With an erroneous estimation of the side information index $\check{u}_k$ the receiver is unable to revert the actual applied mapping on the received OFDM frame, which will lead to the loss of the whole information at the k^{th} antenna.

The idea of simplified SLM is basically the same as of oSLM, i.e., at each of the K transmit antennas exactly U signal candidates are generated and assessed. Unlike oSLM the selection of the "best" signal candidate is not accomplished individually at each transmit antenna, but jointly over all transmit antennas. Hence, at each antenna the same index of signal candidate is chosen for transmission, i.e., only one index $\check{u}$ is necessary, here. The complexity of sSLM is the same as with oSLM, as the same number of signal candidates has to be assessed. The only simplification is that less bits of side information per antenna are required, namely $\mu = \lceil \log_2(U)/K \rceil$.

TABLE I. Required number μ of bits of side information per transmit antenna for different SLM variants.

SLM variant	μ
oSLM	$\lceil \log_2(U) \rceil$
sSLM	$\lceil \log_2(U)/K \rceil$
dSLM (original)	$\lceil \log_2(K(U-1)+1) \rceil$
dSLM (RS coding)	$\lceil \log_2(KU) \rceil$

For the subsequent analysis we need to define the number $\hat{U}$ of required mappings per transmit antenna. For both, ordinary and simplified SLM, this number is evidently given by $\hat{U} = U$.

B. Directed Selected Mapping

1) Original Candidate Generation: In [7], [8] a novel variant of SLM for MIMO point-to-point scenarios was introduced, called *directed SLM (dSLM)*, which utilizes the multiple transmit antennas to improve PAR reduction. This technique considers the K OFDM frames jointly and is able to achieve a significant gain in PAR reduction by distributing complexity adaptively over the antennas.

In particular, the idea of dSLM is as follows. In the initial step all OFDM frames at the K transmit antennas are transformed into time domain and the respective PARs are evaluated. The following assessment of signal candidates works iteratively and concentrates in each iteration on that transmit antenna with the signal exhibiting the highest PAR. This step is reasonable as the maximum PAR over all transmit antennas (according to (1)) should be lowered. For this signal an alternative representation is generated and assessed. This procedure is repeated until in total KU signal candidates have been assessed and hence the same complexity as with oSLM or sSLM is consumed.

Now, the maximum possible number of assessed signal candidates at one transmit antenna can be as large as $K(U-1)+1$, hence $\hat{U} = K(U-1)+1$ different mappings (phase rotations via $\boldsymbol{P}_{\mathrm{si}}$ are required. Consequently, $\mu = \lceil \log_2(\hat{U}) \rceil$ bits of side information have to be transmitted per OFDM frame and per transmit antenna.

With dSLM the probabilities that at one transmit antenna the u^{th} signal candidate is chosen for transmission are not uniformly distributed. For smaller values u_k these probabilities are higher. In particular, the probability that the u^{th} signal candidate is chosen reads[2] [15]

$$\Pr\{u\} = \frac{\binom{KU-u}{K-1}}{U \cdot \binom{KU-1}{K-1}}\,. \tag{3}$$

[1]In this paper, we distinguish between the natural logarithm ($\log(\cdot)$) and the logarithm with respect to basis b ($\log_b(\cdot)$). Furthermore, $\lceil\cdot\rceil$ denotes rounding to the next integer towards infinity.

[2]This probability can be easily calculated by considering a typical urn problem, which is putting KU balls into K urns with at least one ball in each urn (see e.g., [15]).

2) Candidate Generation via Reed-Solomon Coding: In [9], [11] an alternative way to generate alternative signal representations has been proposed. With this scheme the K OFDM frames $\boldsymbol{A}_k$ at the K transmit antennas are combined into a hyper frame and processed jointly. In particular, a (shortened) (N, K) *Reed-Solomon (RS) code* [20] is applied across the hyper frame in order to obtain $KU \leq N$ signal candidates. Due to the MDS property (see [20]) of the Reed-Solomon code it is sufficient to choose any K OFDM frames out of the KU signal candidates. The original K OFDM frames may be obtained through the selected K ones using erasure decoding. After mapping to QAM symbols and transformation into time domain of all KU signal candidates, the K best, i.e., the ones exhibiting the lowest PAR, are chosen for transmission.

After equalization, the receiver is able to decode the information-carrying OFDM frames by performing erasure decoding. However, the decoder has to be aware which symbols have been erased; again some side information is required. In particular, with each OFDM frame its position within the Reed-Solomon codeword has to be transmitted. Hence, in total $\hat{U} = KU$ signal candidates have to be distinguished, and $\mu = \lceil \log_2(\hat{U}) \rceil$ bits of side information (see also Tab. I) are required, which is a little more than with the original approach of dSLM.

With this variant of dSLM, the probability that the u_k^{th} signal candidate is chosen at the k^{th} transmit antenna is given by a hypergeometric distribution [15]

$$\Pr\{u_k\} = \frac{\binom{u_k-1}{k-1} \cdot \binom{\hat{U}-u_k}{K-k}}{\binom{\hat{U}}{K}} \tag{4}$$

for $u_k = k, \ldots, \hat{U} - K + k$ and $k = 1, \ldots, K$.

IV. Transmission of Side Information

As mentioned above, with SLM the transmission of side information (the index of the actual chosen signal candidate) is indispensable. In literature, many approaches have been proposed, in order to transmit the side information in a clever way, see e.g., [5], [13], [14], [1]. Recently, in [18] a technique has been proposed for the original single-antenna SLM approach. Now, with multi-antenna schemes, in total K indices ($\check{u}_k$, $k = 1, \ldots, K$) have to be transmitted. As the embedding of the side information index into the actual OFDM frame may have an effect on its PAR value all indices must already be embedded into the respective signal candidates before the evaluation is accomplished. Hence, a joint embedding over the actual chosen candidates after selection is not possible and the side information has to be embedded into each signal candidate individually. However, applying the idea of embedding the side information from [18] to the multi-antenna extensions of SLM is straightforward.

Subsequently, we give a short review of the side information embedding scheme of [18]. The side information index u is expressed as a word of binary symbols written as a vector $\boldsymbol{b}^{(u)} = [b_i^{(u)}]$, $i = 1, \ldots, \mu$, $b_i^{(u)} \in \{\pm 1\}$. The number μ of required bits depends on the actual chosen SLM approach. An overview about how many bits per transmit antenna are required for the particular SLM extension has already been given in Tab. I. The binary information is now embedded into the (frequency-domain) OFDM frame, whereby each subcarrier represents exactly one bit. This is done in that way, that $b_i^{(u)} = -1$ is represented by the original QAM constellation and $b_i^{(u)} = +1$ by the QAM constellation rotated by $\pi/4$. Now, the receiver has to estimate the combination of unrotated and rotated subcarriers and can therefore determine the index of the applied mapping on the actual OFDM frame.

With this approach, each OFDM frame is able to encode the side information in total with D bits. However, usually the number μ of required bits is much smaller than D. All subcarriers should be used to protect the side information against transmission errors. Subsequently, we use a simple $(r\mu, \mu)$ repetition code [20] with r repetitions. In principal, any block code (e.g., a Hamming code) may be used as well. Subsequently, we will concentrate on the repetition code, because its decoding at the receiver using soft values is easily possible. Moreover, the promising simulation results of Sec. VI justify this approach.

In order that the receiver-sided synchronization algorithm does not get confused by the $\pi/4$ phase rotations, the first position of one coded side information index is never rotated and serves as reference. Now, the synchronization algorithm has to refer its estimation on these unrotated subcarriers. The total number of repetitions is hence limited by $r = \lfloor D/(\mu+1) \rfloor$. At the remaining subcarriers the original constellation is used. The resulting codeword $\boldsymbol{c}_k^{(u)}$ of length D, representing the side information within the signal transmitted at the k^{th} antenna, reads

$$\boldsymbol{c}^{(u)} = [c_d^{(u)}] = [-1 \;\; \boldsymbol{b}^{(u)} \;\; -1 \;\; \boldsymbol{b}^{(u)} \;\; \ldots \;\; -1 \;\; \ldots] \,. \tag{5}$$

The sets of indices, where the i^{th} element of the vector $\boldsymbol{b}^{(u)}$ occurs within $\boldsymbol{c}^{(u)}$, are denoted as

$$\mathcal{I}_{\text{si},i} = \{i + m(\mu+1) | m = 0, \ldots, r-1\} \,, \quad i = 1, \ldots, \mu \,. \tag{6}$$

The embedding of the side information can now be described by an element-wise multiplication of the respective signal candidate (frequency-domain OFDM frame after mapping to QAM symbols) with a phase vector (side information embedding vector)

$$\boldsymbol{P}_{\text{si}}^{(u)} = [\mathrm{e}^{\mathrm{j}\frac{\pi}{8}(c_d^{(u)}+1)}] \tag{7}$$

representing the u^{th} index.

At first, we consider the embedding of the side information index for ordinary, simplified, and original directed SLM. As proposed in the original publication of SLM [3] the u_k^{th} candidate OFDM frame, at the k^{th} antenna, may be generated through an element-wise multiplication with a phase vector $\boldsymbol{P}_{\text{c}}^{(u_k)}$. In this case, the embedding of the side information can be combined with the candidate generating vector to the entire candidate generating vector $\boldsymbol{P}^{(u_k)} = \boldsymbol{P}_{\text{c}}^{(u_k)} \odot \boldsymbol{P}_{\text{si}}^{(u_k)}$, $u_k = 1, \ldots, \hat{U}$.

Now, we regard dSLM with candidate generation using RS coding. After generating the KU signal candidates each of them are mapped to QAM symbols. The index of each signal candidate can now be embedded via an element-wise multiplication with $\boldsymbol{P}_{\text{si}}^{(u)}$, $u = 1, \ldots, \hat{U} = KU$. Then, the PAR of the resulting OFDM frames is assessed and the K best ones are selected for transmission. Noteworthy, we consider the indices of the K selected OFDM frames are sorted over the transmit antennas, i.e., the frame with the lowest index is transmitted at the first antenna, the next higher index at the second antenna, and so on.

V. Decoding of the Side Information

A. Extracting the Side Information

At the receiver, the spatial interferences at each subcarrier are equalized via linear ZF equalization [6], [21]. Any other equalization method is also possible. The received frequency-domain MIMO OFDM frame after the equalization is denoted by $\boldsymbol{Y} = [\boldsymbol{Y}_d]$ (d column vectors of length K) with

$$\boldsymbol{Y}_d = \boldsymbol{A}_d^{(\check{u})} + \boldsymbol{H}_d^{-1}\boldsymbol{N}_d\,, \tag{8}$$

i.e., the actual chosen signal candidate plus a spatially correlated Gaussian noise vector with elements $\boldsymbol{H}_d^{-1}\boldsymbol{N}_d = [\check{N}_{k,d}]$, with $\check{N}_{k,d} \sim \mathcal{CN}(0, \gamma_{k,d}^2\sigma_n^2)$ and $[\gamma_{k,d}^2] = \mathrm{diag}((\boldsymbol{H}_d^{\mathsf{H}}\boldsymbol{H}_d)^{-1})$, $k = 1, \ldots, K$.

In order to extract the information, whether or not one certain subcarrier has been rotated by $\pi/4$, the received signal is taken to the fourth power, scaled by 4, and only the real part is considered. Raising the signal to the fourth power has two effects. On the one hand, the $\pi/4$ rotation via the phase vector $\boldsymbol{P}_{\mathrm{si}}^{(u)}$ is converted into a multiplication by minus one ($(\mathrm{e}^{\mathrm{j}\pi/4})^4 = -1$), whereas the original constellation is not affected. On the other hand scaling and raising the signal to the fourth power removes the 4-QAM modulated information carrying signal, i.e., $4A_{k,d}^4 = -1$ independent from the actual symbol.

The resulting symbols, containing only the information about a possible $\pi/4$ phase rotation at the k^{th} signal within the MIMO OFDM frame are denoted by $\tilde{c}_{k,d}$ and read

$$\begin{aligned}\tilde{c}_{k,d} &= 4 \cdot \mathrm{Re}\left\{Y_{k,d}^4\right\} \\ &= 4 \cdot \mathrm{Re}\left\{\left(A_{k,d}^{(\check{u})} + \check{N}_{k,d}\right)^4\right\} \\ &= \mathrm{Re}\left\{4A_{k,d}^4 \cdot P_{\mathrm{si},k,d}^4\right\} + \tilde{N}_{k,d} \\ &= c_{k,d}^{(\check{u})} + \tilde{N}_{k,d}\,, \end{aligned} \tag{9}$$

with effective noise

$$\begin{aligned}\tilde{N}_{k,d} = \mathrm{Re}\{&16(A_{k,d}^{(\check{u})})^3\check{N}_{k,d} + 24(A_{k,d}^{(\check{u})})^2\check{N}_{k,d}^2 + \\ &16A_{k,d}^{(\check{u})}\check{N}_{k,d}^3 + 4\check{N}_{k,d}^4\}\,. \end{aligned} \tag{10}$$

The probability density function $f_{\tilde{N}}(\tilde{N}_{k,d})$ of the effective noise $\tilde{N}_{k,d}$ cannot be easily derived. However, due to the statistical independence of $A_{k,d}^{(\check{u})}$ and $\check{N}_{k,d}$, it is obvious that $\tilde{N}_{k,d}$ has zero mean. The variance can be calculated to

$$\begin{aligned}\sigma_{\tilde{N}_{k,d}}^2 = \mathrm{E}\left\{\tilde{N}_{k,d}^2\right\} = &16\gamma_{k,d}^2\sigma_n^2 + 144\gamma_{k,d}^4\sigma_n^4 + \\ &384\gamma_{k,d}^6\sigma_n^6 + 192\gamma_{k,d}^8\sigma_n^8 \end{aligned} \tag{11}$$

Noteworthy, the distribution of the values $\gamma_{k,d}$ depends on the dimension of the channel matrix $\boldsymbol{H}_d$, which is determined by K, here. It turns out, that the variance $\sigma_{\tilde{N}_{k,d}}^2$ of the effective noise is smaller for larger values of K. Hence, the transmission of the side information is more reliable for increasing K.

So far, the estimation of the side information is considered only if the information carrying data symbols are drawn from a 4-QAM constellation. However, it is also possible to use larger $(M > 4)$-QAM constellations and embed the side information by combinations of original and $\pi/4$ rotated subcarriers.

Unfortunately, in this case the detection of the side information is not as simple, because the fourth power of such a signal point does not result in a single value. A detailed analysis of the estimation of the side information for larger QAM constellations is given in [18]. It is straightforwardly applicable to all schemes, discussed in this paper, and will therefore not regarded subsequently.

B. Individual Estimation of the Side Information

Out of the extracted symbols $\tilde{c}_{k,d}$, representing the side information, the receiver has to find an estimate $\hat{u}$ for the side information index $\check{u}$ of the actual applied mapping. As mentioned above, in this paper we consider a $(r\mu, \mu)$ repetition code with r repetitions of the binary codeword $\boldsymbol{b}_k^{(u_k)}$ representing the index $\check{u}$. First, we consider an independent individual estimation at each transmit antenna.

The most simple approach of decoding the index $\check{u}_k$ given the vector $\tilde{\boldsymbol{c}}_k$ is a hard decision at each relevant subcarrier and a majority decision.

An improvement compared to hard decision could be achieved if the applied repetition code is decoded using soft-decision *maximum-likelihood (ML)* or *maximum-a-posteriori (MAP)* estimators. In order to estimate the side information index $\check{u}_k$, at the k^{th} antenna, the MAP estimator reads

$$\begin{aligned}\hat{u}_k &= \underset{u_k=1,\ldots,\hat{U}}{\mathrm{argmax}}\ \Pr\{u_k|\tilde{\boldsymbol{c}}_k\} \\ &= \underset{u_k=1,\ldots,\hat{U}}{\mathrm{argmax}}\ f_{\tilde{N}}(\tilde{\boldsymbol{c}}_k|u_k) \cdot \Pr\{u_k\} \\ &= \underset{u_k=1,\ldots,\hat{U}}{\mathrm{argmax}}\ f_{\tilde{N}}(\tilde{\boldsymbol{c}}_k|\boldsymbol{c}_k^{(u_k)}) \cdot \Pr\{u_k\} \\ &= \underset{u_k=1,\ldots,\hat{U}}{\mathrm{argmax}} \prod_{i=1}^{\mu} \prod_{d\in\mathcal{I}_{\mathrm{si},i}} f_{\tilde{N}}(\tilde{c}_{k,d}|b_{i,k}^{(u_k)}) \cdot \Pr\{u_k\}. \end{aligned} \tag{12}$$

An evaluation of this estimate is not easily possible, as we have no analytical expression of $f_{\tilde{N}}(x)$. Moreover, the effective noise $\tilde{N}_{k,d}$ is not Gaussian distributed, whereby it is not possible to formulate an additive decision metric according to squared Euclidean distance as common. In order to overcome these issues, we use the approximation that all $\tilde{N}_{k,d}$ are drawn from a real Gaussian random process with zero mean and variance $\sigma_{\tilde{N}_{k,d}}^2$ according to (11). Evidently, this approximation does not lead to an optimum estimator of the side information bits, but it turns out to be an improvement compared to hard decision. Hence, the estimator further reads

$$\begin{aligned}\hat{u}_k &\overset{\text{Gauss}}{=} \underset{u_k=1,\ldots,\hat{U}}{\mathrm{argmin}} \sum_{i=1}^{\mu} \sum_{d\in\mathcal{I}_{\mathrm{si},i}} \frac{(\tilde{c}_{k,d} - b_{i,k}^{(u_k)})^2}{\sigma_{\tilde{N}_{k,d}}^2} - \log(\Pr\{u_k\}) \\ &= \underset{u_k=1,\ldots,\hat{U}}{\mathrm{argmin}} \sum_{i=1}^{\mu} \sum_{d\in\mathcal{I}_{\mathrm{si},i}} \frac{\tilde{c}_{k,d}^2 - 2\tilde{c}_{k,d}b_{i,k}^{(u_k)} + 1}{\sigma_{\tilde{N}_{k,d}}^2} \\ &\qquad - \log(\Pr\{u_k\}) \end{aligned}$$

$$= \underset{u_k=1,\dots,\hat{U}}{\mathrm{argmax}} \sum_{i=1}^{\mu} b_{i,k}^{(u_k)} \sum_{d\in\mathcal{I}_{\mathrm{si},i}} \frac{\tilde{c}_{k,d}}{\sigma^2_{\tilde{N}_{k,d}}} + \frac{\log(\Pr\{u_k\})}{2} \, . \tag{13}$$

According to this result, we define the metric $\Theta_{u,k}$, which reflects the probability that the u^{th} signal candidate has been transmitted with the signal at the k^{th} antenna and reads for MAP estimation

$$\Theta_{u,k}^{(\mathrm{MAP})} = \sum_{i=1}^{\mu} b_{i,k} \sum_{\forall d\in\mathcal{I}_{\mathrm{si},i}} \frac{\tilde{c}_d}{\sigma^2_{\tilde{N}_{k,d}}} + \frac{\log(\Pr\{u\})}{2} \tag{14}$$

for $u = 1,\dots,\hat{U}$ and $k = 1,\dots,K$.

With ordinary SLM the probabilities that one signal candidate at a certain antenna is chosen for transmission are all equally distributed. Hence, the last term in (14) may be dropped, which results to the ML metric

$$\Theta_{u,k}^{(\mathrm{ML})} = \sum_{i=1}^{\mu} b_{i,k} \sum_{\forall d\in\mathcal{I}_{\mathrm{si},i}} \frac{\tilde{c}_d}{\sigma^2_{\tilde{N}_{k,d}}} \, . \tag{15}$$

Using sSLM for PAR reduction, all K transmit signals are generated via the same mapping, whereas the same index is embedded into each candidate. Therefore, the side information is protected against transmission errors with a $(Kr\mu, \mu)$ repetition code. Hence, in this case it is reasonable to decode the side information index jointly over all K transmit antennas, which is discussed in the next subsection.

With both dSLM variants, it is possible to decode the side information individually at each transmit antenna using either the MAP metric (14) or the ML metric (15).

C. Joint Decoding of the Side Information

For all SLM variants, except of ordinary SLM, a joint decoding over the transmit antennas is possible and will be analyzed subsequently.

1) Simplified SLM: With simplified SLM only one side information index has to be estimated over all K transmit antennas. Therefore, we combine the respective metrics $\Theta_{u,k}^{(\mathrm{ML})}$ (only ML is reasonable) over all transmit antennas. Now, the resulting estimate reads

$$\hat{u} = \underset{u=1,\dots,U}{\mathrm{argmax}} \sum_{k=1}^{K} \Theta_{u,k}^{(\mathrm{ML})} \, . \tag{16}$$

2) Directed SLM (original): With the dSLM algorithm from [7], [8] a joint decoding of the side information is possible. Fig. 1 shows a pseudocode description in MATLAB notation for such an estimator, which works as follows.

Given the metrics $\Theta_{u,k}$ (ML and MAP are possible here), the number k_{b} of the received signal exhibiting the most reliable side information index is determined (line 03) by looking for the largest metric. Next, the respective side information index is obtained (line 04). All metrics belonging to the $k_{\mathrm{b}}^{\mathrm{th}}$ signal are masked out by setting them to $-\infty$ (line 05).

With this variant of dSLM up to $\hat{U} = K(U-1)+1$ candidates may have been assessed at one transmit antenna. However, the total number of assessed signal candidates over all antennas is limited by KU. Now, having decided the first, i.e., most reliable index $\hat{u}_{k_{\mathrm{b}}}$, the maximum feasible number of indices within all

function $[\hat{u}_k] = \texttt{Decode\_dSLM\_orig}\,([\Theta_{u,k}])$
01 $\quad \hat{u}_k = 0,\ k = 1,\dots,K$
02 $\quad$ **for** $\tilde{k} = 1,\dots,K$
03 $\qquad k_{\mathrm{b}} = \mathrm{argmax}_{k=1,\dots,K} \max_{\forall u} \Theta_{u,k}$
04 $\qquad \hat{u}_{k_{\mathrm{b}}} = \mathrm{argmax}_{\forall u} \Theta_{u,k_{\mathrm{b}}}$
05 $\qquad \Theta_{:,k_{\mathrm{b}}} = -\infty$
06 $\qquad \Theta_{\hat{U}+\tilde{k}+1-\sum_{\kappa=1}^{K}\hat{u}_\kappa:\hat{U},:} = -\infty$
07 $\quad$ **end**

Fig. 1. Pseudocode using MATLAB notation for joint decoding of the side information with dSLM (original candidate generation).

$K-1$ remaining signals is $\hat{U} - \hat{u}_{k_{\mathrm{b}}} + 1$, whereas the rest can be masked out (line 06). This procedure is repeated until K estimates are found.

3) Soft Decoding of directed SLM (RS): Using the dSLM algorithm from [9], [11] a joint estimation of the side information similar to Fig. 1 is also possible. The pseudocode of this estimation is given in Fig. 2.

function $[\hat{u}_k] = \texttt{Decode\_dSLM\_RS}\,([\Theta_{u,k}])$
01 $\quad \hat{u}_k = 0,\ k = 1,\dots,K$
02 $\quad \Theta_{1:k-1,k} = -\infty,\ k = 1,\dots,K$
03 $\quad \Theta_{\hat{U}-K+k+1:\hat{U},k} = -\infty,\ k = 1,\dots,K$
04 $\quad$ **for** $\tilde{k} = 1,\dots,K$
05 $\qquad k_{\mathrm{b}} = \mathrm{argmax}_{k=1,\dots,K} \max_{\forall u} \Theta_{u,k}$
06 $\qquad \hat{u}_{k_{\mathrm{b}}} = \mathrm{argmax}_{\forall u} \Theta_{u,k_{\mathrm{b}}}$
07 $\qquad \Theta_{:,k_{\mathrm{b}}} = -\infty$
08 $\qquad \Theta_{k+\hat{u}_{k_{\mathrm{b}}}-k_{\mathrm{b}}+1:\hat{U},k} = -\infty,\ k = 1,\dots,k_{\mathrm{b}}-1$
09 $\qquad \Theta_{1:k+\hat{u}_{k_{\mathrm{b}}}-k_{\mathrm{b}}-1,k} = -\infty,\ k = k_{\mathrm{b}}+1,\dots,K$
10 $\quad$ **end**

Fig. 2. Pseudocode using MATLAB notation for joint decoding of the side information with dSLM (candidate generation via Reed-Solomon coding).

As already mentioned above, the indices of selected OFDM frames are sorted over the antennas. Hence, at the first ($k = 1$) antenna possible candidate frames are $u = 1,\dots,K(U-1)+1$, at the second ($k = 2$) antenna $u = 2,\dots,K(U-1)+2$, and so on. Unfeasible combinations of u and k are masked out in line 02 and 03 of the algorithm in Fig. 2. Noteworthy, this masking is inherently done when using the MAP metric (14). Now, the most reliable side information index is decoded (line 05 and 06) and the unfeasible metrics are masked out (line 07 to 09) in order that the sorting constraint of the indices holds.

D. Influence on the Bit Error Ratio

As performance measure for transmitting the side information indices we consider the *side information error ratio (SIER)*. For ordinary and original directed SLM, an error of the k^{th} side information index within the MIMO OFDM frame (i.e., $\hat{u}_k \neq \check{u}_k$), causes solely the loss of the k^{th} OFDM frame within the MIMO OFDM frame, which results to a *bit error ratio (BER)* of 0.5. With dSLM using RS coding for candidate generation, the loss of one index leads to the loss of the whole MIMO OFDM frame, because the erasure decoding of the RS code fails.

As performance measure for the overall digital transmission we consider the (uncoded) *bit error ratio (BER)*. For the original signal, i.e., where no PAR reduction algorithm is applied, and using 4-QAM this BER can be calculated to [17]

$$\mathrm{BER}_{\mathrm{orig}} = \frac{1}{2}\left(1 - \sqrt{\frac{E_{\mathrm{b}}/N_0}{1+E_{\mathrm{b}}/N_0}}\right), \qquad (17)$$

with E_{b} energy per bit and N_0 (one sided) noise power spectral density.

Having determined the SIER of one certain transmission scheme and given $\mathrm{BER}_{\mathrm{orig}}$ of the original signal, the resulting BER reads

$$\begin{aligned}\mathrm{BER} &= (1-\mathrm{SIER})\cdot \mathrm{BER}_{\mathrm{orig}} + \mathrm{SIER}\cdot\frac{1}{2} \\ &= \frac{1}{2}\left(1-(1-\mathrm{SIER})\cdot\sqrt{\frac{E_{\mathrm{b}}/N_0}{1+E_{\mathrm{b}}/N_0}}\right). \qquad (18)\end{aligned}$$

VI. Numerical Results

The subsequent numerical simulations consider a $l_h = 5$ tap (equal gain) MIMO channel with $K = 4$ transmit and receive antennas. The number of subcarriers in the OFDM system is chosen to $D = 128$ and as modulation scheme $M = 4$-QAM is considered.

Fig. 3 shows the ccdf of PAR for ordinary, simplified, and directed (both variants discussed in this paper perform equal) SLM. Directed SLM offers always the best performance in terms of PAR reduction followed by ordinary and simplified SLM. Noteworthy, the performance of dSLM increases with an increasing number K of transmit antennas, whereas the performance of oSLM or sSLM decreases. This effect is extensively derived and discussed in [10].

Fig. 4 shows the performance of transmitting the side information index in terms of the SIER if the indices are decoded individually according to Sec. V-B. All schemes have in common that soft decision of the side information offers a better decoding performance than hard decision. Furthermore, increasing the number U of (on average) assessed signal candidates per transmit antenna. The SIER increases as the number r of possible repetitions decreases.

Contrary to the PAR reduction performance, the dSLM variants are slightly outperformed by oSLM. The reason is that the number of required bits of side information per transmit antenna is smaller with oSLM than with dSLM (see Tab. I), and hence more repetitions r of the indices within the OFDM frame are possible.

With the RS coding variant of dSLM the K original, information carrying OFDM frame are obtained out of the K received ones via erasure decoding. The decoding will only be successful, if all K side information indices are estimated correctly. Otherwise the whole MIMO OFDM frame (all K individual OFDM frames) will be lost. Hence, a side information error is counted if at least one of the K transmitted indices is estimated incorrectly. With the original variant of dSLM an erroneous estimation of one side information index will only cause the loss of the respective individual OFDM frame within the MIMO OFDM frame. This is the reason why the SIER of the original approach of dSLM is slightly better.

Using MAP estimation with both variants of dSLM offers only neglectable gains (the respective curves are almost congruent), whereas it is not necessary to incorporate the a-priori probabilities into the decoding process.

Fig. 5 shows the side information error rate when using a joint decoding according to Sec. V-C. Here, simplified SLM shows the best performance, which increases significantly with an increasing number K of transmit antennas. The reason for this effect is that the applied $(Kr\mu, \mu)$ repetition code will get stronger if more transmit antennas are available. However, cf. Fig. 3 performance decreases significantly.

With dSLM, the MAP estimation of the side information offers again only neglectable gains. However, comparing these results with the ones of Fig. 4 shows that the algorithms for joint decoding of the side information according to Fig. 1 and 2 help to further improve the SIER, especially at low signal-to-noise ratios.

Noteworthy, comparing all results of Fig. 3, 4, and 5 shows, that the best SLM variant in terms of PAR reduction (dSLM) exhibits the worst performance in terms of SIER and vice versa, the worst PAR reduction variant of SLM (sSLM) offers the least SIER.

VII. Conclusions

In this paper, the SLM extensions for MIMO systems, namely ordinary, simplified, and directed SLM are studied and the approach of transmitting the side information from [18] is extended for these schemes.

Based on the analysis of the transmission channel of the side information indices, it is possible to derive a decoding scheme of the side information based on soft values. Numerical results show, that a soft decision of the side information is very beneficial in terms of the SIER. Moreover, for directed SLM a joint estimation of the side information over the whole MIMO OFDM frame is derived, which offers better performance in terms of SIER.

Moreover, using the variant directed SLM, it is also possible to extend the ML estimation to a MAP estimation. Unfortunately, this approach offers only neglectable gains, hence it is not necessary to consider the a-priori probabilities into the estimation process.

Basically, it can be stated that the PAR reduction performance behaves contrarily to the SIER. For instance dSLM offers significant performance gains in terms of PAR reduction compared to oSLM or sSLM. However, the SIER is slightly higher than with these schemes. Applying the detection schemes, derived in this paper, help to improve the situation for dSLM.

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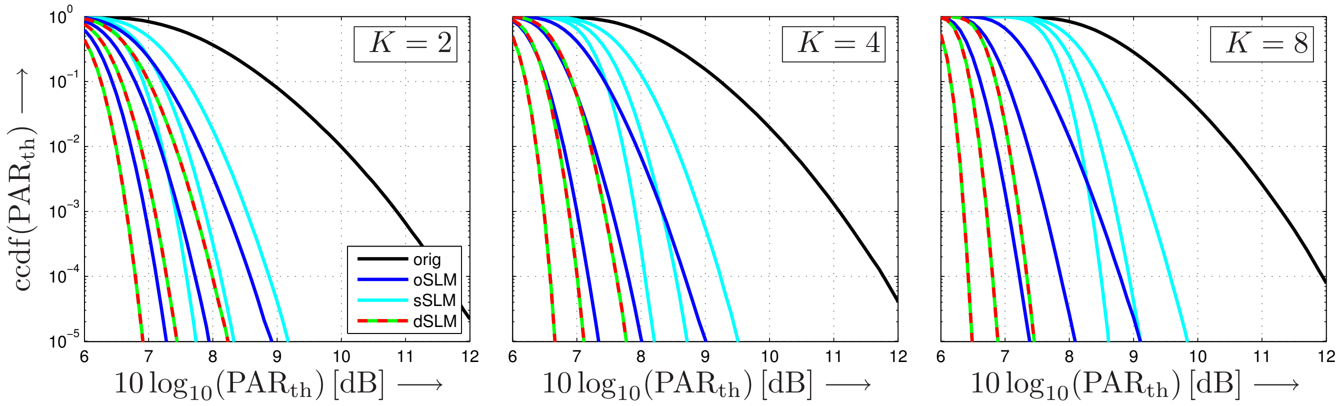

Fig. 3. Ccdf of PAR of the original signal (black) and of ordinary (blue), simplified (light blue), and directed (red green dashed) SLM for various numbers K of transmit antennas; the different curves within each plot represent a different (average) number U of assessed signal candidates per transmit antennas (from left to right: $U = 16$, 8, and 4); $M = 4$; $D = 128$.

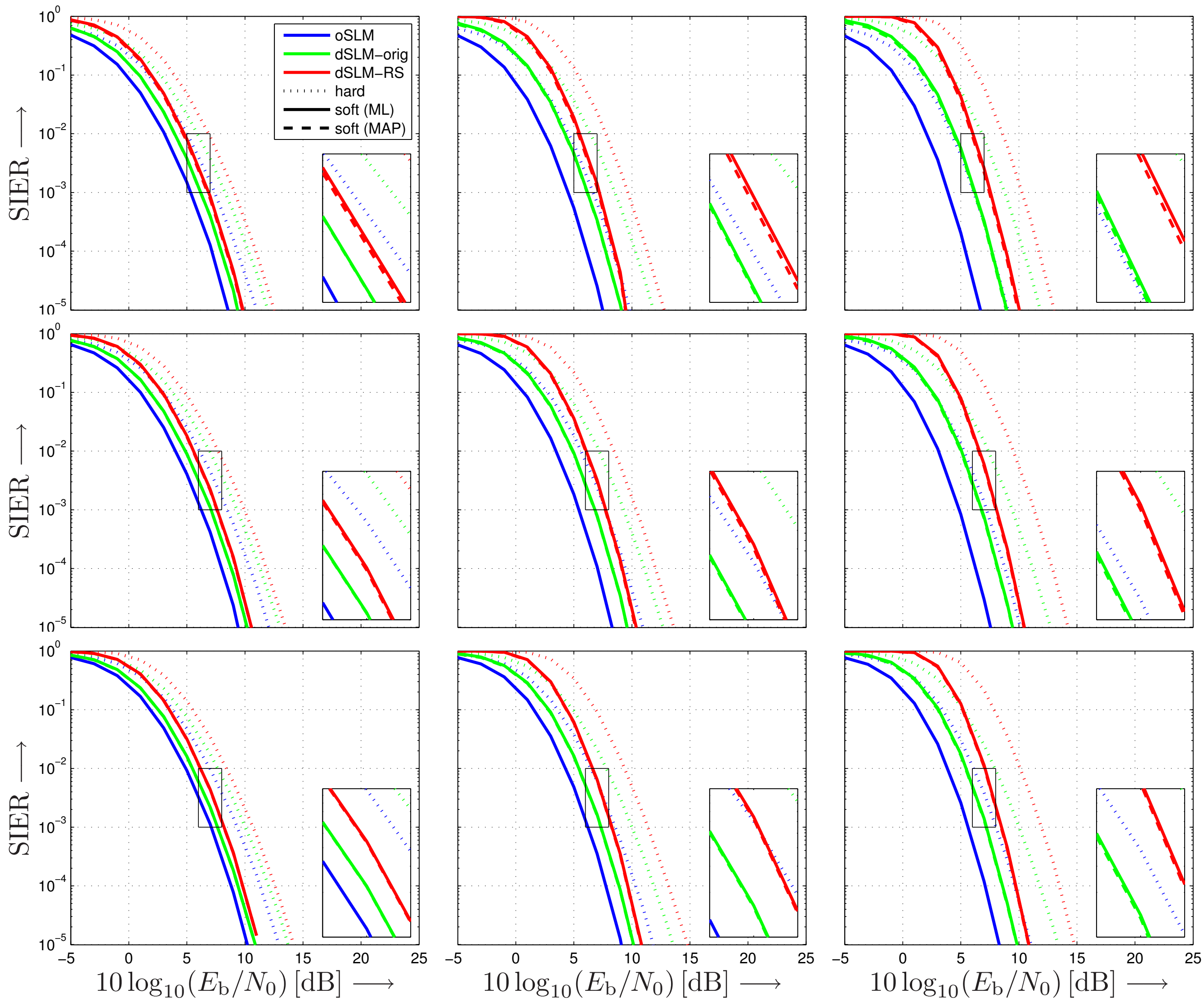

Fig. 4. SIER over signal-to-noise ratio for individual decoding of the side information indices with ordinary (blue) and directed (green: original candidate generation; red: candidate generation via RS coding); The decoding of the repetition code is done based on hard decision (dotted), ML soft decision (solid), or MAP soft decision (dashed, only applicable with directed SLM); columns left to right: $K = 2$, 4, 8; rows top to bottom: $U = 4$, 8, 16; $M = 4$, $D = 128$.

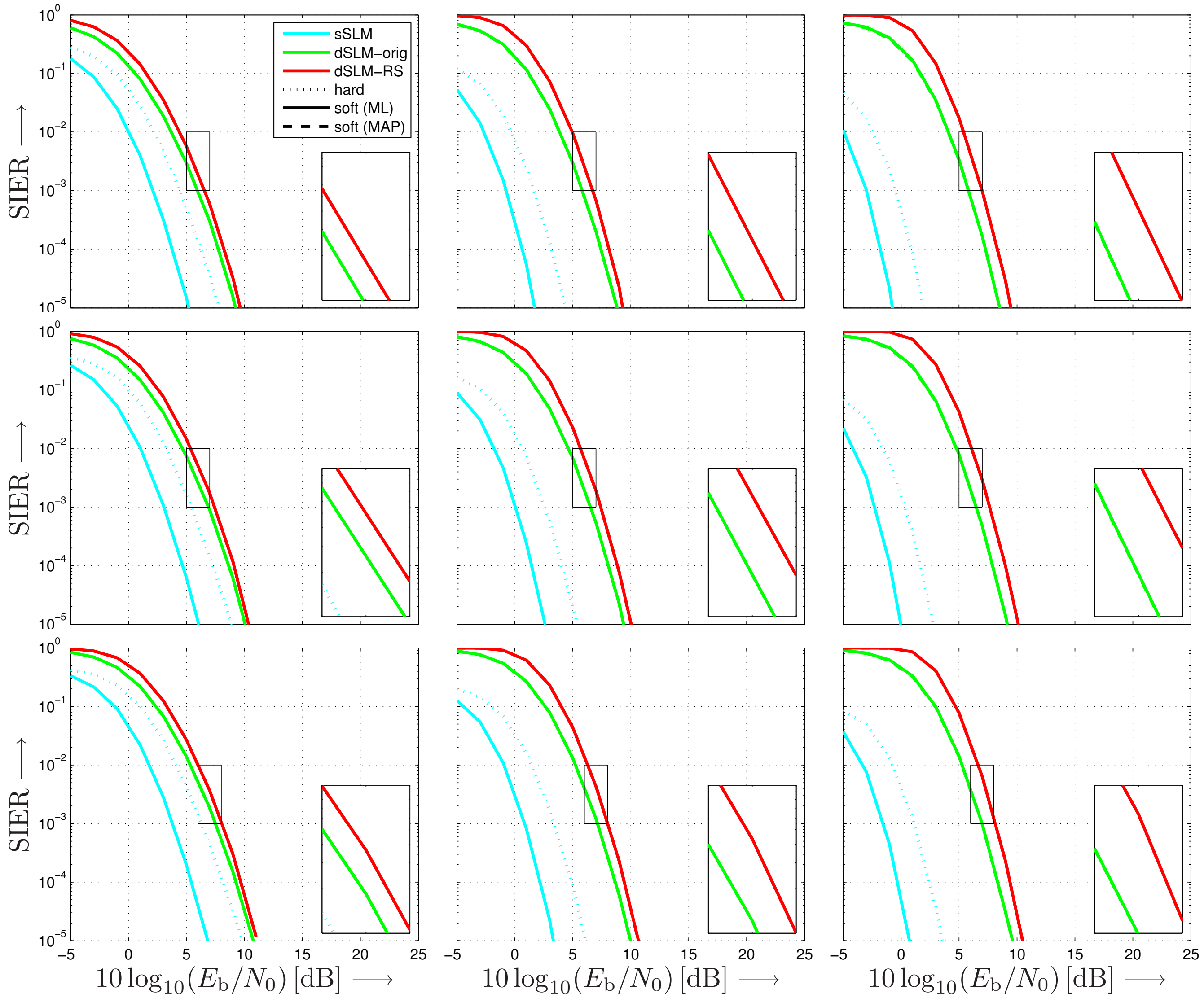

Fig. 5. SIER over signal-to-noise ratio for joint decoding of the side information indices with simplified (light blue) and directed (green: original candidate generation; red: candidate generation via RS coding); The decoding of the repetition code is done based on hard decision (dotted, only applicable with simplified SLM), ML soft decision (solid), or MAP soft decision (dashed, only applicable with directed SLM); columns left to right: $K = 2, 4, 8$; rows top to bottom: $U = 4, 8, 16$; $M = 4$, $D = 128$.

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