SELECTED SORTING FOR PAR REDUCTION IN OFDM MULTI-USER BROADCAST SCENARIOS

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ABSTRACT

OFDM suffers from a high peak-to-average power ratio (PAR) of the transmit signal. This issue becomes even more important when considering multi-antenna systems. In this paper PAR reduction schemes for the multi-antenna broadcast scenarios are assessed. Hereby, the scheme Selected Sorting (SLS) is introduced and analyzed in terms of PAR reduction performance, error performance, and computational complexity. The huge benefit of this scheme is that no side information needs to be signaled to the receiver. Numerical results shown in this paper, demonstrate that SLS offers significant gains in PAR reduction. Moreover, Selected Sorting is compared with simplified Selected Mapping (sSLM), whereby SLS outperforms sSLM with respect to all three parameters, PAR reduction, error performance, and computational complexity.

1. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is a popular scheme for equalizing the temporal interferences caused by frequency-selective channels. One essential drawback of OFDM systems is the high dynamic of the transmit signal. The occurrence of large signal peaks leads to signal clipping at the non-linear power amplifier, which in turn leads to very undesirable out-of-band radiation. In order to avoid violating spectral masks a transmitter sided algorithmic control of the peak power is essential. Moreover, the transmitters of modern communication systems will be equipped with multiple antennas (multiple-input/multiple-output (MIMO) systems). In this case the issue of out-of-band radiation gets even more serious and the reduction of the signal's peak power is more relevant. Recently, peak power reduction schemes, developed for single antenna systems, are extended to the MIMO case. For instance, this has been done for the popular scheme *Selected Mapping (SLM)* [1, 2, 3, 4]. However, in most cases these extensions have only been discussed for multi-antenna point-to-point scenarios where the equalization of the spatial interferences can be accomplished at the receiver side.

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This paper deals with multi-antenna point-to-multipoint transmission (broadcast scenario). Hereby, it is essential to apply a transmitter sided precoding [5, 6] of the channel's spatial (or multi-user) interferences. The combination of transmitter sided precoding with peak-power reduction algorithms is not always possible and may lead to degradation of the error performance or to a significant increase of computational complexity.

In this paper a peak-power reduction technique named *Selected Sorting (SLS)*, originally introduced in [7], is examined in details and further improved. Moreover, it is compared to *simplified Selected Mapping (sSLM)* [2], the extension of SLM to broadcast scenarios, in terms of peak-power reduction, bit error performance, and computational complexity.

This paper is organized as follows: in Section 2 the system model is defined; Section 3 gives a short definition of *sorted Tomlinson-Harashima precoding (sTHP)* [5, 6], the precoding technique which is considered in this paper. Moreover, the impact of different sorting orders is assessed. In Section 4 peak-power reduction for broadcast scenarios are assessed analytically and by numerical simulations. Section 5 draws some conclusions.

2. SYSTEM MODEL

In this paper, we consider transmission over a frequency-selective multi-user broadcast channel. The transmitter (central unit) is equipped with N_C antennas where joint signal processing is possible; the receivers are K distributed users each equipped with a single antenna. For convenience we restrict to the case $N_\mathrm{C} \overset{!}{=} K$. The impulse response of the respective channel in the z-domain is given (in the equivalent complex baseband) by a matrix polynomial $\boldsymbol{H}(z) = \sum_{k=0}^{l_\mathrm{H}-1} \boldsymbol{h}_k \cdot z^{-k}$. The fading coefficient at delay step k is given by the complex matrix $\boldsymbol{h}_k$ which describes the multi-user interferences; l_{H} is the length of the channel impulse response. In order to equalize the temporal interferences OFDM is applied, whereby D subcarriers are assumed. The remaining multi-user interferences at each subcarrier are described by the flat fading channel matrix $\boldsymbol{H}_d = \boldsymbol{H}(\mathrm{e}^{\mathrm{j}2\pi d/D})$, $d = 0, \ldots, D-1$ and have to be equalized by transmitter-sided precoding. In this paper we consider *sorted Tomlinson-Harashima Precoding (sTHP)* [6].

The complex-valued modulation symbols (drawn from an M-ary QAM constellation) are collected in the $K \times D$ matrix $\boldsymbol{A} = [A_{k,d}]$, the frequency-domain MIMO OFDM frame. In order to equalize the multi-user interferences sTHP has to be applied on each column (vector $\boldsymbol{A}_d = [A_{(k=1,\ldots,K),d}]$, $d = 1, \ldots, D$) of $\boldsymbol{A}$ (see Section 3).

The resulting precoded frequency-domain MIMO OFDM frame is denoted by the matrix $\boldsymbol{X}$. The time-domain MIMO OFDM frame (matrix $\boldsymbol{x}$) is obtained via an *inverse discrete Fourier transform (IDFT)* [8] along each row of the matrix $\boldsymbol{X}$.

Assuming that the frequency-domain modulation symbols $A_{k,d}$ and hence $X_{k,d}$ are statistically independent, the time-domain symbols $x_{k,d}$ are (approximately) Gaussian distributed due to the central limit theorem. Hence, the transmit signal at the k^{th} antenna exhibits a large *peak-to-average power ratio (PAR)*. As usual in literature we consider the worst-case PAR[1], i.e., the maximum PAR over all antennas which is defined as

$$\mathrm{PAR} \stackrel{\mathrm{def}}{=} \frac{\max_{\forall d, \forall k} |x_{k,d}|^2}{\mathrm{E}\{|x_{k,d}|^2\}} . \tag{1}$$

As performance measure of the PAR reduction schemes discussed in this paper, we consider the *complementary cumulative distribution function (ccdf)* of the PAR, i.e., the probability that the PAR of a given OFDM frame exceeds a certain threshold $\mathrm{PAR}_{\mathrm{th}}$:

$$\mathrm{ccdf}(\mathrm{PAR}_{\mathrm{th}}) \stackrel{\mathrm{def}}{=} \Pr\{\mathrm{PAR} > \mathrm{PAR}_{\mathrm{th}}\}. \tag{2}$$

Assuming all samples of the time-domain signal $x_{k,d}$ to be i.i.d. Gaussian distributed the ccdf of the original signal is given by [3]

$$\mathrm{ccdf}_{\mathrm{MIMO}}(\mathrm{PAR}_{\mathrm{th}}) \stackrel{\mathrm{Gauss}}{=} 1 - (1 - \mathrm{e}^{-\mathrm{PAR}_{\mathrm{th}}})^{DK}. \tag{3}$$

[1] In this paper we do not consider pulse shaping, modulation to radio frequency and the influence of the cyclic prefix. Moreover, we restrict ourselves to the PAR of the non-oversampled transmit signal. Considering oversampling would have no impact on the relation between the results.

3. SORTED TOMLINSON HARASHIMA PRECODING

3.1. Sorted THP for MIMO Flat Fading Channels

Subsequently, we restrict our considerations to MIMO flat fading channels, i.e., we only regard a certain subcarrier of the OFDM system. To be consistent with the definition of the OFDM system all symbols exhibit the index d to represent the d^{th} subcarrier.

As precoding strategy, we consider sorted *Tomlinson-Harashima Precoding (THP)* in each subcarrier. A block diagram of this scheme is given in Fig. 1. First the precoding order of the K users is affected by the permutation matrix $\boldsymbol{P}_d$. This precoding order can be optimized according to some optimization criterion, which will be further specified in Section 3.2. Then the signals of the users are successively precoded in the feedback-loop with the feedback matrix $\boldsymbol{B}_d$, a lower triangular matrix with unit main diagonal, and modulo reduced into the support of the signal constellation. Finally, the signals are processed via the feedforward matrix $\boldsymbol{F}_d$, a unitary matrix which ensures that the average power is equal at each transmit antenna. The modulo conversion in the feedback-loop leads to a slight increase of the transmit power. This effect is known as the precoding loss [9]. The remaining individual scaling factors, given by the diagonal matrix $\boldsymbol{\Gamma}_d = \mathrm{diag}(g_{k,d})$, $k = 1, \ldots, K$, can be equalized within the receiver's *automatic gain control (agc)*.

Given a suited permutation matrix $\boldsymbol{P}_d$ the feedforward matrix $\boldsymbol{F}_d$ and the feedback matrix $\boldsymbol{B}_d$ can be calculated via QR decomposition [5, 10]

$$\boldsymbol{H}_d^{\mathsf{H}} \boldsymbol{P}_d^{\mathsf{T}} = \boldsymbol{Q}_d \cdot \boldsymbol{R}_d = \boldsymbol{F}_d \cdot \boldsymbol{B}_d^{\mathsf{H}} \boldsymbol{\Sigma}_d. \tag{4}$$

Hereby, $\boldsymbol{Q}_d$ describes a unitary and $\boldsymbol{R}_d$ an upper triangular matrix. The diagonal scaling matrix $\boldsymbol{\Sigma}_d$ ensures that the lower triangular feedback matrix $\boldsymbol{B}_d$ has unit main diagonal. The individual scaling factors at the receiver read

$$\boldsymbol{\Gamma}_d = \boldsymbol{P}_d^{\mathsf{T}} \boldsymbol{\Sigma}_d^{-1} \boldsymbol{P}_d. \tag{5}$$

3.2. Precoding Order

So far, the precoding order given by the permutation matrix $\boldsymbol{P}_d$ has not been specified. According to [6] a reasonable criterion is to maximize the performance of the worst user, i.e., the one exhibiting the minimum signal-to-noise ratio (SNR) as this one dominates the mean bit error ratio (BER) over all users. An almost optimum solution to this criterion can be found using the V-BLAST algorithm [11, 6], which finds the optimum detection order for decision-feedback equalization.

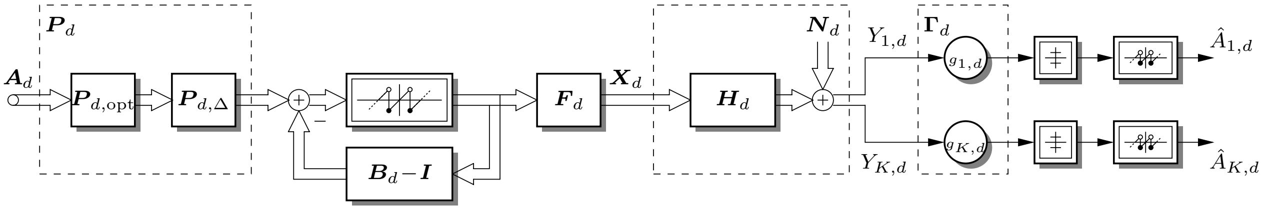

Fig. 1. Block diagram of sorted Tomlinson-Harashima precoding.

Considering the uplink-downlink duality [12], the reverse permutation order found by the V-BLAST algorithm can be applied for sorted THP [6, 13]. The resulting optimum permutation matrix is denoted as $\boldsymbol{P}_{d,\mathrm{opt}}$.

Subsequently, we always consider the application of the optimum encoding order given by $\boldsymbol{P}_{d,\mathrm{opt}}$. Starting from this solution it is possible to change the sorting order by an additional sorting through the matrix $\boldsymbol{P}_{d,\Delta}$, which can be regarded as an offset on the optimum one. The total permutation matrix $\boldsymbol{P}_d$ is then given by (see also Fig. 1)

$$\boldsymbol{P}_d = \boldsymbol{P}_{d,\Delta} \boldsymbol{P}_{d,\mathrm{opt}}. \quad (6)$$

Choosing an additional permutation matrix $\boldsymbol{P}_{d,\Delta}$ other than the identity matrix $\boldsymbol{I}$ will evidently decrease the performance in terms of the mean bit error ratio of the system. In total there exist $K!$ different additional sorting orders and hence permutation matrices $\boldsymbol{P}_{d,\Delta}$.

According to [6] the overall performance (in terms of the bit error ratio) is governed by the user encoded last. Hence, a rearrangement of the sorting with the matrix $\boldsymbol{P}_{d,\Delta}$ without changing the position of the last encoded user will hardly influence the overall performance. On the contrary, the degradation of the bit error ratio induced by $\boldsymbol{P}_{d,\Delta}$ is mainly determined by the new position of the originally (determined by $\boldsymbol{P}_{d,\mathrm{opt}}$) last encoded user. The sorting of all other users will have almost no impact on the overall performance.

Hence, the $K!$ different additional sortings given by the permutation matrices $\boldsymbol{P}_{d,\Delta}$ can be classified into K different classes of additional sortings which lead to different performance results in terms of the bit error ratio. These K classes are defined by the position within the precoding order of the originally (determined by $\boldsymbol{P}_{d,\mathrm{opt}}$) last encoded user. Within each class there exist $(K-1)!$ different sorting orders, which exhibit almost the same performance.

Fig. 2 shows all resulting mean bit error ratios obtained by all $K!$ different permutations for the special case of $K = 4$. The different classes of additional sorting orders are depicted by different colors. The green curve shows the results if the fourth encoded user (after $\boldsymbol{P}_{d,\mathrm{opt}}$) is not influenced by $\boldsymbol{P}_{d,\Delta}$. The blue, cyan, and red curves show the results if the fourth user is rearranged to the third, second, or first position, respectively. As can be seen from this plot, $K = 4$ different classes of bit error ratio results are present, whereby the differences within these classes are neglectable.

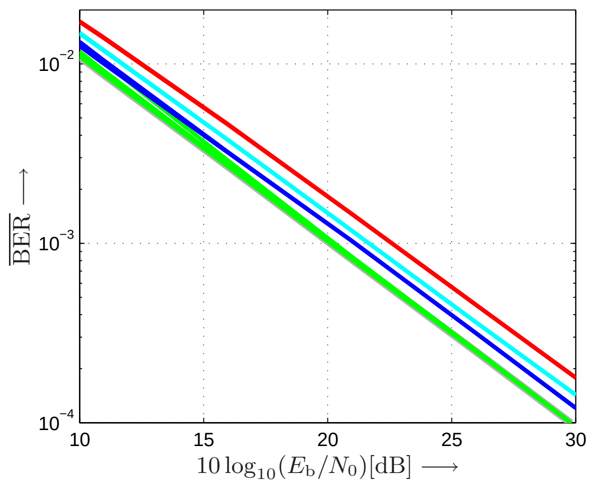

Fig. 2. Mean bit error ratios of sorted THP employing all $K!$ possible additional permutations $\boldsymbol{P}_{d,\Delta}$ considering transmission over a flat (Rayleigh) fading MIMO channel. The resulting curves are classified into four classes whereby the permutation matrix $\boldsymbol{P}_{d,\Delta}$ rearranges the fourth encoded user to the fourth (green), third (dark blue), second (light blue), or first (red) position, respectively. The best result (given with $\boldsymbol{P}_{d,\Delta} = \boldsymbol{I}$) is depicted in gray. $M = 4$-QAM, $K = 4$.

4. SELECTED SORTING FOR PAR REDUCTION

4.1. Review of (Simplified) Selected Mapping

Selected Mapping [1] is one of the most popular techniques for PAR reduction in OFDM systems. The idea behind this scheme is to generate out of the original OFDM frame several, say U_{SLM}, different signal representations via U_{SLM} different bijective mappings $\mathcal{M}^{(u)}$, $u = 1, \ldots, U_{\mathrm{SLM}}$. Out of these signal candidates, the best one, i.e., the one exhibiting the lowest PAR is chosen for transmission. At the receiver after equalization, the original data has to be reconstructed by inverting the applied mapping. Hence, side information, in terms of an index of the applied mapping, has to be transmitted. The required redundancy has to be encoded with at least $\lceil \log_2(U_{\text{SLM}}) \rceil$ bits. However, this index is extraordinary sensitive to transmission errors as the application of the wrong inverse mapping leads to the loss of the whole OFDM frame.

Possible schemes to transmit the side information have been proposed in [14, 15]. For the analysis of the bit error performance of SLM (Section 4.4) we will consider the scheme from [14]. The so-called scrambler variant of SLM distributes the side information inherently over the whole OFDM frame and does not require its explicit transmission.

For multi-antenna scenarios the SLM technique has been extended to the basic schemes *ordinary SLM (oSLM)* [2], *simplified SLM (sSLM)* [2], and *directed SLM (dSLM)* [3, 4]. Following the discussion in [7] it is not feasible to apply ordinary or directed SLM in a broadcast scenario. Due to the transmitter-sided precoding, which influences the data streams of all users, only the simplified approach can be applied here.

Hereby, sSLM is the simplest extension of SLM to MIMO systems. With sSLM the original frequency-domain MIMO OFDM frame $\boldsymbol{A}$ has to be mapped jointly onto U_{sSLM} different signal representations, whereby each row of $\boldsymbol{A}$ has to be mapped the same. Afterwards, each of the resulting signal candidates has to be precoded and transformed into time domain. Out of these the best one, i.e., the one exhibiting the lowest PAR, is then chosen for transmission.

As the individual signal candidates are assumed to be statistically independent, the ccdf of sSLM can be given with respect to the ccdf of the original signal (3) [2, 3, 4]

$$\begin{aligned} \text{ccdf}_{\text{sSLM}}(\text{PAR}_{\text{th}}) &= (\text{ccdf}_{\text{MIMO}}(\text{PAR}_{\text{th}}))^{U_{\text{sSLM}}} \\ &\overset{\text{Gauss}}{=} (1 - (1 - \mathrm{e}^{-\text{PAR}_{\text{th}}})^{DK})^{U_{\text{sSLM}}} . \end{aligned} \quad (7)$$

4.2. Selected Sorting

Another approach to generate different signal representations could be to combine the mapping with the precoding by applying different instances of sTHP in each subcarrier. Hereby, these different instances are generated by considering different permutations $\boldsymbol{P}_d$ of the users. A practical advantage of this approach is that no side information needs to be signaled to the receiver.

Subsequently, we pick a set of V different permutation matrices $\boldsymbol{P}_{d,\Delta}^{(v)}$, $v = 1, \ldots, V$, out of the set of $K!$ possible ones. Starting with the optimum sorting order, the reverse of that obtained according to the V-BLAST criterion [6], we consider the next better (suboptimal, acc. Section 3.2) ones. Now, the information carrying signal $\boldsymbol{A}$ is precoded via all V different precoder instances, the resulting precoded signals are denoted as $\tilde{\boldsymbol{X}}^{(v)}$, $v = 1, \ldots, V$. In oder to generate U_{SLS} different signal candidates $\boldsymbol{X}^{(u)}$, $u = 1, \ldots, U_{\text{SLS}}$, the respective columns (corresponding to the carriers) of $\tilde{\boldsymbol{X}}^{(v)}$ are combined in U_{SLS} different ways. Hence, every column of each of the U_{SLS} signal candidates $\boldsymbol{X}^{(u)}$ is drawn as the column from one of the V possible precoded signals. This is possible as the actual choice of the sorting order of THP at the d^{th} subcarrier influences the precoded signal only at this position.

Noteworthy, with this approach we are able to generate (much) more signal candidates than precoded candidates are present ($U_{\text{SLS}} \geq V$). A principal example how the U_{SLS} signal candidates are generated is depicted in Fig. 3.

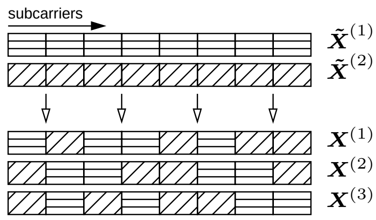

Fig. 3. Generation of $U_{\text{SLS}} = 3$ candidates out of a set of $V = 2$ alternative precoded sequences.

Compared to sSLM, assuming perfect transmission of the side information, this scheme will lead to a loss in bit error performance as suboptimal sorting orders will be used within the generation the signal candidates. However, with SLS much less computational complexity is needed as the precoding has to be performed only V times to generate the U_{SLS} signal candidates.

To further reduce the computational complexity the SLS technique could only be applied on a subset of $D \cdot \rho$, with $0 < \rho \leq 1$, (randomly chosen) influenced subcarriers. All other subcarriers remain unaffected and the optimum sorting order is applied.

The PAR reduction performance of SLS will be the same as that of sSLM (7) if all U_{SLS} resulting signal candidates are statistically independent. However, this is only guaranteed if the number V of alternative precoders is sufficiently large and if the number of influenced subcarriers is adequate, i.e., the factor ρ is near one. Hence, the ccdf of SLS reads

$$\text{ccdf}_{\text{SLS}}(\text{PAR}_{\text{th}}) \overset{\text{large } V,\, \rho}{\longrightarrow} (\text{ccdf}_{\text{MIMO}}(\text{PAR}_{\text{th}}))^{U_{\text{SLS}}} . \quad (8)$$

4.3. Analysis of Computational Complexity

For a reasonable comparison of SLS with sSLM the computational complexity of both schemes has to be taken into account. Subsequently, we refer to complex operations as complexity measure and regard multiplications and divisions equally.

The decomposition of the channel matrix in each subcarrier into feedforward, feedback, and the optimum sorting order has to be accomplished with the V-BLAST algorithm [11]. In [16, 17] low complex implementations have been proposed which reduce the complexity from $\mathcal{O}(K^4)$ to $\mathcal{O}(K^3)$. However, as this decomposition appears in the overall complexities of SLS and sSLM equally, it has not to be specified in more details for a comparison.

The calculation of the feedforward and feedback matrices for the $V-1$ alternative sorting orders is usually implemented as a QR decomposition [10]. Using the result[2] from [18]

$$c_{\mathrm{QR}} = D \cdot \left(2K^3 - \frac{K^2}{2} - \frac{K}{2}\right) \tag{9}$$

complex operations are required here.

In order to calculate the alternative precoding matrices it is not necessary to accomplish $V-1$ times the QR decomposition per influenced subcarrier. Furthermore, it is possible to perform this calculation within the respective implementation of the V-BLAST algorithm by considering in each iteration not only the user exhibiting the lowest SNR but also the one exhibiting the second lowest SNR. Subsequently, we consider the exhaustive complexity of $V-1$ QR decompositions per influenced subcarrier which leads to an upper-bound on the complexity of SLS.

Each precoding of the transmit signal requires

$$c_{\mathrm{prec}} = D \cdot \left(\frac{3}{2}K^2 + \frac{K}{2}\right) \tag{10}$$

complex operations; the transformation into time domain (implemented as fast Fourier transform [8]) and the calculation of the decision metric (PAR) require

$$c_{\mathrm{FFT}} = K \cdot \frac{D}{2}\log_2(D); \qquad c_{\mathrm{met}} = K \cdot D \tag{11}$$

operations.

In the following, we assume that the channel remains constant for the duration of N_{B} OFDM symbols. Hence, for this block of OFDM symbols the calculation of the precoding matrices has to be performed only once, whereas the computation of the precoded signal, the FFT, and the decision metric have to be accomplished for each of the N_{B} OFDM symbols. Table 1 shows the factors with which these individual complexities appear in the overall complexities c_{sSLM} and c_{SLS} of sSLM and SLS.

[2] The result from [18] only considers the pure QR decomposition. In addition to that we need to accomplish the normalization of the feedback matrix to unit main diagonal.

Table 1. Contribution of the individual complexities to the entire complexities of sSLM and SLS.

complexity	sSLM	SLS
$c_{\text{V-BLAST}}$	1	1
c_{QR}	—	$(V-1)\rho$
c_{prec}	$N_{\mathrm{B}} \cdot U_{\mathrm{sSLM}}$	$N_{\mathrm{B}} \cdot [1+(V-1)\rho]$
c_{FFT}	$N_{\mathrm{B}} \cdot U_{\mathrm{sSLM}}$	$N_{\mathrm{B}} \cdot U_{\mathrm{SLS}}$
c_{met}	$N_{\mathrm{B}} \cdot U_{\mathrm{sSLM}}$	$N_{\mathrm{B}} \cdot U_{\mathrm{SLS}}$

Assuming both schemes evaluate the same number $U = U_{\mathrm{SLS}} = U_{\mathrm{sSLM}}$ of alternative signal representations the difference of both complexities is given by

$$\begin{aligned} \Delta c &= c_{\mathrm{sSLM}} - c_{\mathrm{SLS}} \\ &= (1-V)\rho \cdot c_{\mathrm{QR}} + [U - 1 + (1-V)\rho] N_{\mathrm{B}} \cdot c_{\mathrm{prec}}. \end{aligned} \tag{12}$$

Hence, as $c_{\mathrm{QR}} > c_{\mathrm{prec}}$ SLS gains especially for reasonable large block lengths N_{B}.

For a fair comparison of sSLM with SLS both schemes should have the same complexity. Given the parameters V and U_{SLS} for SLS then sSLM assessing

$$U_{\mathrm{sSLM}} = \left\lceil \frac{(V-1)\rho / N_{\mathrm{B}} \cdot c_{\mathrm{QR}} + V c_{\mathrm{prec}} + U_{\mathrm{SLS}}(c_{\mathrm{FFT}} + c_{\mathrm{met}})}{c_{\mathrm{prec}} + c_{\mathrm{FFT}} + c_{\mathrm{met}}} \right\rceil \tag{13}$$

signal candidates will exhibit approximately the same computational complexity. Hereby, the number U_{sSLM} of assessed candidates for sSLM is rounded to the next greater integer, whereby sSLM will exhibit a slightly larger complexity.

4.4. Numerical Results

The subsequent numerical simulations consider a $l_{\mathrm{H}} = 5$ tap (equal gain) MIMO channel with $N_{\mathrm{C}} = 4$ transmit and $K = 4$ receive antennas. The number of subcarriers in the OFDM system is chosen to $D = 512$ whereby all subcarriers are active. As modulation scheme $M = 4$-QAM is considered.

In the top plot of Fig. 4 numerical results of the ccdf of SLS for a various number of assessed signal candidates ($U_{\mathrm{SLS}} = 4$, 8, 16) and number of available alternative sorting orders ($V = 2$, 4, 8) are shown. Hereby all subcarriers are influenced by SLS ($\rho = 1$). Compared to the PAR distribution of the original signal (red curve) SLS has the ability to reduce the peak-power significantly. Moreover, the numerical results are compared with the analytical ones (8) (gray curves). For $V = 2$ explicit differences are visible, which shows that in this case the

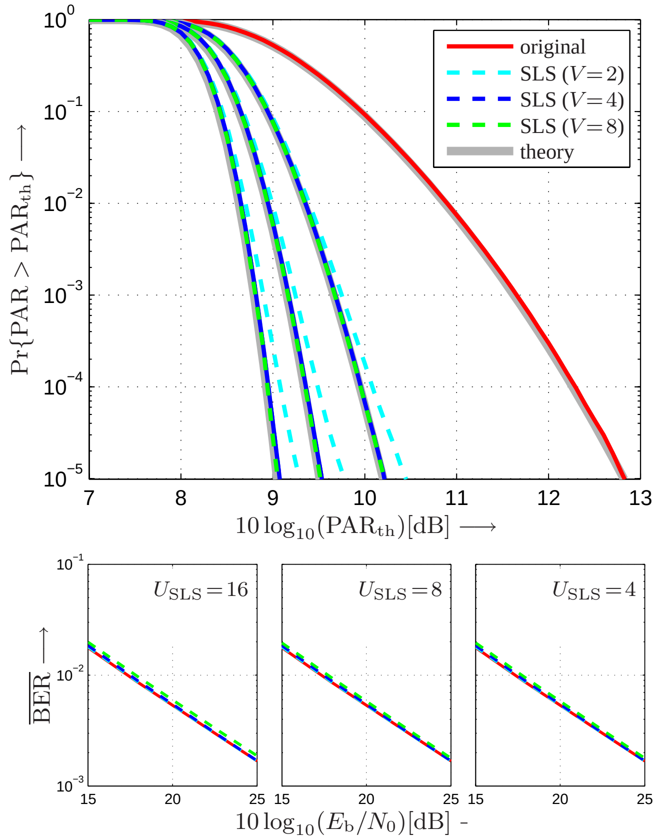

Fig. 4. Top: ccdf of PAR of the original signal (red) and SLS with $V = 2$ (blue), $V = 4$ (green), $V = 8$ (magenta) alternative precoding orders, each assessing (from left to right) $U_{\mathrm{SLS}} = 16, 8, 4$ signal candidates. Bottom: corresponding mean bit error ratios of SLS. As reference serves the result of the original signal (red curve) which represents pure sTHP. M = 4-QAM, $K = 4$, $l_{\mathrm{H}} = 5$, $D = 512$, $\rho = 1$.

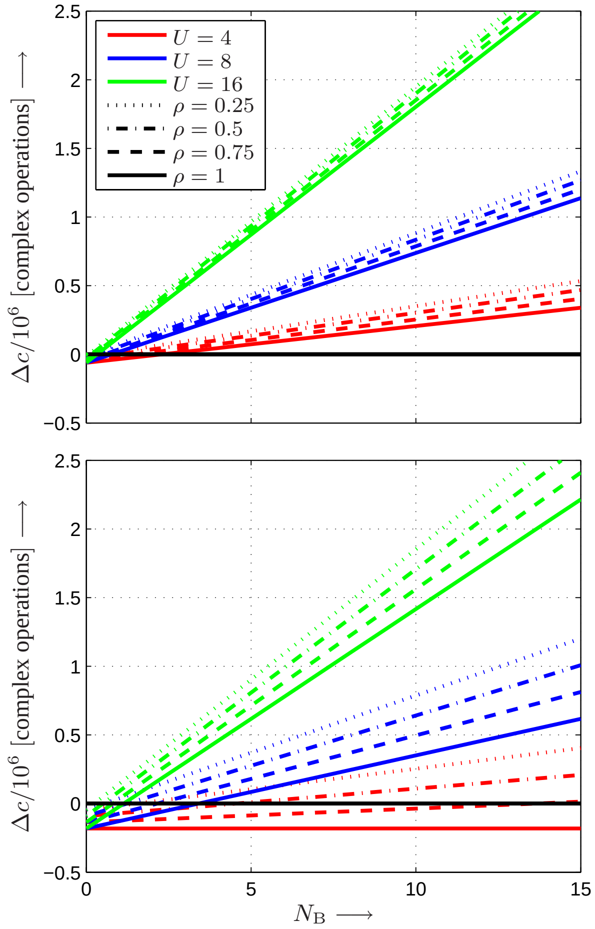

Fig. 6. Difference of complexities of sSLM and SLS (with $V = 2$ (top) and $V = 4$ (bottom)) depending on the block length N_{B}. Both schemes assess an equal number of signal candidates: $U = U_{\mathrm{sSLM}} = U_{\mathrm{SLS}}$. $K = 4$, $D = 512$.

number of degrees of freedom is not large enough to generate statistically independent signal candidates. If V is chosen to $V \geq 4$ the simulation results of SLS fit to the theory very well. The bottom plots of Fig. 4 show the bit error performance for SLS for $U_{\mathrm{SLS}} = 16, 8, 4$ (from left to right). The performance loss in terms of bit error ratios is rather neglectable. Even for $V = 8$ where alternative permutation orders have to be used which lead to significant degradation of the BER (see Section 3.2), the overall loss in BER is very small.

In order to reduce the computational complexity it is possible to apply SLS only on a subset of $D \cdot \rho$ subcarriers. As discussed in Section 4.2, choosing $\rho < 1$ will restrict the possibilities to generate statistical independent signal candidates, whereby the analytical result from (8) is not strict any more. Fig. 5 shows the influence of different values of $\rho = 1, 0.75, 0.5$, and 0.25 on the PAR reduction performance. Hereby it is obvious that only for large values of ρ (here $\rho \geq 0.75$) the performance is comparable with the maximum number of $\rho = 1$.

For the comparison of SLS with sSLM the difference Δc (as given in (12)) of computational complexity over the block length N_{B} is depicted in Fig. 6. Already for small N_{B} (in most scenarios values of $N_{\mathrm{B}} \approx 5$ are sufficient) a gain of SLS compared to sSLM in terms of computational effort can be recognized as the impact of the higher complexity, which occurs through the computation of the precoding matrices with SLS, becomes less important on the overall difference.

In addition to that, Fig. 6 shows the difference Δc for var-

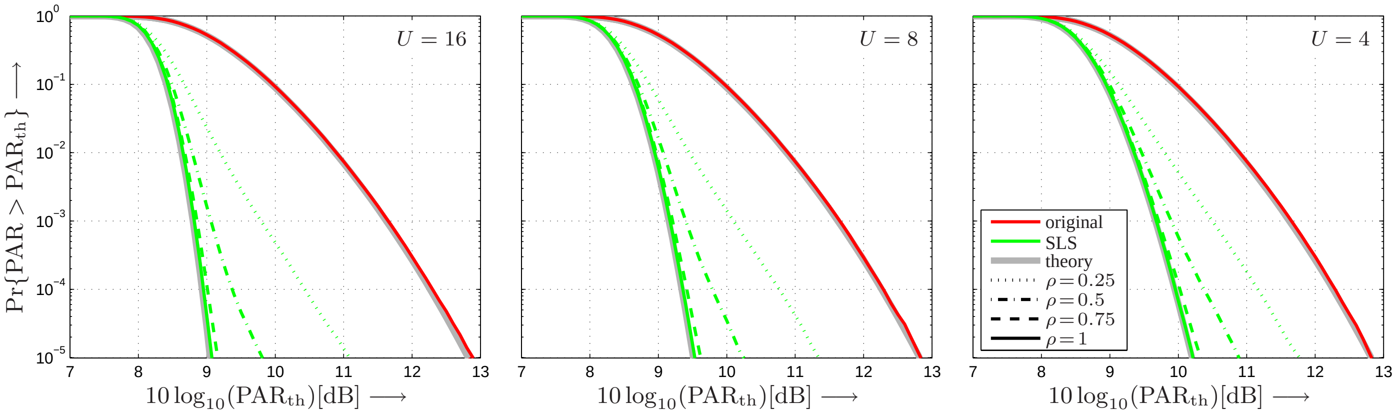

Fig. 5. Ccdf of PAR of the original signal (red) and SLS (green), with $V = 4$ and $U_{\mathrm{SLS}} = 16$ (left), 8 (middle), and 4 (right) for various factors $\rho = 0.25, 0.5, 0.75$, and 1. $M = 4$-QAM, $K = 4$, $D = 512$.

ious ratios ρ of influenced subcarriers. These results show that these gains in computational complexity are rather neglectable. Against the background of the PAR reduction results from Fig. 5 we recommend to choose $\rho = 1$ which is assumed for the analysis of Fig. 4 and 7.

According to (12) the difference $U - V$ determines the slope of Δc. Hence, SLS provides especially gains in terms of complexity if the difference $U - V$ is large. If U and V are chosen equal then SLS will not provide any gains compared to sSLM as in both cases the same number of precoding procedures are processed and SLS suffers from the more complex computation of the precoding matrices. This effect can also be observed form Fig. 6.

The top row of Fig. 7 compares the PAR reduction performance of SLS with sSLM on the bases that both schemes exhibit the same computational complexity. Hereby, the number of assessed candidates for sSLM is chosen according to (13). Providing a good trade-off between loss in bit error performance and computational complexity, the number of alternative precoders is chosen to $V = 2$ (left column) and $V = 4$ (right column). For the most relevant range of values of V and U_{SLS}, SLS offers significant gains in terms of PAR reduction performance.

The bottom row of Fig. 7 shows the corresponding bit error ratios of SLS and sSLM. To achieve a fair comparison, we consider the side information, required with sSLM, to be transmitted through the scrambler variant of SLM [14]. However, the descrambler at the receiver introduces some error propagation in this case. Due to this error propagation SLS outperforms sSLM in terms of error performance as the degradation of the error rate according to the suboptimal precoders is neglectable.

5. CONCLUSIONS

In this paper we have analyzed Selected Sorting for PAR reduction in broadcast scenarios. This scheme combines the channel precoding, which is essential in broadcast scenarios, with peak-power reduction. Hereby, SLS offers significant gains in PAR reduction, whereas the degradation of the error performance is neglectable.

The peak-power reduction performance of SLS is comparable with the one of simplified SLM. Comparing these two schemes shows that SLS exhibits much less complexity than sSLM. On the contrary, choosing the parameters of both PAR reduction schemes such that they exhibit the same complexity SLS will outperform sSLM. These significant gains of SLS are possible as no side information is necessary to be signaled to the receiver.

6. REFERENCES

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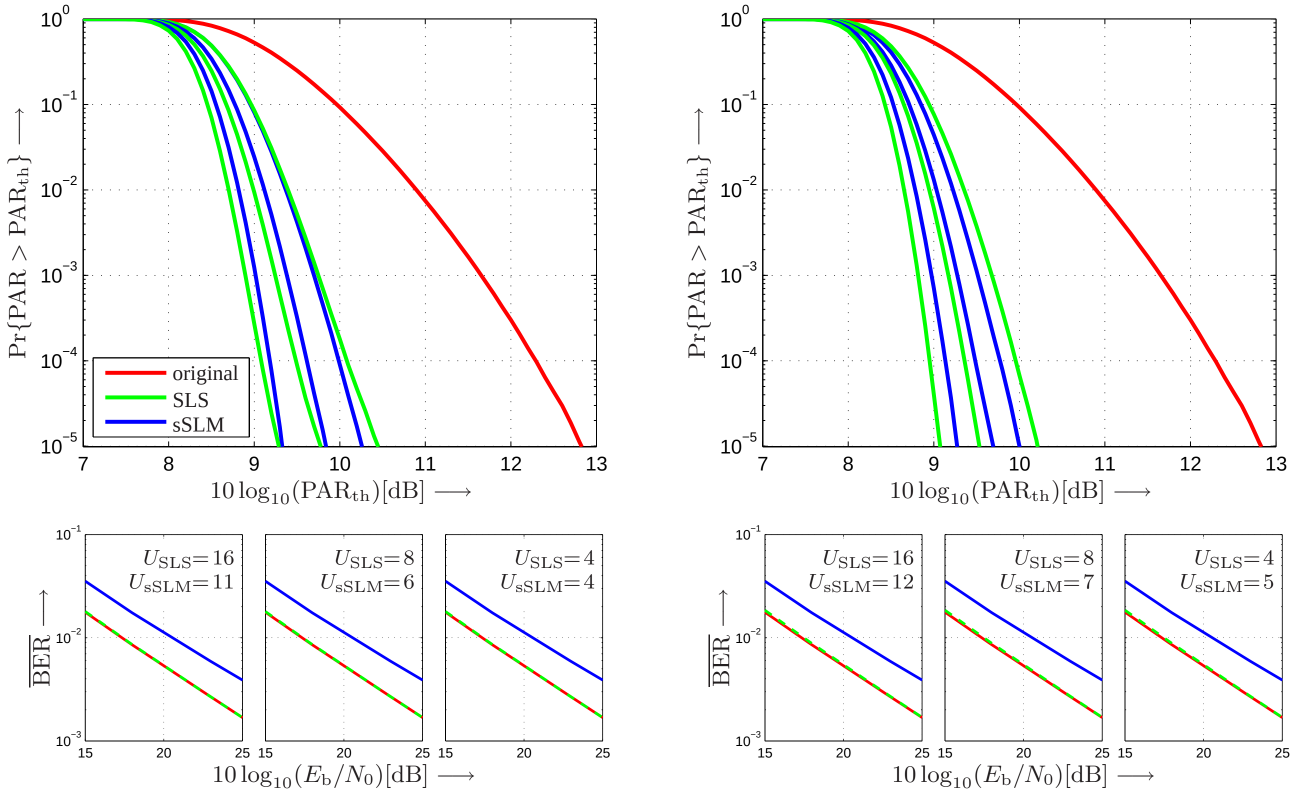

Fig. 7. Comparison ccdf of PAR (top row) and bit error ratio (bottom row) of SLS and sSLM; the number of alternative precoders for SLS is $V = 2$ (left column) or $V = 4$ (right column). The number of assessed signal candidates for SLS is (from right to left) $U_{\mathrm{SLS}} = 4$, 8, and 16; to exhibit almost same computational complexity the number of assessed candidates of sSLM is chosen to $U_{\mathrm{sSLM}} = 4$, 6, and 11 (left column) or $U_{\mathrm{sSLM}} = 5$, 7, and 12 (right column). For the bit error ratio analysis the side information, necessary with sSLM, is transmitted according the scheme from [14], whereby a scrambler with $N_{\mathrm{states}} = 16$ states has been used. $M = 4$-QAM, $K = 4$, $D = 512$, $\rho = 1$.

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