Adaptive Linear Equalization Combined With Noncoherent Detection for MDPSK Signals[1]

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Abstract

A novel noncoherent receiver for M–ary differential phase shift–keying (MDPSK) signals transmitted over intersymbol interference (ISI) channels is presented. The noncoherent receiver consists of a linear equalizer and a decision–feedback differential detector. A significant performance gain over a previously proposed noncoherent receiver can be observed. For an infinite number of feedback symbols, the optimum equalizer coefficients can be calculated analytically and the performance of the proposed receiver approaches that of a coherent linear minimum mean–squared error (MMSE) equalizer. Moreover, a modified least–mean–square (LMS) and a modified recursive least–squares (RLS) algorithm for adaptation of the equalizer coefficients are discussed.

Keywords:

Decision–feedback differential detection (DF–DD), noncoherent detection, linear equalization, intersymbol interference, adaptive algorithms

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1 Introduction

The combination of linear or nonlinear equalization and coherent detection (CD) has been investigated extensively in literature and is standard in modern communication systems [1]. However, only few authors have studied the combination of equalization and noncoherent detection, although such receivers are less complex and more robust against carrier phase variations. Sehier et al. [2] proposed to combine a linear equalizer with conventional differential detection (DD), whereas Masoomzadeh–Fard et al. [3] and Jones et al. [4] considered decision–feedback equalization (DFE) together with conventional DD. All these schemes have in common, that they suffer from a significant loss in power efficiency compared to the coherent case. Here, we focus on linear equalization and it will be shown that the proposed technique can approach the performance of a linear equalizer combined with CD.

In principle, the proposed receiver is similar to that of [2], i.e., a linear equalizer is followed by a noncoherent detector. However, instead of simple DD, decision–feedback differential detection (DF–DD), previously introduced for noncoherent detection of M–ary differential phase shift–keying (MDPSK) transmitted over the additive white Gaussian noise (AWGN) channel, is applied. The reference symbol for DF–DD may be generated either nonrecursively [5, 6] or recursively [7, 8]. For adaptation of the equalizer coefficients modified least–mean–square (LMS) and recursive least–squares (RLS) algorithms are proposed.

This letter is organized as follows. In Section 2, the transmission model and the receiver structure are presented. The modified LMS and RLS algorithms for adaptation of the equalizer coefficients are given in Section 3. In Section 4, it is shown that the proposed noncoherent receiver approaches a coherent linear minimum mean–squared error (MMSE) equalizer if an infinite number of feedback symbols is used. Finally, in Section 5, the performance of the novel receiver is evaluated by computer simulations.

2 Transmission Model and Receiver Structure

Fig. 1 shows a block diagram of the discrete–time transmission model. All signals are represented by their complex–valued baseband equivalents. For simplicity, only T–spaced equalizers are considered here, however, it is straightforward to extend our results to fractionally–spaced equalizers. At the transmitter, the MDPSK symbols $a[\cdot] \in \mathcal{A} = \{\mathrm{e}^{j2\pi\nu/M} | \nu \in \{0, 1, \ldots, M-1\}\}$ are differentially encoded. The resulting MPSK symbols $b[\cdot]$ are given by

$$b[k] = a[k]b[k-1], \quad k \in \mathbb{Z}. \tag{1}$$

The discrete–time received signal, sampled at times kT at the output of the receiver input filter, can be expressed as

$$r[k] = \mathrm{e}^{j\Theta} \sum_{\nu=0}^{L_h-1} h_\nu b[k-\nu] + n[k]. \tag{2}$$

Here, Θ denotes an unknown, constant phase shift and h_ν, $0 \leq \nu \leq L_h - 1$, are the coefficients of the combined discrete–time impulse response of the cascade of continuous–time transmit filter, channel, and receiver input filter and T–spaced sampling; its length is denoted by L_h. We assume a square–root Nyquist frequency response for the receiver input filter, and thus, the zero mean complex Gaussian noise $n[\cdot]$ is white. Due to an appropriate normalization, the noise variance is $\sigma_n^2 = \mathcal{E}\{|n[k]|^2\} = N_0/E_S$, where $\mathcal{E}\{\cdot\}$ denotes expectation. E_S and N_0 are the mean received energy per symbol and the single–sided power spectral density of the underlying passband noise process, respectively. The equalizer output signal $q[k]$ may be written as

$$\begin{aligned} q[k] &= \sum_{\nu=0}^{L_c-1} c_\nu r[k-\nu] \\ &= \mathrm{e}^{j\Theta} g_{k_0} b[k-k_0] + \mathrm{e}^{j\Theta} \sum_{\substack{\nu=0 \\ \nu \neq k_0}}^{L_h+L_c-2} g_\nu b[k-\nu] + \sum_{\nu=0}^{L_c-1} c_\nu n[k-\nu], \end{aligned} \tag{3}$$

where c_ν are the equalizer coefficients and

$$g_\nu \triangleq \sum_{\mu=0}^{L_c-1} c_\mu h_{\nu-\mu} \tag{4}$$

are the coefficients of the discrete–time overall impulse response; L_c is the equalizer length. The decision delay k_0 which can significantly affect performance, especially for small L_c, should be optimized like in the coherent case (cf. e.g. [9]).

In the next stage of our receiver the decision variable $d[k]$ is obtained by noncoherent processing of $q[k]$. For this, the reference symbol $q_{\mathrm{ref}}[k-1]$ is generated. This might be done either nonrecursively (cf. [5, 6])

$$q_{\mathrm{ref}}[k-1] = \frac{1}{N-1} \sum_{\nu=1}^{N-1} q[k-\nu] \prod_{\mu=1}^{\nu-1} \hat{a}[k-k_0-\mu], \tag{5}$$

where N, $N \geq 2$, is the number of equalizer output symbols used for calculation of $d[k]$ (cf. Eqs. (5), (7)), or recursively (cf. [7, 8])

$$q_{\mathrm{ref}}[k-1] = (1-\alpha) q[k-1] + \alpha\, \hat{a}[k-k_0-1] q_{\mathrm{ref}}[k-2], \tag{6}$$

where α, $0 \leq \alpha < 1$, is a forgetting factor. $\hat{a}[\cdot] \in \mathcal{A}$ denotes the estimated symbols. The decision variable $d[k]$ is then obtained from

$$d[k] = q[k] q^*_{\mathrm{ref}}[k-1] \tag{7}$$

($(\cdot)^*$ denotes complex conjugation). Note, that for the special cases $N = 2$ and $\alpha = 0$, $q_{\rm ref}[k-1] = q[k-1]$ results, i.e., $d[k]$ (cf. Eq. (7)) is the decision variable of a conventional differential detector and the proposed receiver is identical with that of [2]. However, for $N > 2$ and $\alpha > 0$ a significant performance improvement can be observed as will be shown in Section 5.

For adjustment of the equalizer coefficients, the MMSE criterion is applied, i.e., the vector $\boldsymbol{c} \triangleq [c_0 \; c_1 \; \ldots \; c_{L_c-1}]^H$ ($[\cdot]^H$ denotes Hermitian transposition) of filter coefficients is tuned to minimize the error variance

$$\sigma_e^2 = \mathcal{E}\{|e[k]|^2\}, \tag{8}$$

where the error $e[k]$ is given by

$$e[k] = \hat{a}[k-k_0] - d[k]. \tag{9}$$

Here, $\hat{a}[k-k_0] = a[k-k_0]$ is assumed. Since the resulting error–performance surface is not quadratic, in general, it is not possible to derive a simple closed–form solution for $\boldsymbol{c}$ from Eq. (8). Thus, in the next section two adaptive algorithms will be provided which allow to find the desired filter settings. A closed–form solution will be given in Section 4 for the special cases $N \to \infty$ and $\alpha \to 1$.

3 Adaptive Algorithms for Equalizer Adjustment

In this section, a modified LMS and a modified RLS algorithm for adaptation of the equalizer coefficients are provided. Using Eqs. (3) and (7), Eq. (9) may be rewritten to

$$\begin{aligned} e[k] &= \hat{a}[k-k_0] - q[k]q^*_{\rm ref}[k-1] \\ &= \hat{a}[k-k_0] - \boldsymbol{c}^H[k]\boldsymbol{u}[k], \end{aligned} \tag{10}$$

with the definitions

$$\boldsymbol{c}[k] \triangleq [c_0[k] \; c_1[k] \; \ldots \; c_{L_c-1}[k]]^H, \tag{11}$$

$$\boldsymbol{u}[k] \triangleq \boldsymbol{r}[k]q^*_{\rm ref}[k-1], \tag{12}$$

$$\boldsymbol{r}[k] \triangleq [r[k] \; r[k-1] \; \ldots \; r[k-L_c+1]]^T \tag{13}$$

($[\cdot]^T$ denotes transposition). Note, that now $q[k]$ is given by

$$q[k] = \boldsymbol{c}^H[k]\boldsymbol{r}[k]. \tag{14}$$

For the modified LMS algorithm, the instantaneous gradient vector has to be calculated applying the method for complex differentiation described in [10][2] :

$$\frac{\partial}{\partial \boldsymbol{c}^*[k]} |e[k]|^2 = -e^*[k]\boldsymbol{u}[k]. \tag{15}$$

$\boldsymbol{u}[k]$ must be considered as a constant for differentiation with respect to $\boldsymbol{c}[k]$ since $q_{\mathrm{ref}}[k-1]$ only depends on $\boldsymbol{c}[k-\nu]$, $\nu \geq 1$ (cf. [2, 3]). The coefficient vector is updated according to

$$\boldsymbol{c}[k+1] = \boldsymbol{c}[k] + \mu e^*[k]\boldsymbol{u}[k], \tag{16}$$

with adaptation constant μ. Note, that for $N = 2$ and $\alpha = 0$, the proposed modified LMS algorithm is identical with that of [2]. Also, the proposed LMS algorithm is similar to the conventional LMS algorithm [10]. The only difference is that for the conventional LMS algorithm $\boldsymbol{r}[k]$ would have to be used in Eqs. (10) and (16) instead of $\boldsymbol{u}[k]$.

For derivation of the modified RLS algorithm, the cost function

$$J[k] = \sum_{\nu=1}^{k} w^{k-\nu} |e[k,\, \nu]|^2, \tag{17}$$

with forgetting factor w, $0 < w \leq 1$, and

$$e[k,\, \nu] = \hat{a}[\nu - k_0] - \boldsymbol{c}^H[k]\boldsymbol{u}[\nu], \tag{18}$$

has to be minimized. Using a similar procedure as proposed for the conventional RLS algorithm in [10] and for a modified RLS algorithm in [3], the following novel modified RLS algorithm can be obtained:

$$\boldsymbol{l}[k] = \frac{\boldsymbol{P}[k-1]\boldsymbol{u}[k]}{w + \boldsymbol{u}^H[k]\boldsymbol{P}[k-1]\boldsymbol{u}[k]}, \tag{19}$$

$$\xi[k] = \hat{a}[k-k_0] - \boldsymbol{c}^H[k-1]\boldsymbol{u}[k], \tag{20}$$

$$\boldsymbol{c}[k] = \boldsymbol{c}[k-1] + \boldsymbol{l}[k]\xi^*[k], \tag{21}$$

$$\boldsymbol{P}[k] = w^{-1}\boldsymbol{P}[k-1] - w^{-1}\boldsymbol{l}[k]\boldsymbol{u}^H[k]\boldsymbol{P}[k-1]. \tag{22}$$

$\boldsymbol{P}[k]$ is initialized by

$$\boldsymbol{P}[0] = \delta^{-1}\boldsymbol{I}, \tag{23}$$

where δ is a small positive constant (typical value: 0.004) and $\boldsymbol{I}$ is the $L_c \times L_c$ identity matrix. Note, that for the special cases $N = 2$ and $\alpha = 0$ the proposed modified RLS algorithm is identical with that of [3]. In addition, the conventional RLS algorithm [10] results if in Eqs. (19)–(22) $\boldsymbol{u}[k]$ is replaced by $\boldsymbol{r}[k]$.

[2]Note, that the vector $\boldsymbol{c}[k]$ contains the complex conjugated filter coefficients (cf. Eq. (11)) as it is customary in literature, cf. e.g. [10].

For the conventional LMS and RLS algorithm it is customary to initialize the equalizer coefficients with the all zero vector, i.e., $\boldsymbol{c}[0] = \mathbf{0}$. This is not possible here, since in this case $\boldsymbol{u}[0] = \mathbf{0}$ is obtained from Eqs. (3), (5), (6), and (12). Hence, $\boldsymbol{c}[k] = \mathbf{0}$, $k \geq 0$, follows. Similar to [3], we propose to initialize the equalizer coefficients with $c_{k_0}[0] = 0.1$ and $c_\nu[0] = 0$, $0 \leq \nu \leq L_c - 1$, $\nu \neq k_0$. With this initial setting, in our simulations, both algorithms always converged to the desired solution for all channels investigated.

4 Performance for $N \to \infty$ and $\alpha \to 1$

Using Eqs. (5) and (6), it can be shown that the nonrecursive scheme and the recursive scheme are identical for $N \to \infty$ and $\alpha \to 1$. Thus, it suffices to consider the nonrecursive scheme. In the following, two different approaches for determination of the filter coefficients will be used. First, the steady–state solutions of the adaptive algorithms proposed in Section 3 will be determined. The second approach is to calculate the optimum filter settings directly by minimizing σ_e^2 (cf. Eq. (8)).

For coherent equalizers it is obvious, that both approaches lead to the same result since in this case the error $e[k]$ at time k depends only on the current filter coefficient vector $\boldsymbol{c}[k]$. Here, however, $e[k]$ at time k also depends on past coefficient vectors $\boldsymbol{c}[k-\nu]$, $\nu \geq 1$. For derivation of the adaptive algorithms, these past coefficient vectors have to be treated as constants since $|e[k]|^2$ has to be differentiated only with respect to $\boldsymbol{c}[k]$ (cf. Section 3). On the other hand, calculation of the optimum coefficient vector $\boldsymbol{c}$, $\boldsymbol{c}[k-\nu] = \boldsymbol{c}$, $\forall\, \nu$, by means of differentiation of σ_e^2 may lead to a different result.

4.1 Steady–State Solution of the Adaptive Algorithms

Under steady–state conditions $\mathcal{E}\{\boldsymbol{c}[k]\} = \mathcal{E}\{\boldsymbol{c}[k-1]\} = \boldsymbol{c}$ holds. Thus, from Eq. (16), the condition

$$\mathcal{E}\{e^*[k]\boldsymbol{u}[k]\} = \mathbf{0} \tag{24}$$

is obtained (a similar condition can be obtained for the modified RLS algorithm using Eqs. (19)–(22) and $w = 1$). Since all stochastic processes involved are mutually independent and $a[\cdot]$ is an independent and identically distributed (i.i.d.) sequence, in the limit $N \to \infty$, Eq. (5) reduces to

$$q_{\mathrm{ref}}[k-1] = \mathrm{e}^{j\Theta} g_{k_0} b[k-k_0-1], \tag{25}$$

where Eq. (3) is applied. Note, that Eq. (25) holds in the mean–squared sense and here, error–free feedback, i.e., $\hat{a}[\cdot] = a[\cdot]$, is assumed. Using Eqs. (10), (12), and (25), from Eq. (24)

$$|g_{k_0}|^2 \boldsymbol{\Phi}\boldsymbol{c} = \mathrm{e}^{-j\Theta} g_{k_0}^* \boldsymbol{\varphi} \tag{26}$$

can be obtained, where the definitions

$$\boldsymbol{\Phi} \stackrel{\triangle}{=} \mathcal{E}\{\boldsymbol{r}[k]\boldsymbol{r}^H[k]\}, \tag{27}$$

$$\boldsymbol{\varphi} \stackrel{\triangle}{=} \mathcal{E}\{\boldsymbol{r}[k]b^*[k-k_0]\} = \mathrm{e}^{j\Theta}[h_{k_0}\ h_{k_0-1}\ \dots\ h_{k_0-L_c+1}]^T, \tag{28}$$

are used. Note, that $h_\nu = 0$ for $\nu < 0$ or $\nu > L_h - 1$. From Eq. (4)

$$g_{k_0} = \mathrm{e}^{-j\Theta}\boldsymbol{c}^H\boldsymbol{\varphi} \tag{29}$$

follows. Thus, if the trivial solution $\boldsymbol{\varphi}^H\boldsymbol{c} = 0$ is excluded[3],

$$\boldsymbol{\Phi}\boldsymbol{c}\boldsymbol{c}^H\boldsymbol{\varphi} = \boldsymbol{\varphi} \tag{30}$$

results from Eq. (26). From Eqs. (29) and (30), $|g_{k_0}|^2 = \boldsymbol{\varphi}^H\boldsymbol{c}\boldsymbol{c}^H\boldsymbol{\varphi} = \boldsymbol{\varphi}^H\boldsymbol{\Phi}^{-1}\boldsymbol{\varphi}$ is obtained. Thus, g_{k_0} is given by

$$g_{k_0} = \sqrt{\boldsymbol{\varphi}^H\boldsymbol{\Phi}^{-1}\boldsymbol{\varphi}}\,\mathrm{e}^{j\phi_0}, \tag{31}$$

where ϕ_0 is an arbitrary phase. Using Eqs. (29), (30), and (31), the equalizer coefficient vector can be calculated to

$$\boldsymbol{c} = \frac{\boldsymbol{\Phi}^{-1}\boldsymbol{\varphi}}{\sqrt{\boldsymbol{\varphi}^H\boldsymbol{\Phi}^{-1}\boldsymbol{\varphi}}}\,\mathrm{e}^{j\phi_1}, \tag{32}$$

where $\phi_1 = -\Theta - \phi_0$ is an arbitrary phase since ϕ_0 is arbitrary, i.e., $\boldsymbol{c}$ is unique only up to a complex factor with magnitude one which is obvious for a noncoherent receiver. Note, that the coefficients of a linear MMSE equalizer combined with CD are given by $\boldsymbol{c} = \boldsymbol{\Phi}^{-1}\boldsymbol{\varphi}$ [1]. This shows, that besides the arbitrary phase, the magnitude of the coefficients of the proposed noncoherent equalizer is scaled differently.

4.2 Optimum Solution

Now, the optimum filter settings are determined from Eq. (8). Using Eqs. (10) (note, that $\boldsymbol{c}[k]$ has to be replaced by $\boldsymbol{c}$), (12), (25), and (27)–(29), Eq. (8) can be rewritten to

$$\sigma_e^2 = 1 - 2\boldsymbol{c}^H\boldsymbol{\varphi}\boldsymbol{\varphi}^H\boldsymbol{c} + \boldsymbol{c}^H\boldsymbol{\varphi}\boldsymbol{c}^H\boldsymbol{\Phi}\boldsymbol{c}\boldsymbol{\varphi}^H\boldsymbol{c}. \tag{33}$$

Thus, the optimum filter settings are determined from

$$\frac{\partial}{\partial\boldsymbol{c}^*}\sigma_e^2 = \boldsymbol{\varphi}^H\boldsymbol{c}(-2\boldsymbol{\varphi} + \boldsymbol{\varphi}\boldsymbol{c}^H\boldsymbol{\Phi}\boldsymbol{c} + \boldsymbol{c}^H\boldsymbol{\varphi}\boldsymbol{\Phi}\boldsymbol{c}) = \boldsymbol{0}. \tag{34}$$

As in Section 4.1, the trivial solution $\boldsymbol{\varphi}^H\boldsymbol{c} = 0$ corresponds to a maximum ($\sigma_e^2 = 1$) and is not considered in the following. With the definitions $p \stackrel{\triangle}{=} \boldsymbol{c}^H\boldsymbol{\Phi}\boldsymbol{c}$ and $g \stackrel{\triangle}{=} \mathrm{e}^{-j\Theta}\boldsymbol{c}^H\boldsymbol{\varphi}$,

$$\boldsymbol{c} = \frac{2-p}{\mathrm{e}^{j\Theta}g}\boldsymbol{\Phi}^{-1}\boldsymbol{\varphi} \stackrel{\triangle}{=} x\,\boldsymbol{\Phi}^{-1}\boldsymbol{\varphi} \tag{35}$$

[3]Note, that $\boldsymbol{\varphi}^H\boldsymbol{c} = 0$ corresponds to a maximum with $\sigma_e^2 = 1$ (cf. Eq. (33)).

is obtained from Eq. (34). $|x|^2$ can be determined by inserting $\boldsymbol{c}$ from Eq. (35) in Eq. (34). This leads to

$$|x|^2 = \frac{1}{\boldsymbol{\varphi}^H \boldsymbol{\Phi}^{-1} \boldsymbol{\varphi}}. \tag{36}$$

Taking into account, that the phase of x is arbitrary, Eq. (32) results from Eqs. (35) and (36), i.e., the filter settings determined by the adaptive algorithms coincide with the optimum solution.

Using Eqs. (3), (8), (10), (12), (25), (26), (29), and (31),

$$\sigma_e^2 = 1 - |g_{k_0}|^2 = 1 - \boldsymbol{\varphi}^H \boldsymbol{\Phi}^{-1} \boldsymbol{\varphi} \tag{37}$$

is obtained, i.e., the error variance is the same as for a coherent linear MMSE equalizer (cf. [1]). Therefore, for error–free feedback and $N \rightarrow \infty$ ($\alpha \rightarrow 1$) the noncoherent receiver for MDPSK approaches the performance of a coherent receiver for MPSK (without differential encoding). If previously detected symbols are fed back, the proposed noncoherent receiver approaches the performance of a coherent receiver for MDPSK, where $b[k-k_0]$ is estimated and differential encoding is inverted subsequently. This will be shown by computer simulations in the next section.

Now, we show another interesting property of the proposed receiver. For this we define the SNR of the noncoherent MMSE equalizer as

$$\mathrm{SNR}_{\mathrm{MMSE}} \triangleq \frac{\mathcal{E}\{|a[k-k_0]|^2\}}{\mathcal{E}\{|e[k]|^2\}} = \frac{1}{\sigma_e^2} \tag{38}$$

and rewrite the decision variable $d[k]$ as [11]

$$d[k] = \text{bias} \cdot a[k-k_0] + \text{ISI term} + \text{noise term}. \tag{39}$$

From Eqs. (3), (7), and (25), it is obvious that for $N \rightarrow \infty$ the bias is given by

$$\text{bias} = |g_{k_0}|^2. \tag{40}$$

Therefore, the SNR of the (biased) MMSE equalizer and the bias are related by

$$\frac{\mathrm{SNR}_{\mathrm{MMSE}} - 1}{\mathrm{SNR}_{\mathrm{MMSE}}} = \text{bias}, \tag{41}$$

where Eqs. (37), (38), and (40) are used. Note, that for the coherent linear MMSE equalizer the same relation is valid, however, in this case the bias is different, of course [11, 12]. Like in the coherent case, we could remove the bias and the resulting SNR would be $\mathrm{SNR}_{\mathrm{unbiased}} = \mathrm{SNR}_{\mathrm{MMSE}} - 1$ [12]. However, since no amplitude information is used for the MDPSK decision rule, the bit error rate cannot be improved by removing the bias.

5 Simulation Results

In this section, the performance of our noncoherent receiver is evaluated by computer simulations. For all simulations, the same channel and equalizer length as in [2] is used, i.e., $h_0 = h_2 = 0.304$, $h_1 = 0.903$, $L_c = 7$. In addition, k_0 is always set to 3 since this value yields the best performance. However, instead of BDPSK ($M = 2$) like in [2], QDPSK ($M = 4$) is discussed because of the higher bandwidth efficiency and more significant performance difference between conventional DD and CD for $M = 4$. We compare the performance of our receiver with that of a coherent receiver for MDPSK. The coherent receiver consists of a linear equalizer followed by a coherent detector, which determines an estimate $\hat{b}[k - k_0]$ for the MPSK symbol $b[k - k_0]$. The estimate for the MDPSK symbol $a[k - k_0]$ is obtained subsequently by differential decoding ($\hat{a}[k - k_0] = \hat{b}[k - k_0]\hat{b}^*[k - k_0 - 1]$).

In Figs. 2a) and b), the learning curves [10] for the proposed modified LMS and RLS algorithms, respectively, are given. A training sequence is used and the nonrecursive receiver is applied. For the LMS algorithm ($\mu = 0.005$), $J'[k] \stackrel{\triangle}{=} \mathcal{E}\{|e[k]|^2\}$, whereas $J'[k] \stackrel{\triangle}{=} \mathcal{E}\{|\xi[k]|^2\}$ is valid for the RLS algorithm ($w = 1.0$). Note, that in both cases the averaging was done over 1000 adaptation processes, $\boldsymbol{c}[\cdot]$ was initialized with $c_{k_0}[0] = 0.1$ and $c_\nu[0] = 0$, $\nu \neq k_0$, and $10\log_{10}(E_b/N_0) = 12$ dB ($E_b = E_S/2$ is the mean received energy per bit) is valid. As in the coherent case, the RLS algorithm converges faster than the LMS algorithm. In the steady state, $J'[k]$ decreases with increasing N for both modified algorithms and is lower bounded by the coherent case. In Fig. 2a) it can be observed that the convergence speed of the modified LMS algorithm increases considerably with increasing N. This might be attributed to the fact that the reference symbol $q_{\mathrm{ref}}[k-1]$ is the less noisy the larger N (cf. Eq. (5)). For $N = 5$ and $N = 10$, the algorithm converges even faster than the conventional LMS algorithm of the coherent receiver since in our noncoherent receiver the phase of $\boldsymbol{c}$ is arbitrary and has not to be adjusted whereas it is unique in the coherent case. In Fig. 2b), however, it can be seen that the convergence speed of the modified RLS algorithm is lower for $N = 10$ than for $N = 5$. Here, another effect has to be taken into account. Since the RLS algorithm converges much faster than the LMS algorithm, $c_\nu[k]$, $0 \leq \nu \leq L_c - 1$, changes considerably in the interval $[k,\ k+N-1]$ if N is large and steady state is not yet reached. Therefore, in this case the reference symbol $q_{\mathrm{ref}}[k - 1]$ becomes more noisy if N is too large (cf. Eq. (5)) and the convergence speed decreases. For the same reason it can be expected that the convergence speed of the modified LMS algorithm will also decrease if N is chosen too large. It can also be seen, that the convergence speed of the conventional RLS algorithm combined with CD is always higher than that of the novel noncoherent scheme.

For the following simulations, the modified LMS algorithm and the conventional LMS are used for the noncoherent and the coherent receiver, respectively. In both cases $\mu = 0.001$ is chosen. After transmission of a training sequence, a transition to the decision–directed mode occurs.

In Figs. 3a) and b), BER in steady state vs. $10\log_{10}(E_b/N_0)$ is shown for the proposed noncoherent receiver with nonrecursively and recursively generated reference symbol $q_{\mathrm{ref}}[k-1]$, respectively. Clearly, the receiver proposed in [2] ($N = 2$, $\alpha = 0.0$) can be improved considerably by the technique proposed here. For $N \gg 1$ and $\alpha \to 1$, the performance of the coherent receiver is approached. The presented results demonstrate, that the algorithm always converges to a 'reasonable' solution also for $N < \infty$, $\alpha < 1$. However, no analytical proof for the absence of spurious local minima in the error performance surface could be obtained so far.

For Figs. 2 and 3, an unknown but constant phase is assumed. However, practical receivers often have to cope with frequency offset. In Figs. 4a) and b), BER vs. normalized frequency offset ΔfT is shown for the nonrecursive ($10\log_{10}(E_b/N_0) = 12$ dB) and the recursive ($10\log_{10}(E_b/N_0) = 14$ dB) scheme, respectively. Obviously both noncoherent receiver structures are robust against frequency offset. Robustness increases with decreasing N (α). Thus, as typical for noncoherent receivers, there is a trade–off between performance under pure AWGN conditions and robustness against frequency offset. This has to be taken into account for receiver design in a practical application. Note, that a coherent receiver applying the simple LMS algorithm degrades severely even for frequency offsets as small as $\Delta fT = 0.0005$.

6 Conclusions

Two novel noncoherent receiver structures have been proposed in this work. They consist of a linear equalizer and either a nonrecursive or a recursive DF–DD scheme. Both noncoherent receiver structures can approach the performance of a comparable coherent receiver if an infinite number of feedback symbols is used as has been shown analytically. So far, no other combination of an equalizer and a noncoherent detector has been reported in literature, which can approach the performance of a corresponding coherent receiver. Computer simulations confirm that even for a finite number of feedback symbols the proposed receivers perform considerably better than a comparable state–of–the–art scheme. For adaptation of the equalizer coefficients a modified LMS algorithm with low complexity and a fast converging modified RLS algorithm are provided.

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Figures:

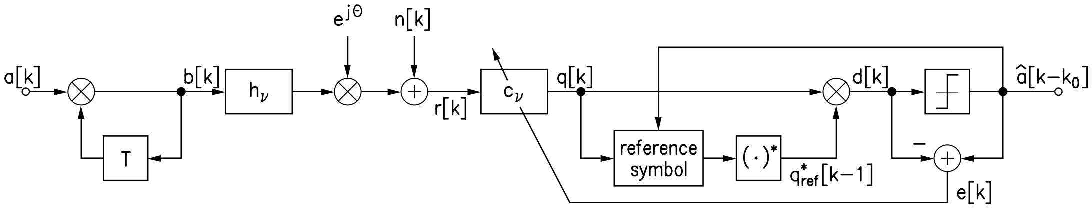

Figure 1: Block diagram of the discrete–time transmission model under consideration with linear equalization combined with noncoherent DF–DD.

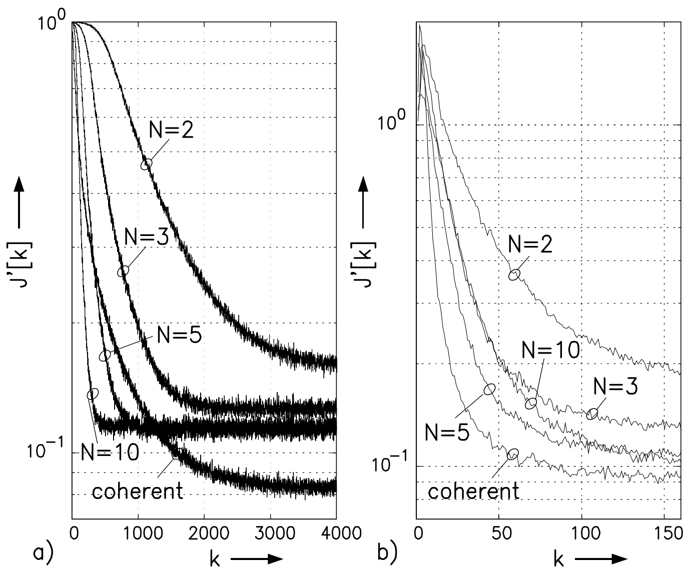

Figure 2: Learning curves for a) modified LMS and conventional (coherent) LMS algorithm; b) modified RLS and conventional (coherent) RLS algorithm.

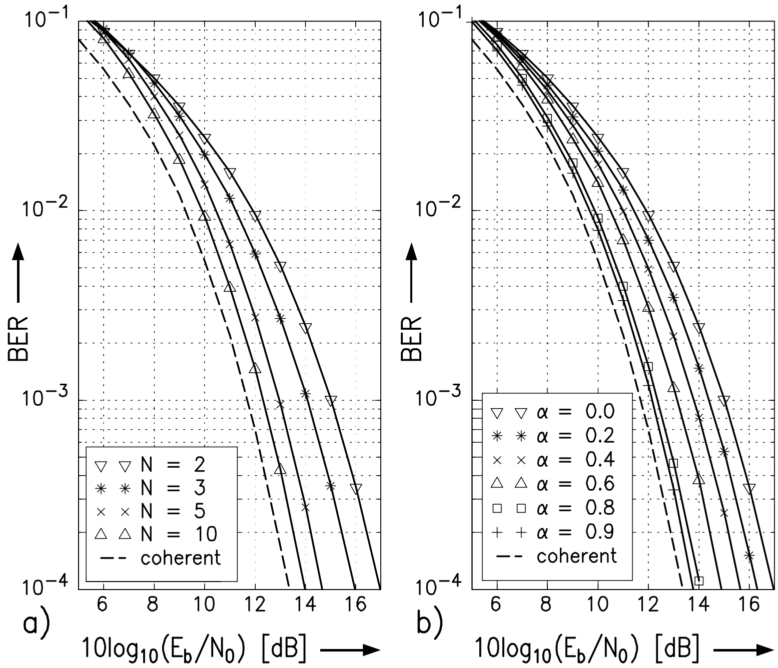

Figure 3: BER vs. $10\log_{10}(E_b/N_0)$ for the noncoherent receiver with a) nonrecursively and b) recursively generated reference symbol $q_{\mathrm{ref}}[k-1]$. For comparison the BER for a coherent receiver is also shown.

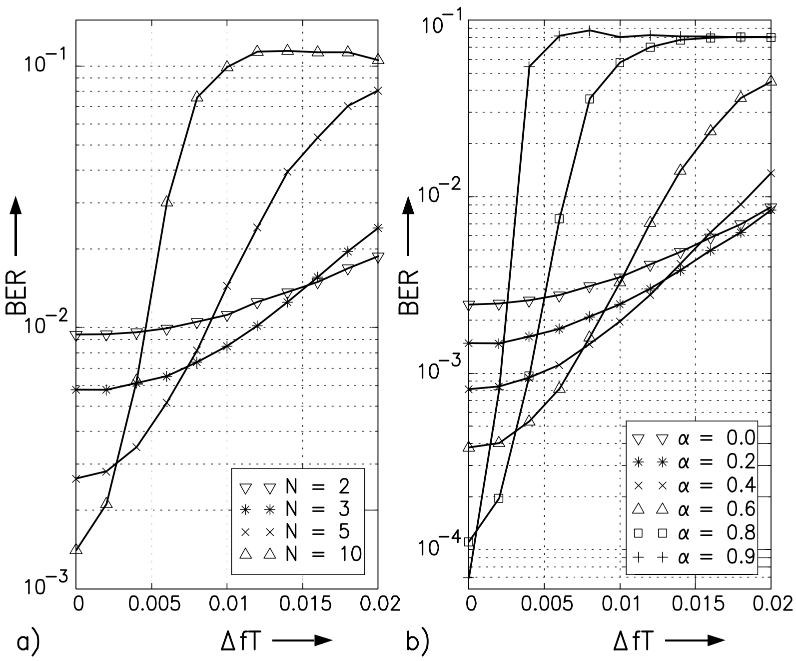

Figure 4: BER vs. ΔfT for the noncoherent receiver with a) nonrecursively ($10\log_{10}(E_b/N_0) = 12$ dB) and b) recursively ($10\log_{10}(E_b/N_0) = 14$ dB) generated reference symbol $q_{\mathrm{ref}}[k-1]$.