Reed–Solomon and Simplex Codes for Peak-to-Average Power Ratio Reduction in OFDM

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Abstract

New schemes for peak-to-average power ratio reduction in OFDM systems are proposed. Reed–Solomon and Simplex codes are employed to create a number of candidates, from which the best are selected. Thereby, in contrast to existing approaches, the codes are arranged over a number of OFDM frames rather than over the carriers, hence a combination of the principles of multiple signal representation with selection (as done in selected mapping) and the use of channel coding is present. In particular in multi-antenna transmission the proposed schemes do not cause any additional delay, but due to the utilization of the dimension space, additional gains can be achieved. Moreover, the schemes are very flexible; due to the selection step any criterion of optimality can be taken into account. Besides multi-antenna transmission, packet transmission is briefly considered, which, moreover, covers the appealing similarities with incremental redundancy check schemes in automatic repeat request (ARQ) applications and with decoding of codes transmitted over the erasure channel. The performance of the schemes is (using some approximations) derived analytically and is covered by numerical results which are in very good agreement with the theory. Significant gains can be achieved with these very flexible and versatile methods.

Index Terms

OFDM, Peak-to-Average Power Ratio Reduction, Reed–Solomon Code, Simplex Code

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I. Introduction and System Model

Over the last years, *orthogonal frequency-division multiplexing (OFDM)* [5], [34], has become very popular for transmission over frequency-selective channels. Main ideal of these *multicarrier techniques* is the transmission of a data stream by splitting it into an (often large) number of sub-streams, each modulating an individual carrier. In OFDM, this frequency multiplexing is done very efficiently by employing the *(inverse) discrete Fourier transform ((I)DFT)*, in particular its fast implementation *fast Fourier transform (FFT)* [32].

Unfortunately, due to the superposition of a large number of individual components, the complex amplitude OFDM transmit signal is almost Gaussian distributed. Hence it exhibits a large *peak-to-average power ratio (PAR)* [5]. This fact significantly complicates implementation of the analog radio frequency (RF) frontend since amplifiers operating linearly over a wide dynamic range are required. Clipping of signal peaks by non-linear amplifiers will cause distortion and, even worse, untolerable out-of-band radiation which leads to a violation of spectral masks. In order not to operate with large power back-offs, an algorithmic control of the transmit signal for lowering the PAR is required.

Since the mid 1990s, a variety of methods for *peak-to-average power ratio reduction* have been developed. Even though they are all based on (i) introducing new degrees of freedom for restricting to or selection of suited candidate OFDM signals and (ii) implicitly or explicitely adding redundancy, very different approaches are present in literature, cf. [11]. The most relevant PAR reduction schemes are

- *Redundant Signal Representations*

  Multiple transmit signals are created which represent the same data, from which the "best" representation is selected. Here, in particular *selected mapping (SLM)*, e.g., [4], [42], [27], [7], [3], [10] and *partial transmit sequences (PTS)*, e.g., [28], [8], [1], [38] have to be mentioned.
- *(Soft) Clipping*

  The transmit signal (preferably the discrete-time symbols prior to pulse shaping) is passed through a non-linear, memoryless device. Combinations of clipping and filtering (maybe iterative) are also popular, e.g., [31], [19], [33], [2].
- *Coding Techniques*

  An algebraic code construction is adopted to code over the frequency-domain symbols (possibly combined with channel coding), e.g., [15], [35], [36] or to consider an OFDM symbol as a codeword of a Reed–Solomon code defined over the complex numbers. [12].

- *Tone Reservation*
  Some carriers are not used for data transmission but are selected via an algorithmic search (sometimes in an iterative way between frequency and time domain), e.g., [41], [18].
- *Constellation Expansion*
  The signal constellations in the carriers are warped. Points at the perimeter of the constellation may be allowed to have (any) amplitude larger than the original one, e.g., [17].
- *(Trellis) Shaping*
  A signal shaping algorithm is adopted to operate over the frequency domain symbols, e.g., [21], [13], [29].

In this paper, we introduce a new method for PAR reduction in OFDM. It is based on a combination of the principles of multiple signal representation with selection (especially SLM) and using channel coding. However, instead of designing codes for a specific situation, we use "general purpose" channel codes to create a number of (independent) candidates, from which (after transformation into time domain) the best are selected. This enables new applications; two of them, multi-antenna PAR reduction and packet transmission with low PAR, will be discussed.

Main difference to existing techniques is that coding is done over a number of OFDM frames. In particular in multi-antenna transmission this does not cause any additional delay but due to the utilization of the dimension space, additional gains can be achieved, i.e., similar to error performance in multiple-input/multiple-output transmission, a diversity gain can be achieved. Moreover, the scheme is very flexible; due to the selection step any criterion of optimality can be taken into account. Besides PAR reduction other signal design aims can be incorporated immediately.

The proposed PAR reduction scheme has appealing similarities with incremental redundancy check schemes in automatic repeat request (ARQ) applications, cf. [9], [14]. However, in the present case only erasure decoding is required at the receiver, and puncturing (or "packet loss") is caused intentionally at the transmitter rather than by the channel.

The paper is organized as follows: In Section II, the system model is established and the performance measure is introduced. A new PAR reduction scheme based on Reed–Solomon codes is presented in Section III. The basic operation is given, the combination with other types of channel coding is discussed, and (using some assumptions) an analytic expression for its performance is derived. Section IV discusses a second scheme based on the Simplex code. Again the main operation and a theoretical expression for its performance are given. The proposed PAR reduction schemes are assessed via numerical simulations in Section V, and Section VI draws some conclusions.

II. System Model and Main Concept

In this paper, we consider a standard discrete-time OFDM system model [5], [34], employing an (I)DFT of length D. For brevity, we expect all D carriers to be active. In each carrier, binary data is first mapped to complex-valued data symbols A_d, $d = 0, \ldots, D-1$, taken from zero-mean M-ary QAM or PSK constellations (not necessarily the same over the carriers), all with the same variance $\sigma_a^2 = \mathrm{E}\{|A_d|^2\}$. Then, the frequency-domain OFDM frame composed of D data symbols, denoted by $\boldsymbol{A} = [A_0, \ldots, A_{D-1}]$, is transformed into the time-domain OFDM frame $\boldsymbol{a} = [a_0, \ldots, a_{DI-1}]$ via I-times oversampled IDFT, i.e., $a_k = \frac{1}{\sqrt{D}} \sum_{d=0}^{D-1} A_d \cdot \mathrm{e}^{\mathrm{j}2\pi kd/(DI)}$, $k = 0, \ldots, DI-1$. The correspondence is written in short as $\boldsymbol{a} = \mathrm{IDFT}_I\{\boldsymbol{A}\}$. For $I \approx 4 \ldots 8$ the samples a_k very closely reflect the continuous-time signal; for details see [46], [45].

A. Performance Measure

Due to the summation of a large number of statistically independent carriers, the time-domain samples a_k exhibit a high *peak-to-average power ratio (PAR)*

$$\xi \overset{\text{def}}{=} \max_{k=0,\ldots,DI-1} |a_k|^2/\sigma_a^2 \,. \tag{1}$$

In this definition the average (long-term) energy of an OFDM frame is considered, hence basically the peak power is taken into account. Other definitions, e.g., using the actual energy of the considered OFDM frame (short-term), are also possible, cf. [30]. However, using 4PSK in each carrier, the energy of each OFDM frame is constant and both definitions of PAR coincide.

As performance measure for PAR reduction the probability that the PAR of an OFDM frame exceeds a certain threshold ξ_{th} is studied, i.e., the *complementary cumulative distribution function (ccdf)*

$$\mathrm{ccdf}(\xi_{\mathrm{th}}) \overset{\text{def}}{=} \Pr\{\xi > \xi_{\mathrm{th}}\} \tag{2}$$

is considered.

Although not strictly true (cf. [47], [22], [43]), in some situations it is convenient to consider the time-domain signal to be approximately complex Gaussian distributed. Thereby, analytic results can be obtained. In particular, assuming Gaussian samples at Nyquist rate ($I = 1$), the ccdf of the original OFDM signal is well approximated by [4], [27]

$$\mathrm{ccdf}_{\mathrm{org,Gauss}}(\xi_{\mathrm{th}}) = 1 - (1 - \mathrm{e}^{-\xi_{\mathrm{th}}})^{D} \; . \tag{3}$$

B. Main Concept

For PAR reduction, subsequently, K OFDM frames are treated jointly. Thereby, these frames may be taken over a number of antennas, they may be consecutive frames over time, or may be formed by grouping over any other dimension. However, in all cases we expect that the frames are processed jointly at transmitter and receiver (point-to-point transmission).

In contrast to other PAR reduction schemes based on code constructions, e.g., [12], [15], [35], we do not code across the carriers (i.e., coding within one OFDM frame) but we encode over the K OFDM frames. Thereby, these K frames are regarded as the K "information symbols" to be encoded by a (linear) block code of length N. Out of the N "code symbols", a number of (at least) K exhibiting the lowest PAR (or, in general, showing the best performance measure) are selected for actual transmission. Hence, *puncturing* takes place at the transmitter; erasures are deliberately caused. Only codes together with the particular selection strategy are suited, for which it is guaranteed that via erasure decoding at the receiver the K initial information symbols (OFDM frames) can be recovered. This question is tightly related to the decodability of a code over the erasure channel, a field recently gained much attention in the context of network coding, e.g., [24], [37]. However, as the transmitter has control on the selection, new aspects, usually not treated in coding literature, arise.

Subsequently we study two approaches: Firstly, the use of Reed–Solomon codes, which are *maximum distance separable* (MDS code). Here it is guaranteed that any selection of K symbols suffice to recover data.

Secondly, we identify the OFDM frames (binary data in frequency domain) as the information symbols and generate all possible linear combinations (over the binary field) thereof. Noteworthy, writing this procedure in usual vector/matrix notation, the generator matrix corresponding to this encoding is identical to the generator matrix of the *Simplex code*, the dual to the (binary) Hamming codes [6]. However, as this code is not MDS, some caution has to be taken at the selection (erasure) step.

III. Reed–Solomon Codes Based PAR Reduction

A. Proposed Scheme

In order to perform PAR reduction on K OFDM frames jointly, a Reed–Solomon (RS) code over the Galois field $\mathbb{F}_{2^m}$ with (maximum) code length $N = 2^m - 1$ is employed [6]. Its dimension (number of information symbols) is chosen equal to K. In the sequel, we will exploit the property that RS codes are MDS, i.e., they meet the *Singleton bound* on the minimum distance of the code with equality, i.e., we have $d_{\min} = N - K + 1$ [6].

In frequency domain, blocks of m bits (not necessarily belonging to adjacent carriers) are formed and constitute one RS code symbol. Using (on average) an $(M = 2^\mu)$-ary modulation scheme per carrier, m/μ carriers are combined into one RS symbol (typically $m \geq \mu$ will hold). E.g., using 4PSK modulation and RS codes over $\mathbb{F}_{256} = \mathbb{F}_{2^8}$, four carriers are combined into one RS symbol (8 bits). Assuming all D carriers to be active, $\Delta = D\mu/m$ RS code words are present in parallel.

Via the RS codes, given each K information symbols, $n \leq N$ coded symbols are calculated. Thereby systematic encoding (the information symbols appear unchanged at the first K positions; $P \leq N - K$ parity symbols are calculated) may be used preferably but non-systematic encoding works as well. Using an RS generator polynomial of degree $N - K$ (designing the code for codelength N and dimension K), a *punctured code* is used and not a shorted one (where a generator polynomial of degree $n - K$ would have been used and the missing $N - n$ information symbols would have been assumed to be zero).

Mapping the bits in the n RS symbols of the Δ parallel code words to data symbols $A_{\nu,d}$, n OFDM frames $\boldsymbol{A}_\nu = [A_{\nu,0}, \ldots, A_{\nu,D-1}]$, $\nu = 1, \ldots, n$, are obtained. Transforming them to time domain, their respective ξ_ν can be calculated. Neglecting the complexity of RS encoding (done in Galois field arithmetic) the required effort is dominated by the $n > K$ (oversampled) IDFTs and PAR calculations (done with complex numbers). From the n OFDM frames, the K frames with the lowest PAR are selected for transmission; any other selection criterion is possible as well. Hence, the new scheme could be adopted very flexibly to other scenarios and optimization aims. Figure 1 sketches the arrangement of the code words over the OFDM frames and the selection process.

Due to the MDS property of RS codes, any selection of K code symbols out of N is sufficient to recover data (additional symbol errors are ignored for the moment). At the receiver, using Δ parallel RS decoders,[1] erasure decoding is performed and the K initial OFDM frames are reconstructed, from which the binary data is obtained. Thereby, the RS decoder has to gain knowledge which K symbols (position within the code word) are present. This can be accomplished by tagging each OFDM frame with its number ν. Hence, some small amount of side information has to be transmitted as well, which will be ignored in the following. Schemes, avoiding explicit side information, e.g., [16], can be used at this stage.

B. Combination with Other Channel Coding Schemes

In the above exposition, the RS code solely serves for the generation of candidates. If, additionally, channel errors have to be corrected basically two approaches are possible. First, more than K OFDM frames may be transmitted. Each increase in the number of frames by two leads to an extra correction capability of one RS symbol [6]. However, since soft-decision decoding of RS codes is still a challenging task, as an alternative, the combination of the above scheme with other types of channel codes is considered.

In this first step, the RS codes are applied across the frames and constitute an outer code. Then, using any linear code (with respect to the bits), the n OFDM frames are encoded separately (across the carriers, inner code). For example, convolutional codes, Turbo codes, low-density parity check codes, etc. are possible, whereby it does not matter whether systematic or non-systematic encoding is employed. In summary a product code, cf. [20, Fig. 15.2], is established, illustrated in Figure 2. Due to selection, only the redundancy of the channel code has to be additionally transmitted.

At the receiver, first the K inner channel codes over the OFDM frames are decoded as usual (using soft information). At this stage, no additional complexity compared to conventional coded transmission of K OFDM frames is required. In the second step, via RS erasure decoding, the K initial OFDM frames are reconstructed, cf. Figure 2.

C. Complementary Cumulative Distribution Function

In order to obtain an analytic expression for the ccdf of PAR for the proposed scheme, we assume that the ccdf of the original OFDM frames, $\mathrm{ccdf}_{\mathrm{org}}(\xi_{\mathrm{th}})$, (no PAR reduction) is known. Assuming Gaussian time-domain samples at Nyquist rate, an analytic expression

[1] Any implementation of an RS decoder can be used. However, for successive decoding, the scheme based on Newton's interpolation [40] is preferable.

(cf. (3)) can be given. In case of QAM modulation and I-times oversampling bounds and approximations on the ccdf can be found, e.g., in [47], [22], [23].

Moreover, we assume that the $P = n - K$ via RS encoding additionally generated OFDM frames are also i.i.d. and statistically independent of the K initial OFDM frames. This assumption does not hold in practice, since due to properties of the RS code, only any selection of K frames is statistically independent; the remaining frames are dependent. However, this approximation proves to be useful in the following and seems to be justified since we are only considering the PAR of the frames and not the frames itself. Hence, actually, we demand that the PARs of the n OFDM frames are independent.

Assuming the candidate OFDM frames (before selection) are sorted according to their PAR (ξ_λ is the PAR according to (1) of the λ^{th} best OFDM frame), i.e.,

$$\xi_1 \leq \xi_2 \leq \ldots \leq \xi_n \,, \tag{4}$$

the ccdf of the PAR of the λ^{th} best OFDM frame reads ($\xi_0 \stackrel{\text{def}}{=} -\infty$)

$$\begin{aligned} \text{ccdf}_{\text{parRS},\lambda}(\xi_{\text{th}}) &= \Pr\{\xi_\lambda > \xi_{\text{th}}\} = \Pr\{\xi_{\text{th}} < \xi_\lambda\} \\ &= \sum\nolimits_{l=0}^{\lambda-1} \Pr\{\xi_l \leq \xi_{\text{th}} < \xi_{l+1}\} \,. \end{aligned} \tag{5}$$

Due to the assumed independence, the above individual probabilities are given by a binomial distribution with a single event probability of $1 - \text{ccdf}_{\text{org}}(\xi_{\text{th}})$ (the original frame does not exceed the threshold) and l events out of n come true (l frames have PAR not exceeding the limit), hence

$$\Pr\{\xi_l \leq \xi_{\text{th}} < \xi_{l+1}\} = \binom{n}{l} (1 - \text{ccdf}_{\text{org}}(\xi_{\text{th}}))^l \text{ccdf}_{\text{org}}^{n-l}(\xi_{\text{th}}) \tag{6}$$

and in turn

$$\text{ccdf}_{\text{parRS},\lambda}(\xi_{\text{th}}) = \sum\nolimits_{l=0}^{\lambda-1} \binom{n}{l} (1 - \text{ccdf}_{\text{org}}(\xi_{\text{th}}))^l \text{ccdf}_{\text{org}}^{n-l}(\xi_{\text{th}}) \,. \tag{7}$$

In the algorithm, out of the n OFDM frames with their respective PAR, the K best ($\lambda = 1, \ldots, K$) are selected for transmission.

D. Applications

1) Multi-Antenna Transmission: The first situation where the proposed scheme is preferably applied is *multi-antenna transmission*. Here, K is chosen equal to the number N_{T} of transmit antennas.[2] From the K initial OFDM frames, K with lower PAR are calculated. These K frames are then actually radiated from the antennas.

Main advantage of this approach is that, to the most extent, it can be implemented in parallel; the Δ RS codes can be encoded in parallel, and then the n IDFTs and PAR calculations can also be carried out in parallel. No successive calculations and decisions as in [10] are required. The same is true for the receiver, where optional channel decoding, calculation of the DFTs and RS decoding can respectively be performed in parallel.

In multi-antenna OFDM it is usually assumed that the *worst-case PAR* over the antennas determines performance. The frames are radiated from the antennas at the same time, hence any distortion (e.g., out-of-band radiation) caused by the the worst-case OFDM frame masks the others. Then, the ccdf of PAR for multi-antenna OFDM using the RS scheme is given by

$$\mathrm{ccdf}_{\mathrm{multi-antenna}}(\xi_{\mathrm{th}}) = \mathrm{ccdf}_{\mathrm{parRS},K}(\xi_{\mathrm{th}}) \tag{8}$$

2) Packet Transmission: Another application for the presented PAR reduction scheme is *packet transmission*, where information is divided into a number of packets, which are transmitted in sequel or even in parallel. We may consider mesh or sensor networks, where the packets are possible routed via different paths through the network and hence have different delays or even packets may get lost. In this paper we assume that each OFDM frame constitutes one packet and only packets with low PAR are desired.

In packet transmission, a new aspect for the proposed PAR reduction scheme may be of interest: sending a number of F, $K \leq F \leq n$, packets, a loss of $F-K$ packets is tolerable. The transmission of the $F-K$ extra packets may be initiated by ARQ, cf. [9], [14]. Transmitting the packets sorted according to increasing PAR (the K best first), an exchange between robustness against packet loss and PAR reduction is obtained.[3] Since now F OFDM frames are transmitted in sequel, the average ccdf of PAR is given as

$$\begin{aligned}
\mathrm{ccdf}_{\mathrm{packet}}(\xi_{\mathrm{th}}) &= \frac{1}{F}\sum_{\lambda=0}^{F-1} \mathrm{ccdf}_{\mathrm{parRS},\lambda}(\xi_{\mathrm{th}}) \\
&= \frac{1}{F}\sum_{\lambda=0}^{F-1}\sum_{l=0}^{\lambda-1} \binom{n}{l} \left(1-\mathrm{ccdf}_{\mathrm{org}}(\xi_{\mathrm{th}})\right)^{l} \mathrm{ccdf}_{\mathrm{org}}^{n-l}(\xi_{\mathrm{th}})
\end{aligned}$$

[2]It is also possible to use more transmit antennas than initial OFDM frames leading to a transmission of redundant OFDM frames and hence some error correction capability.

[3]A successive generation of additional packets is possible. If the PAR of a packet is below a given, tolerable threshold, it may be used for transmission, otherwise it will be dropped.

$$= \frac{1}{F} \sum\nolimits_{l=0}^{F-1} (F-l) \binom{n}{l} (1 - \mathrm{ccdf}_{\mathrm{org}}(\xi_{\mathrm{th}}))^l \mathrm{ccdf}_{\mathrm{org}}^{n-l}(\xi_{\mathrm{th}}) \;, \qquad (9)$$

where again $\mathrm{ccdf}_{\mathrm{org}}(\xi_{\mathrm{th}})$ is again the ccdf of the original OFDM scheme without PAR reduction.

IV. PAR REDUCTION BASED ON THE SIMPLEX CODE

A. Simplex Code

An even simpler approach as the above RS scheme is as follows: Given the OFDM frames (binary data in frequency domain), linear combinations over the binary field are generated. Mapping them to PSK/QAM symbol and transforming them to time domain, their PAR can be calculated. From these, K with low PAR are selected. Thereby, it has to be checked that the original frames can be reconstructed.

Denoting the binary representation of the OFDM frames by $\boldsymbol{Q}_\kappa$, $\kappa = 1, \ldots, K$, all possible (non-zero) binary linear combination $\boldsymbol{C}_\nu$, $\nu = 1, \ldots, 2^K - 1$, may be generated via

$$[\boldsymbol{C}_1, \ldots, \boldsymbol{C}_{2^K-1}] = [\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_K]\boldsymbol{G} \qquad (10)$$

where the $K \times 2^K - 1$ matrix $\boldsymbol{G}$ contains as columns all $2^K - 1$ binary K tuples $\boldsymbol{t}_\nu$ except the all-zero word, i.e.,

$$\boldsymbol{G} = [\boldsymbol{t}_1, \, \boldsymbol{t}_2, \, \ldots, \, \boldsymbol{t}_{2^K-1}] \qquad (11)$$

and $\boldsymbol{t}_1 = [1, 0, \ldots, 0]^\mathsf{T}$ through $\boldsymbol{t}_{2^K-1} = [1, 1, \ldots, 1]^\mathsf{T}$, where, w.l.o.g, we expect the first K vectors to be the unit vectors (generator matrix in systematic form). Noteworthy, $\boldsymbol{G}$ is the generator matrix of the *simplex code*, the dual to the (binary) Hamming codes [26], [6].

Given $\{\boldsymbol{C}_\nu\}$ (or a randomly selected subset of $n \leq 2^K - 1$ combinations thereof), after IDFT their PARs are calculated and sorted in ascending order. We denote the sequence of indices of the sorted OFDM frames as $s_1, \ldots, s_{2^K-1}$ with $\xi_{s_1} \leq \xi_{s_2} \leq \cdots \leq \xi_{s_{2^K-1}}$. In the optimum, the best K frames are selected. However, to be able to recover data, it has to be guaranteed that the coefficient matrix composed of the corresponding vectors $\boldsymbol{t}_{s_\nu}$, $\nu = 1, \ldots, K$, has full rank, i.e., is non-singular and hence an inverse (a binary matrix, too) exists. This property can be achieved by successively going through the columns of the sorted generator matrix $\boldsymbol{G}_{\mathrm{s}} = [\boldsymbol{t}_{s_1}, \, \boldsymbol{t}_{s_2}, \, \ldots, \, \boldsymbol{t}_{s_{2^K-1}}]$ (starting from $\boldsymbol{t}_{s_1}$) and accepting a new column (and hence new OFDM frame) only if it is linearly independent of the already selected columns (can efficiently be done via Gaussian elimination). The selection stops if K linearly independent vectors are found; the corresponding OFDM frames are then actually transmitted. Thereby, as above, the respective vector $\boldsymbol{t}_{s_\mu}$ is incorporated as side information; again we will ignore this small fraction of redundancy. Figure 3 illustrates the procedure of candidate generation and selection. Of course, additional channel coding can be performed, too.

As above, complexity of this scheme is dominated by the n (oversampled) IDFTs and PAR calculations. Since linear combinations are formed over the binary field before mapping to complex symbols and IDFT is carried out over complex numbers, no interchange of these operations for reducing complexity is possible.

A slight disadvantage is that in some situations ($\boldsymbol{t}_{s_1}$ through $\boldsymbol{t}_{s_K}$ are not linearly independent) not the K best OFDM frames can be selected for actual transmission. As long as the $K+1$ (or $K+2$, ...) best OFDM frame has a PAR not significantly worse than that of the K^{th} best, only a slight loss will occur.

At the receiver, the initial data is recovered from the K received OFDM frames as follows: Given the numbers (equal to the vectors $\boldsymbol{t}_{s_\nu}$) of the OFDM frames (after optimal channel decoding and inverse mapping to the binary representation $\boldsymbol{C}_{s_\nu}$), the matrix $\tilde{\boldsymbol{G}}$ is composed and its inverse is calculated. This can be done easily, as only binary matrices with (typically) small dimensions have to be handled. The initial data is then obtained as (also binary) linear combinations of the received OFDM frames, i.e.,

$$[\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_K] = [\boldsymbol{C}_{s_1}, \ldots, \boldsymbol{C}_{s_K}]\tilde{\boldsymbol{G}}^{-1} \, . \tag{12}$$

Contrary to, e.g., fountain codes [25], no successive decoding without the need for matrix inversion is possible.

Please note, that contrary to the RS scheme, the present approach is not very well suited for packet transmission where $F \geq K$ OFDM frames should be transmitted. Typically, with very high probability the F actually best frames are chosen. However, as here it is not guaranteed that any K received packets are sufficient to recover data, we will not consider the simplex code scheme in case of packet transmission but solely for multi-antenna systems.

Finally it should be noted that a generalization of the scheme to non-binary Simplex codes (the dual to non-binary Hamming codes [44]; preferably 2^q-ary symbols, $q \in \mathbb{N}$) is easily possible. The described PAR reduction algorithm is equally valid in this case.

B. CCDF of PAR for the Simplex Code Construction

In order to calculate the ccdf of PAR for the proposed scheme based on the simplex code, we have to calculate the probability that a random selection of K, $K+1$, ..., 2^{K-1} distinct binary vectors (columns $\boldsymbol{t}_\nu$) spans K dimensions,[4] but when deleting the last vector/column it spans only $K-1$ dimensions. As above, we expect the PARs of the selected OFDM frames to be independent, which means that the choice of the columns $\boldsymbol{t}_\nu$ of the generator matrix $\boldsymbol{G}$ is random.

We first note, that any two distinct, non-zero binary vectors are independent of each other. Given already a set of n_{li} linearly independent vectors and additionally n_{ld} vectors, linearly dependent on the first set, out of the $2^K - 1 - n$, $n \stackrel{\text{def}}{=} n_{\mathrm{li}} + n_{\mathrm{ld}}$, remaining possible non-zero vectors of dimension K,

$$N_{\mathrm{ld}}(n, n_{\mathrm{li}}) \stackrel{\text{def}}{=} \sum_{l=2}^{n_{\mathrm{li}}} \binom{n_{\mathrm{li}}}{l} - (n - n_{\mathrm{li}}) = 2^{n_{\mathrm{li}}} - 1 - n \tag{13}$$

are linearly dependent on the first set. Hence, the probability that the next chosen vector (corresponding to the candidate OFDM frame) is linearly dependent on the given vectors is

$$P_{\mathrm{ld}}(n, n_{\mathrm{li}}) \stackrel{\text{def}}{=} \frac{N_{\mathrm{ld}}(n, n_{\mathrm{li}})}{2^K - 1 - n} = \frac{2^{n_{\mathrm{li}}} - 1 - n}{2^K - 1 - n} \tag{14}$$

and $P_{\mathrm{li}}(n, n_{\mathrm{li}}) = 1 - P_{\mathrm{ld}}(n, n_{\mathrm{li}}) = (2^K - 2^{n_{\mathrm{li}}})/(2^K - 1 - n)$.

To obtain the probability that given K, the selection of $\kappa \geq K$ vectors contains exactly K linearly independent vectors (the remaining are dependent), all possible combinations have to be studied. For selection (written left to right in a vector with κ elements) the following combinations of choosing a next vector linearly dependent (“0”) or independent (“1”) on the already given ones are possible ($\mathrm{wt}(\cdot)$: Hamming weight)

$$\mathcal{D}(\kappa, K) \stackrel{\text{def}}{=} \{\boldsymbol{d} = [1\,1 \mid \cdots 1/0 \cdots \mid 1] \;\mid\; \boldsymbol{d} \text{ has } \kappa \text{ elements},\ \mathrm{wt}(\boldsymbol{d}) = K\}\ , \tag{15}$$

where the first two “1” reflect the independence of any two distinct binary vectors, the last “1” the final selection of the K^{th} linearly independent vector, and in the middle part any permutation of $K-3$ “1” and $\kappa - K + 3$ “0” is possible.[5]

Due to the assumed statistical independence of the selection, the probability of a particular combination described by a vector $\boldsymbol{d}$ is given by the product of the respective probabilities $P_{\mathrm{li/ld}}$. In total, combining the above equations, we have ($\boldsymbol{d}[l]$ denotes the l^{th} element of $\boldsymbol{d}$

[4] At least K vectors are required to span K dimensions; at maximum 2^{K-1} binary vectors have to be considered, since, in the worst case, $2^{K-1} - 1$ distinct binary vectors may span only $K-1$ dimensions.

[5] Example: the following successive choice $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$ of the column vectors $\boldsymbol{t}_\nu$ of dimension $K=4$ is represented as vector $\boldsymbol{d} = [110101]$; here, $\kappa = 6$ steps are required to span 4 dimensions.

and $\boldsymbol{d}[1:l]$ the vector containing elements 1 through l)

$$p_{\kappa,K} \overset{\text{def}}{=} \frac{\prod_{l=2}^{K-1}(2^K - 2^l)}{\prod_{l=3}^{\kappa}(2^K - l)} \cdot \sum_{\forall \boldsymbol{d} \in \mathcal{D}(\kappa,K)} \prod_{\forall l, \boldsymbol{d}[l]=0} (2^{\mathrm{wt}(\boldsymbol{d}[1:l])} - l) \tag{16}$$

$$= \frac{\prod_{l=2}^{K-1}(2^K - 2^l)}{\prod_{l=3}^{\kappa}(2^K - l)} \cdot \sum_{\substack{\forall \boldsymbol{v} \in \mathbb{F}_2^{\kappa-3} \\ \mathrm{wt}(\boldsymbol{v})=K-3}} \prod_{\forall l, \boldsymbol{v}[l]=0} (2^{\mathrm{wt}(\boldsymbol{v}[1:l])+2} - (l+2)) \; . \tag{17}$$

In case of multi-antenna transmission, the ccdf of the new scheme is then finally the average over the ccdfs given in (7) for $\lambda = K,\ K+1,\ \ldots$ according to the probabilities (16), i.e., we have

$$\mathrm{ccdf}_{\mathrm{multi-antenna}}(\xi_{\mathrm{th}}) = \sum_{\kappa \geq K} p_{\kappa,K} \cdot \mathrm{ccdf}_{\mathrm{parRS},\kappa}(\xi_{\mathrm{th}}) \; . \tag{18}$$

V. NUMERICAL EXAMPLES

In order to assess the performance of the proposed PAR reduction schemes, numerical simulations have been conducted. OFDM frames with $D = 512$ carriers, each modulated with 4PSK (unless otherwise stated) are considered.

A. RS Codes

1) Multi-Antenna Transmission: First, multi-antenna transmission with $K = N_{\mathrm{T}} = 4$ antennas is assumed. RS codes over $\mathbb{F}_{2^8}$ are employed ($\Delta = 512 \cdot 2/8 = 128$ codes in parallel), and the code length is chosen either to $n = 255$ or punctured to $n = 127$, 63, 31, 15, and 7. Figure 4 displays the simulation results for oversampling factors $I = 1$ (Nyquist-rate sampling), 2, 4, and 8. For comparison, the theoretical ccdf (8) (gray solid), the ccdf for OFDM without PAR reduction (rightmost curve), and the ccdf of parallel, independent application of SLM to each antenna (called “ordinary SLM” in [3], gray dashed) are shown.

All theoretical curves start from the ccdf of the original OFDM scheme ($D = 512$ carriers, 4PSK modulation). However, as an exact analytical expression is not known, numerical simulations were performed and an approximation (via curve fitting) was derived. Starting from the Gaussian approximation (3) and introducing two free parameters (shifting and scaling the abscissa) for optimization, we derived the following correspondences: $\mathrm{ccdf}_{\mathrm{org},I=1}(\xi_{\mathrm{th}}) \approx 1 - (1 - \mathrm{e}^{-1.02\xi_{\mathrm{th}}+0.13})^{512}$, $\mathrm{ccdf}_{\mathrm{org},I=2}(\xi_{\mathrm{th}}) \approx 1 - (1 - \mathrm{e}^{-1.01\xi_{\mathrm{th}}+0.67})^{512}$, $\mathrm{ccdf}_{\mathrm{org},I=4}(\xi_{\mathrm{th}}) \approx 1 - (1 - \mathrm{e}^{-0.98\xi_{\mathrm{th}}+0.79})^{512}$, $\mathrm{ccdf}_{\mathrm{org},I=8}(\xi_{\mathrm{th}}) \approx 1 - (1 - \mathrm{e}^{-0.97\xi_{\mathrm{th}}+0.80})^{512}$.

It is clearly visible that increasing the code length n, i.e., selecting from more candidates, leads to better results (lower PAR). This is paid with complexity, as n IDFTs and PAR calculations have to be carried out. The same complexity is required for parallel, independent SLM schemes [4], [27] each with $U = n/K$ candidates, but performance is noticeably inferior. It is worth to note, that the proposed scheme performs the same as *"directed SLM"* [10]. Hence, the potential of the multi-antenna transmission, visible in the slope of the ccdf curve and corresponding to a diversity gain [39], is utilized with the RS code construction.

Simulations and (semi-)analytic results are in very good agreement. This fact hold for all oversampling factors. Moreover, the results are essentially the same for all oversampling factors. Since, increasing the factor I, basically the ccdf of the original scheme simply moves to the right by some tenth of dB, the curves for the proposed (and the competing) scheme simply move towards higher thresholds by the same amount. Finally, $I = 4$ is sufficient to predict the performance of the continuous-time signal; almost the same curves are also obtained for $I = 8$.

In Figure 5, the ccdf of PAR for $n = 255$ and $K = 4$ and 4PSK, 16QAM, and 256QAM modulation (left) and for 16QAM modulation and $D = 128$, 512, 2048, and 8192 (right) are compared.[6] Increasing the size of the modulation alphabet or the number D of carriers gives results even closer to the derived theoretical curve. Here, the Gaussian approximation becomes more and more accurate.

2) Packet Transmission: Next, packet transmission is considered. Again an OFDM scheme with $D = 512$ carriers and 4PSK modulation are assumed. K packets (OFDM frames) are encoded jointly using RS codes over $\mathbb{F}_{2^8}$ with code length $n = 255$ and out of them F are selected for transmission. Figure 6 plots the ccdf of the numerical simulations for $F = 4$, 7, 15, 31, 63, 127 together with the theory (9). Rightmost, the ccdf for OFDM without PAR reduction is shown for reference. Noteworthy, in spite choosing $K = 4$ in the simulations, the shown results are independent of K, and valid for all $F \geq K$, cf. (9), where no variable K occurs.

As can be seen, sending $F > K$ packets causes almost no loss in PAR if F is not too large. Even for $F = 127$, the ccdf curve exhibits a very steep decay, as $n = 255$ is chosen as large as possible. Again, numerical simulations and theoretical results are in good agreement.

To see the performance for other pairs of n (number of considered frames) and F (number of actual transmitted frames), in Figure 7, a contour plot of the threshold ξ_{th} (in dB),

[6] The ccdf of the original OFDM scheme is here approximated by $\mathrm{ccdf}_{\mathrm{org},I=1,D=128}(\xi_{\mathrm{th}}) \approx 1-(1-\mathrm{e}^{-1.04\xi_{\mathrm{th}}+0.20})^{128}$, $\mathrm{ccdf}_{\mathrm{org},I=1,D=2048}(\xi_{\mathrm{th}}) \approx 1-(1-\mathrm{e}^{-1.01\xi_{\mathrm{th}}+0.05})^{2048}$, $\mathrm{ccdf}_{\mathrm{org},I=1,D=8192}(\xi_{\mathrm{th}}) \approx 1-(1-\mathrm{e}^{-1.00\xi_{\mathrm{th}}+0.01})^{8192}$. As expected, for large D the Gaussian assumption (3) becomes more and more accurate.

minimally required for clipping probabilities lower than 10^{-4} is shown (only combinations in the lower right part, $F \leq n$, are possible).

Interestingly, similar to what is known from channel coding theory, fixing the ratio n/F and choosing larger n and F gives some gain, i.e., lower thresholds; joint processing of more candidates is rewarding. For $F \approx n/2$ gains of 4–5 dB in clipping level over conventional OFDM are achievable. Selecting F even only slightly smaller than n already gives significant gains as the worst-case OFDM frames can be eliminated. E.g. for ($n = 240, F = 210$) the clipping level (clipping probability 10^{-4}) is 9.39 dB, for ($n = 112, F = 128$) 9.49 dB, and for ($n = 14, F = 16$) 10.21 dB ($F/n = 7/8$ in all cases, marked with circles in Figure 7). However, conventional OFDM requires a level of 11.89 dB.

B. Simplex Codes

Now, multi-antenna transmission with $K = N_{\mathrm{T}}$ antennas and PAR reduction using simplex codes is assessed. The code parameters are chosen either to $K = 4$ and $n = 15$, $K = 7$ and $n = 127$, or $K = 8$ and $n = 255$, i.e., a search over all possible binary combinations out of the K OFDM frames is performed.

Figure 8 displays the simulation results for $I = 1$ (no oversampling) and $I = 4$. For comparison, the ccdf for OFDM without PAR reduction (rightmost curve), the ccdf of parallel, independent application of SLM to each antenna (dashed), the theoretical ccdf (8) for PAR reduction using RS codes (gray dashed), and the theoretical ccdf from (18) for the simplex code schemes (gray solid) are shown.

For all three sets of parameters, the ccdf of the simplex code scheme is close to that of the RS code scheme; gain of 0.5 to 1.0 dB can be obtained over conventional (independent) SLM. Again, for small K, theoretical curve and simulations fit very well; for larger K some deviations due to the above given reasons are visible. Except of a shift, the results for $I = 1$ and $I = 4$ are basically the same. In summary, using the simplex code scheme, significant gains in PAR reduction can be achieved; complexity is mainly determined by the number of required (oversampled) IDFTs.

VI. Conclusions

New schemes for PAR reduction in OFDM have been presented. In contrast to existing approaches of designing codes for a specific situation, general purpose channel codes, in particular Reed–Solomon codes and Simplex codes are employed to create a number of candidates. From that the best—any criterion is possible—are selected, i.e., the use of coding and the strategy of selecting from alternative signal representations are combined. The codes are thereby arranged over a number of OFDM frames rather than over the carriers. In particular in multi-antenna systems this approach does not cause any additional delay, but uses the dimension space appropriately and achieves significant gains. As a second example, packet transmission (each OFDM frame constitutes a packet) over lossy channels is considered.

Future work has to study the effect of the proposed schemes on the error rate. Due to erasure decoding/signal reconstruction error multiplication may occur. However, as the inner channel code operates at sufficiently low error rates, this effect is of minor importance. Moreover, iterative (turbo) decoding of the inner channel code and the outer RS codes/Simplex codes may be performed as they constitute a product code. This may lead to additional gains when employing this combined PAR reduction/channel coding scheme.

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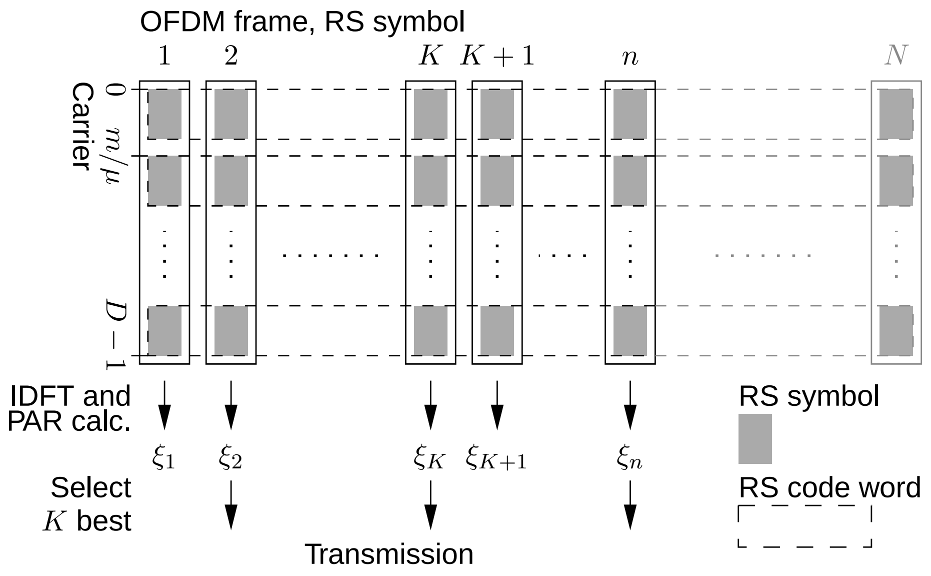

Fig. 1. Illustration of arrangement of the RS code words over the K OFDM frames. N: code length; m/μ carriers (here assumed to be adjacent) are combined into one RS symbol.

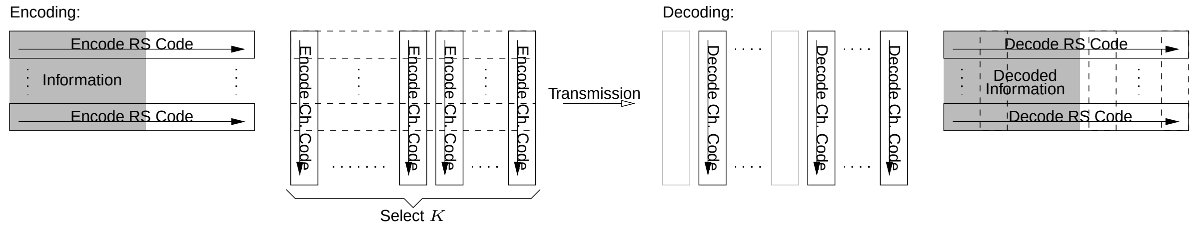

Fig. 2. Sketch of combination of RS code with inner channel code. Left: encoding plus selection; Right: decoding. Gray shaded: rectangular arrangement of initial data.

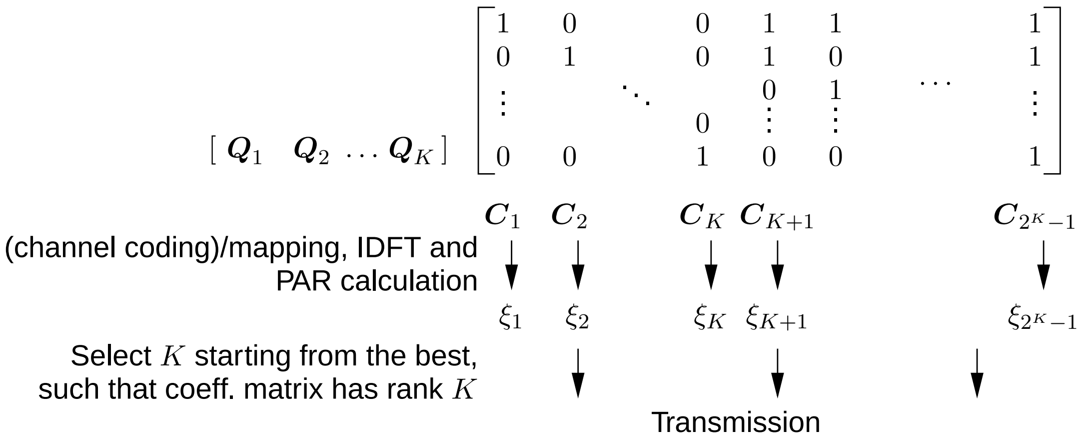

Fig. 3. Illustration of candidate generation and selection in PAR reduction based on the simplex code.

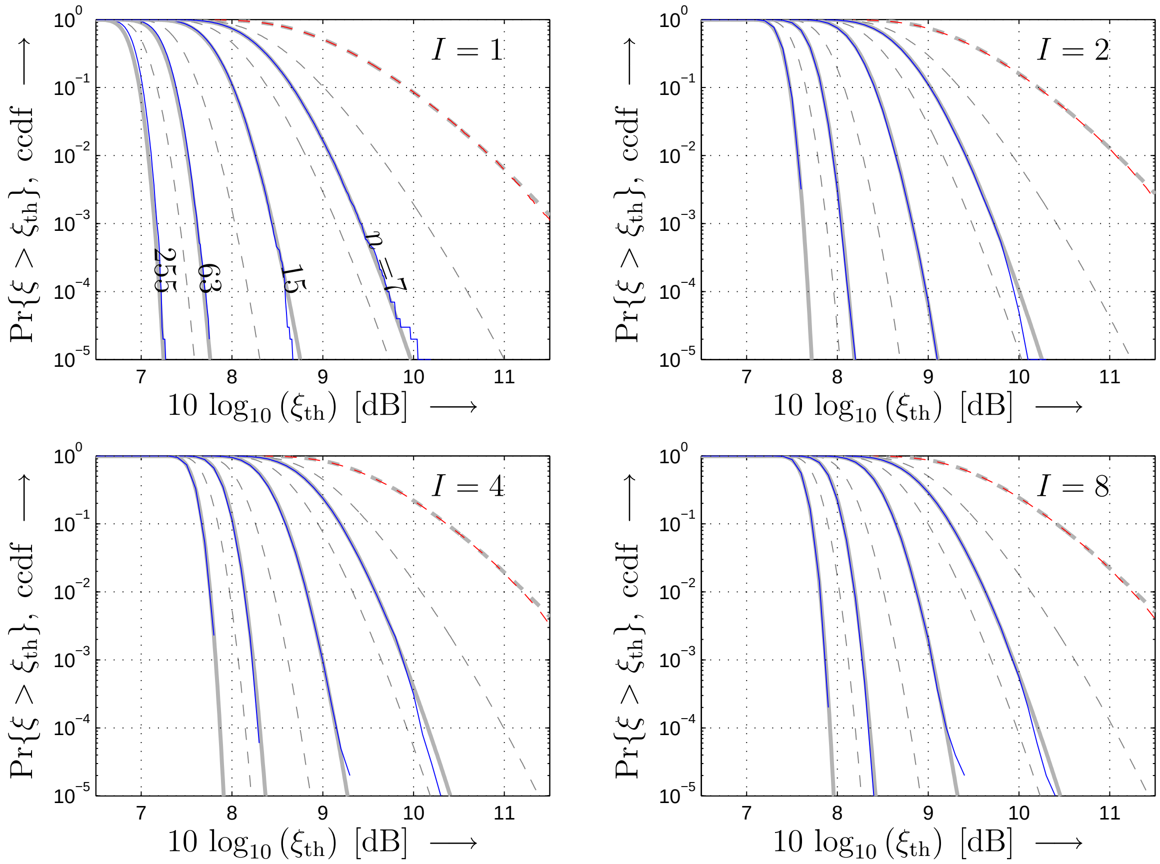

Fig. 4. Ccdf of PAR for the proposed scheme. Oversampling factor: top left $I = 1$; top right $I = 2$; bottom left $I = 4$; bottom right $I = 8$; $D = 512$, 4PSK, $K = 4$, $n = 255, 63, 15, 7$ (left to right). Rightmost curve: OFDM without PAR reduction. Gray solid: theory (8); gray dash: parallel, independent application of SLM with $U = 64, 16, 4, 2$ candidates per antenna.

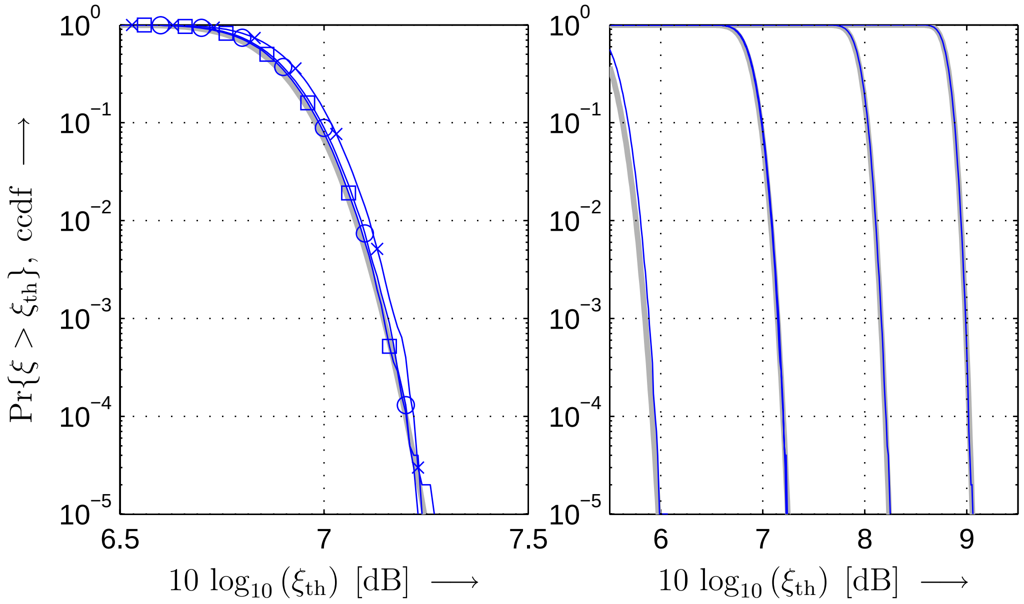

Fig. 5. Ccdf of PAR for the proposed scheme (dark solid). $K = 4$, $n = 255$, $I = 1$. Left: $D = 512$; 4PSK (crosses), 16QAM (circles), 256QAM (squares). Right: 16QAM, $D = 128$, 512, 2048, 8192 (left to right). Gray solid: theory (8).

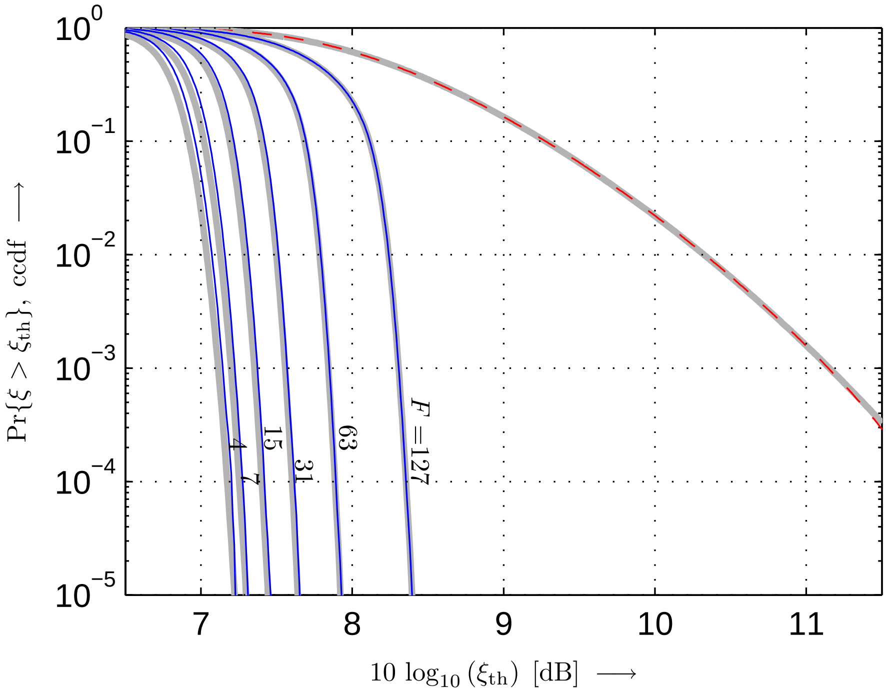

Fig. 6. Ccdf of PAR for the proposed scheme. $D = 512$, 4PSK, $K = 4$, $n = 255$. $F = 4$, 7, 15, 31, 63, 127 (left to right). Rightmost curve: OFDM without PAR reduction (equal to $F = 255$). Gray solid: theory (9).

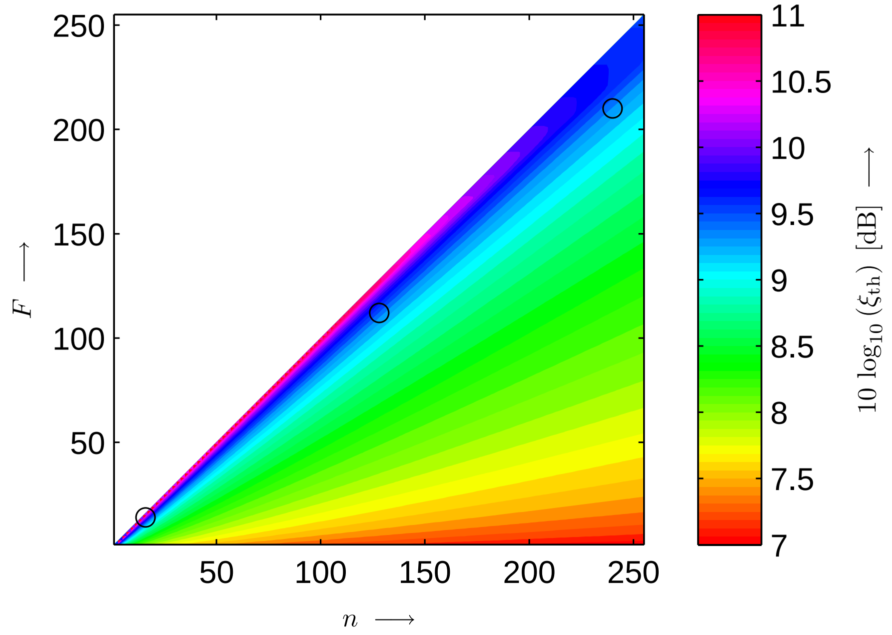

Fig. 7. Contour plot of ξ_{th} at clipping probability 10^{-4} over n and F derived from the theoretical expression (9). The examples discussed in the text are marked with circles.

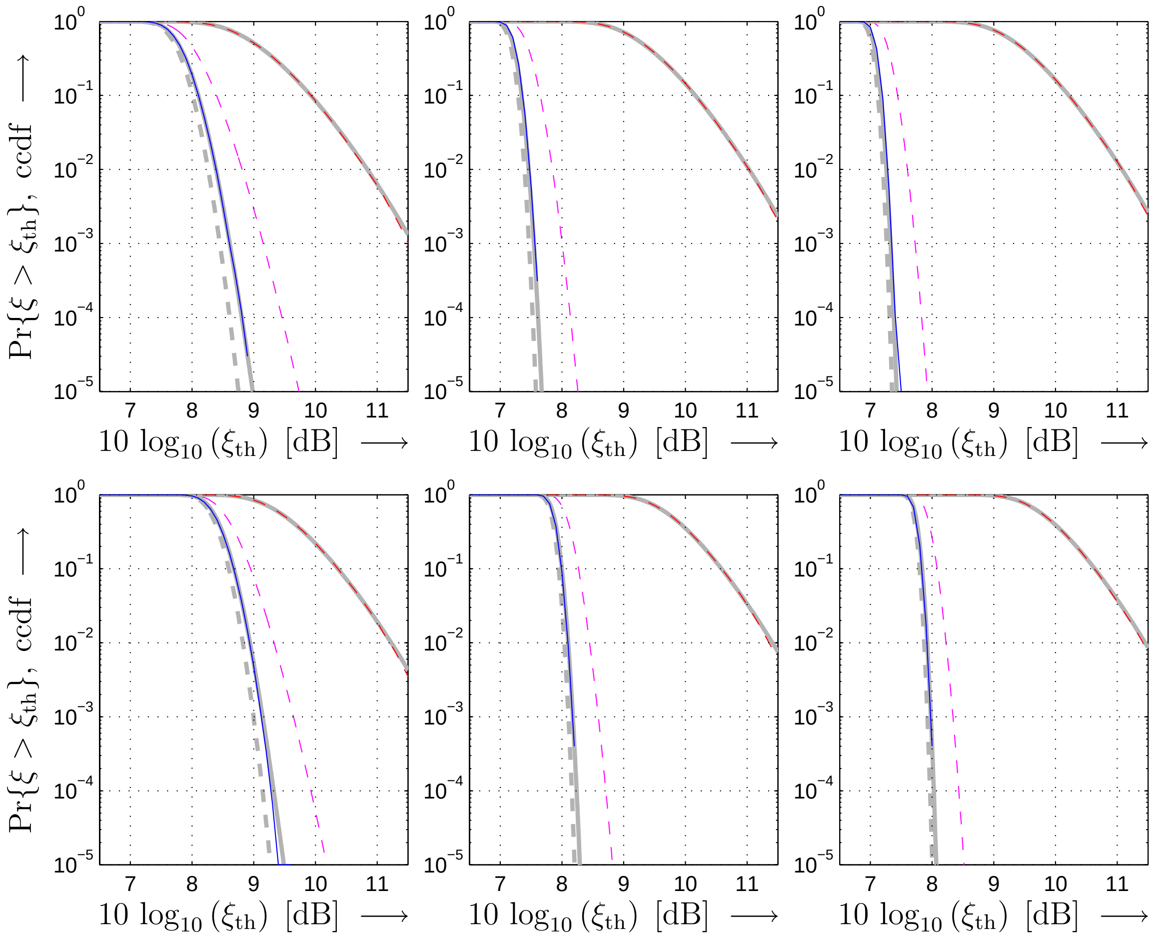

Fig. 8. Ccdf of PAR for the proposed scheme using simplex codes. $D = 512$, 4PSK. Top row: $I = 1$; bottom row: $I = 4$. $K = 4$, $n = 15$ (left), $K = 7$, $n = 127$ (middle), $K = 8$, $n = 255$ (right). Rightmost curve: OFDM without PAR reduction. Dash: parallel, independent application of SLM with $U = 4$ (left), $U = 18$ (middle), or $U = 32$ (right) candidates per antenna. Gray dashed: theory (8); Gray solid: theory (18).