

Pilot Symbol Assisted BPSK on Rayleigh Fading Channels with Diversity: Performance Analysis and Parameter Optimization

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Abstract—In this paper, the loss due to imperfect channel estimation is derived for pilot symbol assisted BPSK on fading channels with diversity. The presented approach, which holds for both narrowband and spread spectrum communication, further provides an analytical optimization of pilot symbol spacing. The loss due to channel estimation is demonstrated to be low if the maximum Doppler frequency is significantly less than the bit rate.

Keywords—Code-division multiple access, spread spectrum, phase-shift keying, channel estimation, convolutional coding, Rayleigh fading channel

I. INTRODUCTION

BINARY phase shift keying (BPSK) is an attractive modulation scheme for mobile radio due to its high power efficiency at moderate bit error rates (BERs). However, an appropriate pilot signal is required to achieve reliable transmission with coherent modulation schemes on fading channels.

Two attractive methods of using a pilot signal are pilot channel assisted BPSK (PCA-BPSK), and pilot symbol assisted BPSK (PSA-BPSK). PCA-BPSK has been proposed frequently for the downlink in spread spectrum multiple-access (SSMA) systems. A performance analysis is presented in [1]. PSA-BPSK has already been considered in [2,3,4,5]. In [2], bit error probability (BEP) of uncoded PSA-BPSK over frequency-nonselective Rayleigh fading channels is derived. The channel estimation filters are assumed as Wiener filters according to the Doppler spectrum. The parameter pilot symbol spacing is optimized by explicitly computing BEP for several values. In particular for direct sequence SSMA systems, PSA-BPSK with convolutional coding is discussed in [3,4,5]. However, power efficiency is estimated by means of simulations. The problem of optimizing pilot symbol spacing is not yet sufficiently solved. In [4], only an approximate solution is given.

In this paper, the loss due to imperfect channel estimation will be derived analytically. The presented approach further provides an optimization of the system parameter pilot symbol spacing. The analysis is valid for SS communication as well as for narrowband PSA-BPSK on fading channels with and without diversity. Moreover, the presented analysis can easily be generalized to ($M > 2$)-ary PSK.

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II. COMMUNICATION SYSTEM

The binary source (data) symbols with bit interval T_b are multiplexed in time with pilot symbols, cf. Fig. 1. The pilot symbols are a-priori known at receiver site. The pilot symbol spacing is denoted by M . Due to insertion of pilot symbols, the modulation interval T_s of the multiplexed symbols is given by

$$T_s = \frac{M-1}{M} T_b. \quad (1)$$

The mobile radio channel is modelled as a Rayleigh fading channel. Furthermore, we assume that diversity reception is possible. This is justified in narrowband systems using more than one antenna, or in SS systems if there are several resolvable propagation paths. In order to enable analytical analysis we assume the L diversity paths as statistically independent with same average power. For SS systems, this is reasonable if the differences between the path delays exceed the inverse of the signal bandwidth. Therefore, the discrete-time channel is characterized by L complex-valued Gaussian random processes with arbitrary spectrum bandlimited to the normalized Doppler frequency f_D , and L complex-valued white Gaussian noise processes with power density N_0 modelling the fading and the additive noise, respectively.

In Fig. 1, the channel estimation procedure is illustrated for one path of the equivalent diversity model. The demultiplexer separates data from pilot symbols. The latter are fed into low-pass filters generating path weight estimates which are required for maximum ratio combining. $M-1$ filters operating with pilot symbol frequency $1/(MT_s)$ are needed. In order to avoid aliasing effects, it is necessary to fulfill the sampling theorem. The corresponding condition $M \leq 1/(2f_D T_s)$ gives an upper bound on pilot symbol spacing. In practical implementations, the Doppler spectrum is not known in the receiver. Therefore, the channel estimation filters are designed to approximate a brick-wall magnitude response up to the maximum Doppler frequency, and linear phase.

In this paper, exclusively the loss due to imperfect estimation of path weights is considered. I.e., it is assumed that the receiver has perfect knowledge of the delays in the channel.

III. LOSS DUE TO IMPERFECT CHANNEL ESTIMATION

BEP of uncoded BPSK with perfect channel estimation on L -path Rayleigh fading channels is derived e.g. in [6, Chapter 7] as

$$P_b = \frac{1}{2} \left[1 - \varrho \sum_{\lambda=0}^{L-1} \binom{2\lambda}{\lambda} \left(\frac{1-\varrho^2}{4} \right)^\lambda \right] \quad (2)$$

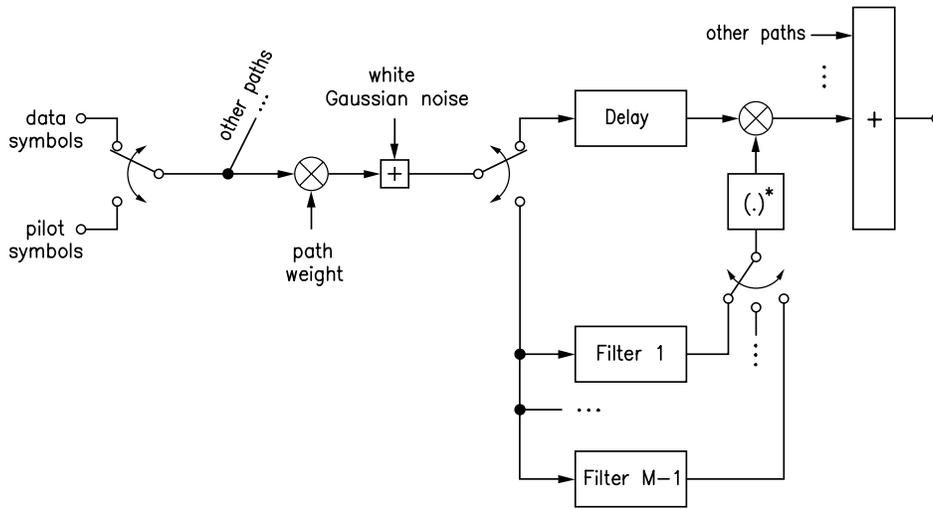


Fig. 1. Transmission model and channel estimation procedure for PSA-BPSK. For simplicity, only one propagation path is depicted.

Furthermore, it is shown in [6, Appendix 7A] that BEP for uncoded BPSK with imperfect channel estimation is also given by (2) if ϱ is defined as the crosscorrelation coefficient of the correlator output samples and the path weight estimates. This result is used in [2] to derive BEP for uncoded PSA-BPSK.

It should be remarked that this derivation of BEP for imperfect channel estimation is only exact if the path weights are not distorted by the channel estimation filters [1]. For the following analysis, this condition is assumed to be fulfilled. Following [6], the crosscorrelation coefficient can be given by

$$\varrho = \frac{1}{\sqrt{(1 + \text{SNR}^{-1})(1 + \text{SNR}'^{-1})}} \quad (3)$$

where SNR and SNR' denote the signal-to-noise ratio at the output of the correlators and at the output of the channel estimation filters, respectively. Each period comprising M transmitted symbols includes $M - 1$ data symbols. Thus,

$$\text{SNR} = \frac{M-1}{M} \gamma \triangleq \frac{M-1}{M} \cdot \frac{E_b}{LN_0} \quad (4)$$

with implicate definition of γ denoting signal-to-noise ratio per path. The channel estimation filters cause a reduction of noise power, but only one out of M symbols serves as pilot. The signal-to-noise ratio at filter output can be given by

$$\text{SNR}' = \frac{1}{M} \cdot \frac{\gamma}{\tilde{\sigma}_c^2}, \quad (5)$$

with $\tilde{\sigma}_c^2$ denoting the equivalent noise bandwidth (ENB) of the channel estimation filters normalized to the bit interval (NBI). The maximum noise suppression without any signal distortion would be possible by using ideal low-pass filters (brickwall magnitude response with cut-off frequency f_D) with appropriate phase. In this case, we get

$$\tilde{\sigma}_c^2 = 2f_D T_b = 2f_D T_s \frac{M}{M-1} \triangleq \sigma_c^2 \frac{M}{M-1} \quad (6)$$

with implicate definition of σ_c^2 denoting ENB normalized to the modulation interval (NMI). The two normalizations provide ENB to be independent of pilot symbol spacing if its respective time interval is given.

Inserting Eqs. (4) and (5) into (3) leads to

$$\varrho = \frac{1}{\sqrt{\left(1 + \frac{M}{M-1}\gamma\right) \left(1 + M\tilde{\sigma}_c^2/\gamma\right)}}. \quad (7)$$

BEP could be calculated by inserting (7) into (2). In this context however, we are particularly interested in the loss due to imperfect channel estimation, which will be derived analytically in the following.

BEP given in (2) is a monotonous function of the crosscorrelation coefficient ϱ [1]. Thus, the correlation coefficient of imperfect channel estimation

$$\varrho = \frac{1}{\sqrt{\left(1 + \frac{M}{M-1}\gamma\right) \left(1 + M\tilde{\sigma}_c^2/\gamma\right)}} \stackrel{!}{=} \frac{1}{\sqrt{1 + V/\gamma}}. \quad (8)$$

has to equal its corresponding counterpart for perfect channel estimation given in [6], with SNR decreased by power loss V . Involving (6) yields

$$\begin{aligned} V &= \frac{M}{M-1} \left(1 + (M-1)\tilde{\sigma}_c^2 + \frac{M\tilde{\sigma}_c^2}{\gamma} \right) \\ &= \frac{M}{M-1} \left(1 + M\sigma_c^2 + \frac{M^2}{M-1} \cdot \frac{\sigma_c^2}{\gamma} \right). \end{aligned} \quad (9)$$

Note that this is the loss in terms of signal-to-noise ratio γ referring to imperfect channel estimation. This may be inconvenient. But the optimum pilot symbol spacing is independent of V from this point of view, and allows insertion of this parameter, which is derived in the next section, into (9).

IV. OPTIMUM PILOT SYMBOL SPACING

The loss due to imperfect channel estimation derived above is a function of signal-to-noise ratio per path, ENB

of the channel estimation filters, and pilot symbol spacing. The latter is a system parameter that is to be chosen to minimize the loss. Our approach provides an analytical approach to this optimization problem.

In (6), ENB-NMI is independent of M if the modulation interval T_s is fixed, and ENB-NBI is independent of M if the bit interval T_b is given, respectively. Both methods may be of interest in practice. On the one hand, given bit interval means, that data rate is fixed and required bandwidth is increased by pilot symbols. On the other hand, given modulation interval means fixed signal bandwidth, e.g. restricted by regulation authorities. The latter implies that pilot symbols are inserted at the cost of lower data rate. In SS systems however, both strategies can be easily exchanged by adapting the processing gain.

Optimization for given bit interval is closely related to PCA-BPSK analyzed in [1], which becomes obvious by the following considerations: An additional pilot channel is introduced by allocating additional dimensions of the signal space. Hereby, it does not matter whether the additional pilot channel is orthogonal in code (PCA-BPSK) or in time (PSA-BPSK), as in both schemes the bit interval remains constant. The single slight difference in performance results from the fact that the pilot-to-data power ratio is continuous-valued for PCA-BPSK, while it is integer for PSA-BPSK given by the symbol spacing.

Due to the equivalence of PSA-BPSK for given bit interval and PCA-BPSK, the following considerations focus on the case of given modulation interval.

As mentioned before, pilot symbol spacing is integer. However, in order to allow analytical optimization of this parameter, it is assumed in the following that M is a real number. The optimum *integer* pilot symbol spacing cannot generally be calculated by rounding the following analytical results towards the nearest integer. Instead, both nearest integers greater and less than the results have to be checked. That one leading to the smaller loss is the optimum value.

In order to find the optimum pilot symbol spacing M_o , the first derivative of the power loss V given by the right hand side of (9) is set equal to zero. This leads to a cubic equation with the single meaningful solution

$$M_o = 1 + 2 \sqrt{\frac{\gamma (1 + 1/\sigma_c^2) / 3 + 1}{\gamma + 1}} \cos \left(\frac{1}{3} \arccos \sqrt{\frac{\gamma + 1}{(\gamma (1 + 1/\sigma_c^2) / 3 + 1)^3}} \right). \quad (10)$$

Note that for validity of (10) the condition established by sampling theorem, cf. Section 2, has to be fulfilled.

The analytical results derived above will be illustrated by some numerical examples. The optimum pilot symbol spacing is depicted in Fig. 2 as a function of ENB. It is steeply decreasing for low ENBs, and saturates at three, if ENB tends towards one. The minimum loss due to imperfect channel estimation is computed by inserting the optimum integer pilot symbol spacing into (9), see Fig. 3.

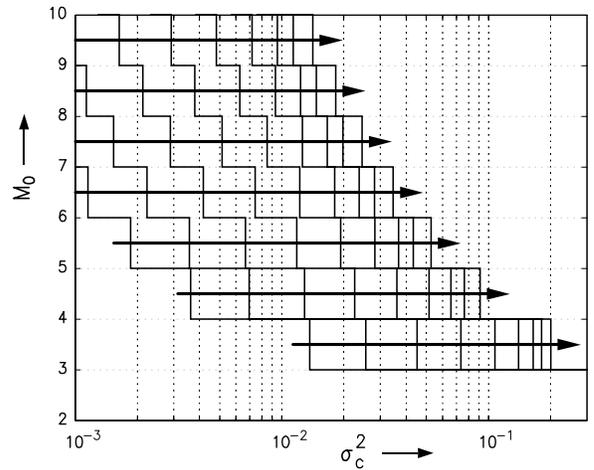


Fig. 2. Optimum integer pilot symbol spacing as a function of ENB-NMI; parameter: $10 \log_{10}(\gamma) = -15(3)6 \text{ dB}$, ∞ (following arrows)

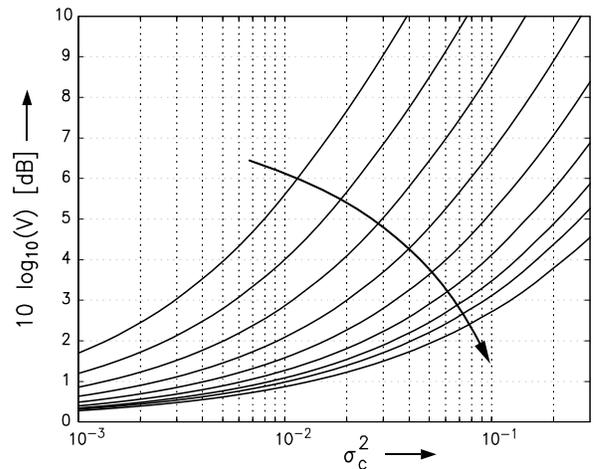


Fig. 3. Minimum loss due to imperfect channel estimation as a function of ENB-NMI; parameter: $10 \log_{10}(\gamma) = -15(3)6 \text{ dB}$, ∞ (following arrow)

The performance of PSA-BPSK appears to be very sensitive to variation of ENB. For ENB-NMI exceeding $1/3$, sampling theorem determines pilot symbol spacing which results in an additional loss.

V. SUMMARY AND CONCLUSIONS

In this paper, PSA-BPSK on Rayleigh fading channels with diversity is considered. The loss due to imperfect channel estimation is derived analytically. The presented approach provides analytical calculation of pilot symbol spacing. The loss due to imperfect channel estimation depends exclusively on the signal-to-noise ratio per propagation path, and the ENB of the channel estimation filters. Numerical examples show that the performance of PSA-BPSK is very sensitive to variation of ENB, and the impact of signal-to-noise ratio is less crucial.

The derivation of the loss due to imperfect channel estimation and the optimization of pilot symbol spacing are based on BEP calculated in [6]. Also in [6], symbol error probability of $(M > 2)$ -ary PSK with imperfect channel estimation is derived. The result is again a function of the

correlation coefficient given in (3). Therefore, the loss due to channel estimation is exactly the same for BPSK and ($M > 2$)-ary PSK, and the results presented in this paper can also be applied to the latter case.

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