Analytical Results on the Statistical Distribution of the Zeros of Mobile Channels

Wolfgang H. Gerstacker and Robert Schober

Lehrstuhl für Nachrichtentechnik II
Universität Erlangen-Nürnberg
Cauerstraße 7/NT, D-91058 Erlangen, Germany
E-mail: gersta@LNT.de

ABSTRACT

In this paper, the distribution of zeros of mobile channels is investigated and the results obtained are applied to the GSM (Global System for Mobile Communications)/EDGE (Enhanced Data Rates for GSM Evolution) system. The taps of the discrete–time overall impulse response can be modeled as correlated complex Gaussian random variables, where the correlations depend on the transmit filter, the power delay profile of the channel, and the receiver input filter. For calculation of the density of zeros of the overall transfer function a result from the mathematical literature on the zeros of polynomials with correlated Gaussian coefficients is used. From this density two cumulative distributions which are relevant for the design of suboptimum receivers are derived and specialized to the case of uncorrelated taps. Finally, practical equalizer design rules for the GSM/EDGE system are deduced from the calculated statistical distributions.

1. INTRODUCTION

Typically, for the design of digital receivers the discrete–time overall channel is modeled as a finite impulse response (FIR) filter resulting from symbol–spaced sampling of the continuous–time receiver input signal. Here, the discrete–time overall impulse response depends on the impulse response of transmit filter, continuous–time equivalent baseband channel, and receiver input filter. As long as the overall channel is flat, receiver design is relatively simple, however, for frequency–selective channels sophisticated equalizers are necessary. If optimum maximum–likelihood sequence estimation (MLSE) [1] employing a full–state Viterbi algorithm (VA) is used in the receiver, the minimum Euclidean distance associated with the discrete–time impulse response is the most important parameter for receiver performance. However, in many situations where the allowable computational complexity is limited which is e.g. the case in mobile communications [2, 3] suboptimum equalization strategies such as delayed decision–feedback sequence estimation (DDFSE) [4] or reduced–state sequence estimation (RSSE) [5], impulse response truncation using a prefilter before the VA, e.g. [6, 7], decision–feedback equalization (DFE) [8], or even linear equalization (LE) have to be employed. This is especially the case for third generation (3G) mobile communication systems, where high–level modulation is used in order to achieve an increased spectral efficiency, e.g. the EDGE (Enhanced Data Rates for GSM Evolution) system [9]. For such systems, MLSE receivers would require a VA with an excessive number of states.

For suboptimum receivers the location of the zeros of the z–transform of the discrete–time overall channel impulse response is very important. For example, LE does not perform well if zeros are located close to the unit circle of the complex plane, whereas DFE and DDFSE/RSSE degrade if zeros are located outside the unit circle, i.e., if the discrete–time impulse response is not minimum–phase.

For static channels, which may be found e.g. in cable transmission, the zeros of the impulse response are known in advance and the receiver can be designed accordingly. In mobile communications, however, the channel is time–variant due to the motion of the mobile station, i.e., the overall impulse response is not known a priori. In many cases, the coefficients of the resulting discrete–time overall impulse response can be modeled as correlated Gaussian random variables. Clearly, for mobile channels the location of the zeros of the overall transfer function is also random and the receiver cannot be designed for one particular channel. However, the *distribution* of the zeros of the overall impulse response which is influenced by the transmit filter, the power delay profile of the mobile channel, the receiver input filter, and the sampling phase, can provide important information for receiver design.

In Section 2, the covariance matrix of the impulse response coefficients is calculated as a function of transmit filter, power delay profile, receiver input filter, and sampling phase. In Section 3, the density of zeros in the complex plane is derived for an overall transfer function whose coefficients are zero–mean correlated complex Gaussian random variables. Two cumulative distribution functions are derived from the density of zeros and discussed in Section 4. In Section 5, some results are shown for the power delay profiles specified for GSM and EDGE and used for the formulation of equalizer design rules.

2. THE CORRELATION MATRIX OF THE COEFFICIENTS OF THE OVERALL IMPULSE RESPONSE

A block diagram of the equivalent baseband continuous–time channel model is shown in Fig. 1. Here, $h_T(t)$ and $h_R(t)$ denote the transmit and the receiver input filter impulse response, respectively. $h_C(t)$ is the (random) equivalent baseband channel impulse response. Usually, $h_C(t)$ is characterized by its power delay profile

$$P(t) = \sum_{\nu=0}^{N-1} \sigma_\nu^2 \, \delta(t - \tau_\nu), \tag{1}$$

where N, σ_ν^2, and τ_ν, $0 \le \nu \le N-1$, denote the number of different paths, the variance and the delay of path ν, respectively. $\delta(\cdot)$ is the Dirac delta function. $h_C(t)$ may be modeled as

$$h_C(t) = \sum_{\nu=0}^{N-1} g_\nu \, \delta(t - \tau_\nu), \tag{2}$$

with mutually independent zero–mean coefficients g_ν, $0 \le \nu \le N-1$, i.e.,

$$\mathcal{E}\{g_\nu g_\mu^*\} = \begin{cases} \sigma_\nu^2, & \nu = \mu, \\ 0, & \nu \neq \mu \end{cases} \tag{3}$$

($\mathcal{E}\{\cdot\}$ and $(\cdot)^*$ denote expectation and complex conjugation, respectively). In order to avoid mathematical problems the coefficients g_ν are assumed to be time–invariant. Such a model is valid for example for burst transmission and low vehicle speeds. In this case, g_ν is approximately constant during one burst but varies from burst to burst. The overall continuous–time impulse response $h(t)$ is given by

$$h(t) \triangleq h_T(t) * h_C(t) * h_R(t), \tag{4}$$

where $*$ denotes convolution. Using the combined impulse response of transmit and receiver input filter

$$h_G(t) \triangleq h_T(t) * h_R(t), \tag{5}$$

the overall impulse response $h(t)$ can be calculated to

$$\begin{aligned} h(t) &= h_G(t) * h_C(t) \\ &= \sum_{\nu=0}^{N-1} g_\nu \, h_G(t - \tau_\nu). \end{aligned} \tag{6}$$

The received signal is sampled at times $kT + t_0$, where $k \in \mathbb{Z}$, T, and t_0 denote discrete time, symbol duration, and sampling phase, respectively. The discrete–time overall impulse response $h[\mu, t_0]$ ($\mu \in \mathbb{Z}$ denotes the coefficient number) can be written as

$$\begin{aligned} h[\mu, t_0] &\triangleq h(\mu T + t_0) \\ &= \sum_{\nu=0}^{N-1} g_\nu \, h_G(\mu T + t_0 - \tau_\nu). \end{aligned} \tag{7}$$

Obviously, the discrete–time overall impulse response depends on the sampling phase t_0 which has to be chosen properly. We propose to choose t_0 for maximization of the average energy of $h[\mu, t_0]$, $0 \le \mu \le L-1$, with L such that pre- and postcursor intersymbol interference ($h[\mu, t_0]$ for $\mu < 0$ and $\mu > L-1$) can be neglected. Then, $h[\cdot, t_0]$ can be truncated to finite length L.

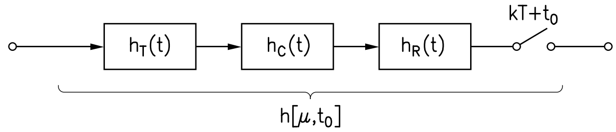

Figure 1: Block diagram of the equivalent baseband continuous–time channel model.

The overall transfer function $H(z, t_0)$, $z \in \mathbb{C}$, i.e., the z–transform of the discrete–time overall impulse response, is given by

$$\begin{aligned} H(z, t_0) &= \sum_{\nu=0}^{L-1} h[\nu, t_0] z^{-\nu} \\ &= z^{-(L-1)} \sum_{\nu=0}^{L-1} h[L-1-\nu, t_0] z^{\nu} \\ &= z^{-(L-1)} \, \boldsymbol{h}^T(t_0) \boldsymbol{v}(z), \end{aligned} \tag{8}$$

where $(\cdot)^T$ denotes transposition and the definitions

$$\boldsymbol{h}(t_0) \triangleq [h[L-1, t_0] \; h[L-2, t_0] \; \dots \; h[0, t_0]]^T \tag{9}$$

$$\boldsymbol{v}(z) \triangleq [1 \; z \; \dots \; z^{L-1}]^T \tag{10}$$

are used.

For the following, the covariance matrix

$$\boldsymbol{C} \triangleq \mathcal{E}\{\boldsymbol{h}(t_0) \, \boldsymbol{h}^H(t_0)\} \tag{11}$$

($(\cdot)^H$ denotes Hermitian transposition) of coefficient vector $\boldsymbol{h}(t_0)$ which has zero mean is required. Using Eqs. (3), (7), and (11), the elements $c_{\mu\nu}$, $0 \le \mu, \nu \le L-1$, of $\boldsymbol{C}$ can be calculated to

$$\begin{aligned} c_{\mu\nu} &\triangleq \mathcal{E}\{h[L-1-\mu, t_0] h^*[L-1-\nu, t_0]\} \\ &= \sum_{\xi=0}^{N-1} \sigma_\xi^2 \, h_G((L-1-\mu)T + t_0 - \tau_\xi) \cdot \\ &\qquad h_G^*((L-1-\nu)T + t_0 - \tau_\xi). \end{aligned} \tag{12}$$

3. THE DENSITY OF ZEROS OF THE OVERALL TRANSFER FUNCTION

In [10] the more general problem of calculating the distribution of the zeros of a system of equations with zero–mean coefficient matrix is addressed. If we specialize Theorem 8.1 of [10] to the problem at hand, the expected number $N_{\mathcal{U}}$ of zeros of the overall transfer function $H(z, t_0)$ that lie in the set (region) $\mathcal{U}$ is

$$N_{\mathcal{U}} = \int_{\mathcal{U}} f_z(z) \, \mathrm{d}x \, \mathrm{d}y, \tag{13}$$

where x and y denote the real and the imaginary part of $z = x + jy$, respectively, while

$$f_z(z) \triangleq \frac{1}{\pi} \frac{\partial^2}{\partial z \, \partial z^*} \log\left(\boldsymbol{v}(z)^T \boldsymbol{C} \boldsymbol{v}(z^*)\right) \tag{14}$$

is referred to as the *density* of zeros in the complex plane [10]. Note that $\log(\boldsymbol{v}(z)^T \boldsymbol{C} \boldsymbol{v}(z^*))$ is not a holomorphic function, of course. For the density $f_z(z) \ge$

$0, \forall z$, is valid and thus, $\log(\boldsymbol{v}(z)^T \boldsymbol{C} \boldsymbol{v}(z^*))$ is a subharmonic function [11].

$f_z(z)$ may be calculated to [12]

$$f_z(z) \triangleq \frac{1}{\pi |z|^2} \left(\frac{l_1(z)}{l_0(z)} - \left(\frac{|l_2(z)|}{l_0(z)} \right)^2 \right) \tag{15}$$

with the definition

$$l_\xi(z) \triangleq \boldsymbol{v}(z)^T \boldsymbol{C}_\xi \boldsymbol{v}(z^*), \quad \xi \in \{0, 1, 2\}. \tag{16}$$

The elements $c_{\mu\nu}[\xi]$, $0 \leq \mu, \nu \leq L-1$, of matrices $\boldsymbol{C}_\xi$ are given by

$$c_{\mu\nu}[0] = c_{\mu\nu}, \tag{17}$$
$$c_{\mu\nu}[1] = \mu\nu c_{\mu\nu}, \tag{18}$$
$$c_{\mu\nu}[2] = \mu c_{\mu\nu}. \tag{19}$$

4. RELEVANT CUMULATIVE DISTRIBUTIONS FOR EQUALIZER DESIGN

In this Section, two cumulative distributions relevant for equalizer design are defined and calculated for the special case of a discrete–time overall impulse response with uncorrelated coefficients.

A. RELEVANT CUMULATIVE DISTRIBUTIONS

For equalizer design the phase of the zeros of the overall transfer function is not important. The important parameter is the magnitude of the zeros. Therefore, we change to polar coordinates r, φ, i.e., $z = r \cdot e^{j\varphi}$, $0 \leq r < \infty$, $0 \leq \varphi < 2\pi$, is valid, and define the marginal density $f_r(r)$:

$$f_r(r) \triangleq r \int_0^{2\pi} f_z(r\cos(\varphi) + jr\sin(\varphi))\, d\varphi. \tag{20}$$

Now, the expected number of zeros in the disc $|z| = r \leq R$ is given by

$$n(R) = \int_0^R f_r(r)\, dr. \tag{21}$$

$n(R)$ is also referred to as the unnormalized cumulative distribution function of the magnitude of the zeros [10]. Here, unnormalized means that $\lim_{R\to\infty} n(R)$ yields $L-1$ (which is the total number of zeros) and not 1 as it is usual for cumulative distribution functions. For the design of reduced–state trellis–based equalizers such as DFE [8] and DDFSE/RSSE [4, 5] receivers $n(R=1)$, i.e., the expected number of zeros inside the unit circle is an important figure of merit. If zeros are located outside the unit circle, the corresponding impulse response is not minimum–phase, and without minimum–phase transformation a loss in performance is inevitable. Thus, if $n(R=1) \not\approx L-1$, the receiver designer should insert an allpass prefilter [13, 14, 15].

For LE and impulse response truncation for the VA using a linear prefilter [6, 7] the expected number of zeros $d(\rho)$ inside the region $\rho \leq |z| \leq 1/\rho$, $0 < \rho < 1$, is of high significance. It can be obtained from

$$d(\rho) = n(1/\rho) - n(\rho) \tag{22}$$

and may also be viewed as an unnormalized cumulative distribution since $\lim_{\rho\to 1} d(\rho) = 0$ and $\lim_{\rho\to 0} d(\rho) = L-1$ are valid. LE performs bad if there are zeros close to the unit circle. Thus, if $d(\rho) \not\approx 0$ for ρ close to 1 (say $\rho = 0.9$), LE cannot be employed. Although many different prefilter optimization criteria have been proposed for impulse response truncation, the performance of these schemes is only satisfactory if the number of zeros of the truncated impulse response is not smaller than the number of zeros close to the unit circle of the original impulse response since otherwise the prefilter enhances the channel noise considerably. Therefore, $d(\rho)$ provides important information for the proper choice of the length of the truncated impulse response.

B. UNCORRELATED IMPULSE RESPONSE COEFFICIENTS

In general, $f_r(r)$, $n(R)$, and $d(\rho)$ have to be evaluated using numerical integration. This will be done for the parameters of the GSM/EDGE system in Section 5. For the special case of uncorrelated impulse response coefficients $h[\mu, t_0]$, $0 \leq \mu \leq L-1$, however, closed–form results can be obtained. Although in practice uncorrelatedness will be fulfilled only approximately, this simple special case offers some important insights.

If the impulse response coefficients are uncorrelated, matrix $\boldsymbol{C}$ simplifies to

$$\boldsymbol{C} = \mathrm{diag}(\sigma^2_{\mathrm{h},L-1}, \sigma^2_{\mathrm{h},L-2}, \ldots, \sigma^2_{\mathrm{h},0}), \tag{23}$$

where $\mathrm{diag}(x_1, x_2, \ldots, x_S)$ denotes an $S \times S$ diagonal matrix with main diagonal elements $x_1, x_2, \ldots, x_S$ and $\sigma^2_{\mathrm{h},\mu}$, $0 \leq \mu \leq L-1$, is the variance of coefficient $h[\mu, t_0]$. Thus, Eq. (15) can be simplified to

$$f_z(z) = \frac{1}{\pi |z|^2} \left(\frac{\sum_{\nu=0}^{L-1} \nu^2 \sigma^2_{\mathrm{h},L-1-\nu} |z|^{2\nu}}{\sum_{\nu=0}^{L-1} \sigma^2_{\mathrm{h},L-1-\nu} |z|^{2\nu}} - \left(\frac{\sum_{\nu=0}^{L-1} \nu \sigma^2_{\mathrm{h},L-1-\nu} |z|^{2\nu}}{\sum_{\nu=0}^{L-1} \sigma^2_{\mathrm{h},L-1-\nu} |z|^{2\nu}} \right)^2 \right). \tag{24}$$

Note that for this special case the density of zeros $f_z(z)$ depends only on the magnitude $|z|$ of the complex variable z, i.e., it is rotationally symmetric. From Eqs. (20) and (24) the marginal density $f_r(r)$ can be calculated to

$$f_r(r) = \frac{2}{r} \left(\frac{\sum_{\nu=0}^{L-1} \nu^2 \sigma^2_{\mathrm{h},L-1-\nu} r^{2\nu}}{\sum_{\nu=0}^{L-1} \sigma^2_{\mathrm{h},L-1-\nu} r^{2\nu}} - \left(\frac{\sum_{\nu=0}^{L-1} \nu \sigma^2_{\mathrm{h},L-1-\nu} r^{2\nu}}{\sum_{\nu=0}^{L-1} \sigma^2_{\mathrm{h},L-1-\nu} r^{2\nu}} \right)^2 \right). \tag{25}$$

For uncorrelated impulse response coefficients it is also possible to obtain a closed–form result for the cumulative distribution functions $n(R)$ and $d(\rho)$. Using Eqs. (21), (25) and basic integration rules, it can be shown easily that the expected number of zeros in the disc $|z| \leq R$ is given by

$$n(R) = \frac{\sum_{\nu=0}^{L-1} \nu \sigma^2_{\mathrm{h},\,L-1-\nu} R^{2\nu}}{\sum_{\nu=0}^{L-1} \sigma^2_{\mathrm{h},\,L-1-\nu} R^{2\nu}}. \tag{26}$$

Finally, from Eqs. (22) and (26)

$$d(\rho) = \frac{\sum_{\nu=0}^{L-1} \nu \sigma^2_{\mathrm{h},\,L-1-\nu} \rho^{-2\nu}}{\sum_{\nu=0}^{L-1} \sigma^2_{\mathrm{h},\,L-1-\nu} \rho^{-2\nu}} - \frac{\sum_{\nu=0}^{L-1} \nu \sigma^2_{\mathrm{h},\,L-1-\nu} \rho^{2\nu}}{\sum_{\nu=0}^{L-1} \sigma^2_{\mathrm{h},\,L-1-\nu} \rho^{2\nu}}$$

$$= \frac{\sum_{\nu=0}^{L-1} (L-1-\nu) \sigma^2_{\mathrm{h},\,\nu} \rho^{2\nu}}{\sum_{\nu=0}^{L-1} \sigma^2_{\mathrm{h},\,\nu} \rho^{2\nu}} - \frac{\sum_{\nu=0}^{L-1} \nu \sigma^2_{\mathrm{h},\,L-1-\nu} \rho^{2\nu}}{\sum_{\nu=0}^{L-1} \sigma^2_{\mathrm{h},\,L-1-\nu} \rho^{2\nu}} \tag{27}$$

can be obtained.

Impulse Response Coefficients with Equal Variances

In the following, we further specialize the overall impulse response and assume that all taps have equal variances, i.e., $\sigma^2_{\mathrm{h},\,\nu} = \sigma^2_{\mathrm{h}}$, $0 \leq \nu \leq L-1$. Thus, Eq. (26) can be simplified to[1]

$$n(R) = \begin{cases} \frac{(L-1)R^{2(L+1)} - LR^{2L} + R^2}{R^{2(L+1)} - R^{2L} - R^2 + 1}, & R \neq 1, \\ \frac{L-1}{2}, & R = 1. \end{cases} \tag{28}$$

As was to be expected, $\lim_{R\to\infty} n(R) = L-1$ follows from Eq. (28). Furthermore, $n(R=1) = (L-1)/2$ implies that for this special case on average there are as many zeros inside the unit circle as outside the unit circle. Thus, for uncorrelated impulse response coefficients having equal variances a minimum–phase transformation is mandatory if DDFSE/RSSE is applied. Using Eq. (28), the relation

$$n(R) = L - 1 - n(1/R) \tag{29}$$

can be proved. From Eqs. (21), (28), and (29) the symmetry relations

$$\int_0^R f_r(r)\,\mathrm{d}r = \int_{1/R}^{\infty} f_r(r)\,\mathrm{d}r \tag{30}$$

and

$$\int_R^1 f_r(r)\,\mathrm{d}r = \int_1^{1/R} f_r(r)\,\mathrm{d}r, \qquad 0 < R \leq 1, \tag{31}$$

follow directly. Eq. (30) means that for uncorrelated impulse response coefficients with equal variances the average numbers of zeros in the regions $0 \leq |z| \leq R$ and $1/R \leq |z| < \infty$ are equal. According to Eq. (31) the same statement is true for the regions $R \leq |z| \leq 1$ and $1 \leq |z| \leq 1/R$.

For the expected number of zeros in the region $\rho \leq |z| \leq 1/\rho$, $0 < \rho < 1$,

$$d(\rho) = L\,\frac{1+\rho^{2L}}{1-\rho^{2L}} - \frac{1+\rho^2}{1-\rho^2} \tag{32}$$

can be obtained. $\lim_{\rho\to 0} d(\rho) = L-1$ results, since integration is performed over the whole complex plane.

For $L \gg 1$ and $\rho < 1$, ρ^{2L} approaches zero. Thus, Eq. (32) simplifies to

$$d(\rho) \approx L - \frac{1+\rho^2}{1-\rho^2} = L - 1 - \frac{2\rho^2}{1-\rho^2}. \tag{33}$$

This means that for long overall impulse responses, on average only $2\rho^2/(1-\rho^2)$ zeros are located outside the region $\rho \leq |z| \leq 1/\rho$. For example, if $\rho = 0.9$ is assumed, Eq. (33) yields $d(0.9) \approx L - 9.5$. To become more specific, we assume $L = 30$ and obtain 20.5 for the expected number of zeros in the region $0.9 \leq |z| \leq 1.11$. From this example, we conclude that for very long impulse responses with approximately uncorrelated coefficients having equal variances (as they may be encountered e.g. in underwater acoustic communications), most zeros are close to the unit circle. Therefore, the attainable complexity reduction using impulse response truncation [6, 7] is limited since the truncated impulse response still has to be quite long (cf. Section 4.A). On the other hand, DFE and DDFSE/RSSE combined with a minimum–phase transformation [13, 14, 15] might be used. However, it should be noted that the effect of the minimum–phase transformation is also limited. Since most zeros are close to the unit circle, transforming the zeros which are located outside the unit circle into it will not significantly increase the energy of the first coefficients of the overall impulse response. Therefore, also for DFE and DDFSE/RSSE a considerable loss in performance compared to MLSE has to be expected.

In order to illustrate our results for uncorrelated impulse response coefficients with equal tap variances, Fig. 2a) shows the (normalized) density $f_z(z = x + jy)/(L-1)$ for an impulse response length of $L = 5$. Clearly, uncorrelated coefficients cause a rotationally symmetric density. In Fig. 2b), $f_r(r)/(L-1)$ is depicted for $L = 2$, 5, and 30. Note that for large L, $f_r(r)/(L-1)$ has a peak at $r = 1$, i.e., the density of zeros is very high at this point. A similar observation can be made from the (normalized) cumulative distributions $n(R)/(L-1)$ and $d(\rho)/(L-1)$ which are shown in Figs. 3a) and b), respectively. Fig. 3a) confirms that there are as many zeros outside the unit circle as inside, while Fig. 3b) illustrates that for $L = 2$ and $L = 5$ on average only 10 and 20 percent of all zeros lie in the region $0.9 \leq |z| \leq 1.11$, respectively. For $L = 30$, however, more than 70 percent of all zeros lie in this region and impulse response truncation is not expected to perform well unless the truncated impulse response is quite long.

[1] Note that this result can also be found in [16].

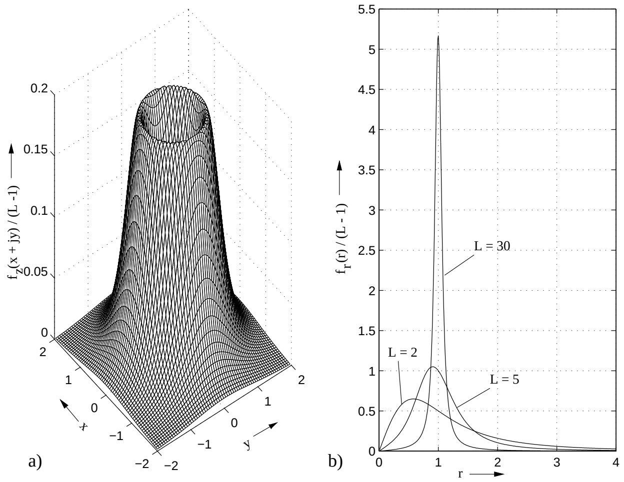

Figure 2: a) Normalized density of zeros $f_z(z)/(L-1)$ for $L = 5$. b) Normalized marginal density of zeros $f_r(r)/(L-1)$ for different impulse response lengths L.

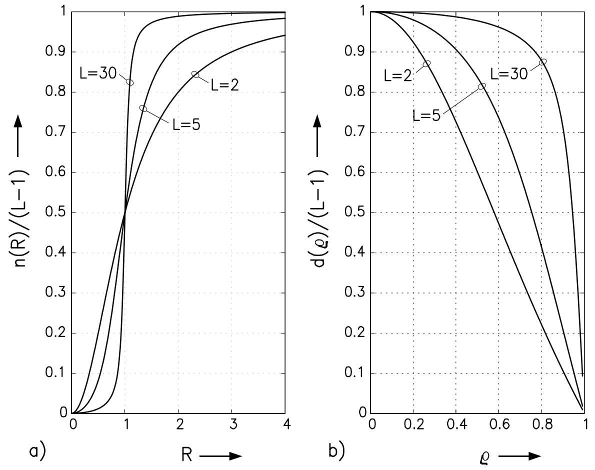

Figure 3: a) Normalized average number $n(R)/(L-1)$ of zeros inside the disc $|z| \le R$ for different impulse response lengths L. b) Normalized average number $d(\rho)/(L-1)$ of zeros inside the region $\rho \le |z| \le 1/\rho$ for different impulse response lengths L.

5. RESULTS FOR GSM/EDGE POWER DELAY PROFILES

For the GSM/EDGE system four different power delay profiles are specified [17]: rural area (RA), hilly terrain (HT), typical urban area (TU), and equalizer test (EQ). For HT, TU, and EQ it is assumed that the amplitudes of all propagation paths are zero–mean complex Gaussian random variables and thus, the distribution of zeros of the corresponding overall transfer functions may be analyzed using the methods presented in the previous sections. Note that the channel specified for RA is essentially flat, i.e., the zeros of the overall transfer function are mainly influenced by transmit and receiver input filter.

For calculation of the correlation matrix $\boldsymbol{C}$ transmit and receiver input filter impulse response are required. For the transmit filter $h_T(t)$ a linearized Gaussian minimum phase–shift keying (GMSK) pulse is standardized for EDGE in order to obtain full compatibility with GSM where GMSK modulation is employed [9]. Thus, the linearized GMSK pulse is also used in this paper. The choice of the receiver input filter $h_R(t)$ is up to the receiver designer. Here, we use a fixed filter, namely a square–root raised cosine (SRC) filter with roll–off factor 0.3. This filter offers a similar performance like the optimum whitened matched filter (WMF) [1]. However, the implementation of the SRC filter is much simpler since, in contrast to the WMF, it has not to be adapted to a particular channel impulse response (cf. [2, 3]). The power delay profiles are taken from [17] (for HT alternative (1) is used). As already mentioned, now $f_r(r)$, $n(R)$, and $d(\rho)$ cannot be calculated in closed form but have to be obtained by numerical integration.

In Fig. 4, the EQ profile is considered. Here, for the length of the overall impulse response $L = 6$ is chosen. In contrast to the case of uncorrelated impulse response coefficients, the density of zeros $f_z(z)$ is not rotationally symmetric. This is because transmit and receiver input filter introduce correlations between different impulse response coefficients, i.e., $\boldsymbol{C}$ is not a diagonal matrix. $f_z(z)$ has a peak at approximately $z = -1$. Note that a zero at $z = -1$ results if e.g. $h[0, t_0]$ and $h[1, t_0]$, $h[2, t_0]$ and $h[3, t_0]$, and $h[4, t_0]$ and $h[5, t_0]$ are equal, respectively. Due to the strong correlations (introduced by transmit and receiver input filter) between neighboring taps of the overall impulse response it is quite likely that neighboring taps are approximately equal which leads to the peak of $f_z(z)$ at $z = -1$. From $f_r(r)$ of Fig. 4b), $n(R = 1)$ can be calculated to $n(1) = 3.5$. Thus, for application of DFE and DDFSE/RSSE an allpass prefilter which transforms the overall impulse response in its minimum–phase equivalent should be applied. Furthermore, it can be derived from $f_r(r)$, that on average there are 1.2 zeros in the region $0.9 \le |z| \le 1.1$ ($d(0.9) = 1.2$). Thus, for impulse response truncation the resulting overall impulse response should have at least a length of 3 (i.e., two zeros) in order to avoid excessive noise enhancement. In Fig. 5, the HT profile is considered. The length of the overall impulse response is $L = 7$. Although it is obvious from Fig. 5 that most zeros are located inside the unit circle, on average 0.8 zeros are located in the area $|z| \ge 2$ ($n(R = 2) = 5.2$). Transforming these zeros inside the unit circle will enhance the energy of the first taps of the impulse response considerably and thus, improve the performance of DFE and DDFSE/RSSE schemes significantly. Hence, we also recommend the use of an allpass prefilter for HT. Calculation of $d(\rho)$ from $f_r(r)$ shows, that on average only one zero lies in the region $0.9 \le |z| \le 1.1$ ($d(0.9) = 1.0$). Therefore, we expect that the overall impulse response can be truncated (via a suitable prefilter) to a length of 3 without significant loss in performance.

Corresponding results for the TU profile and plots of $n(R)$ and $d(\rho)$ for all profiles are given in [12].

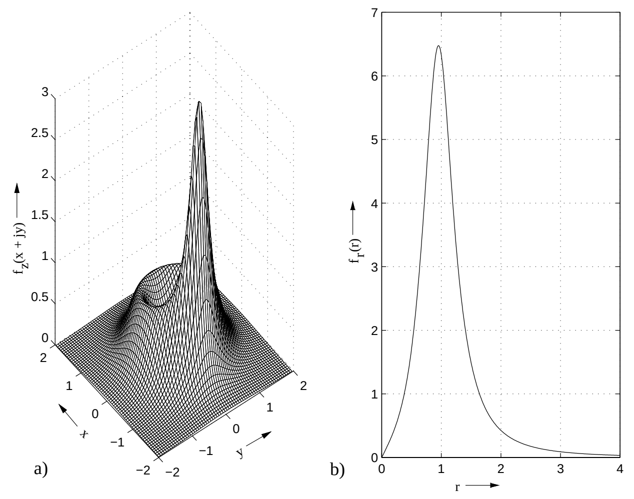

Figure 4: a) Unnormalized density of zeros $f_z(z)$ and b) unnormalized marginal density of zeros $f_r(r)$ for the EQ profile with $L = 6$.

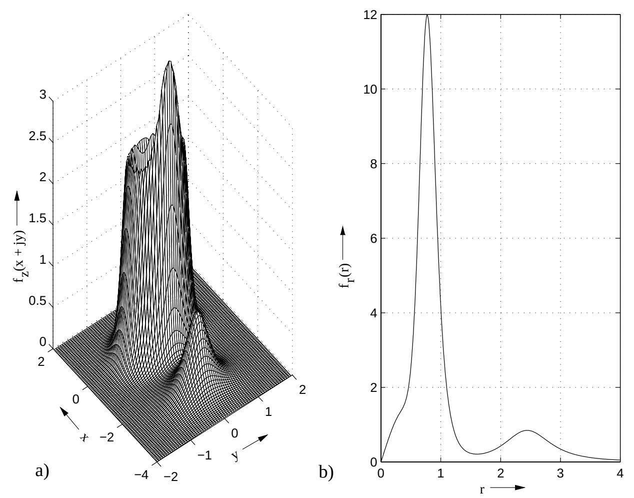

Figure 5: a) Unnormalized density of zeros $f_z(z)$ and b) unnormalized marginal density of zeros $f_r(r)$ for the HT profile with $L = 7$.

6. CONCLUSIONS

In this paper, the distribution of zeros of mobile channels characterized by a power delay profile is analyzed. Using a result from the mathematical literature, the density of zeros of the overall transfer function is calculated in closed form. From this density two cumulative distributions are derived and their importance for the design of suboptimum receivers is shown.

For the special case that all taps of the overall impulse response are mutually uncorrelated closed–form expressions for both cumulative distributions are obtained.

Finally, the power delay profiles specified for GSM and EDGE are investigated in this paper. It is shown that if DFE or DDFSE/RSSE are employed, an all-pass prefilter should be inserted for all profiles. In addition, it turns out that for impulse response truncation a length of 3 of the truncated impulse response is sufficient in all cases.

REFERENCES

[1] G.D. Forney, Jr. Maximum–Likelihood Sequence Estimation of Digital Sequences in the Presence of Intersymbol Interference. *IEEE Transactions on Information Theory*, IT-18:363–378, 1972.

[2] W.H. Gerstacker and R. Schober. Equalisation for EDGE mobile communications. *Electronics Letters*, 36(2):189–191, 2000.

[3] W. H. Gerstacker and R. Schober. Equalization Concepts for EDGE. *Submitted to IEEE Journal on Selected Areas in Communications*, November 1999.

[4] A. Duel-Hallen and A. C. Heegard. Delayed Decision–Feedback Sequence Estimation. *IEEE Trans. on Commun.*, COM-37:428–436, 1989.

[5] M.V. Eyuboğlu and S.U. Qureshi. Reduced–state sequence estimation with set partitioning and decision feedback. *IEEE Trans. on Commun.*, COM-36:13–20, January 1988.

[6] S.U.H. Qureshi and E.E. Newhall. An Adaptive Receiver for Data Transmission over Time-Dispersive Channels. *IEEE Transactions on Information Theory*, IT-19:448–457, July 1973.

[7] D. D. Falconer and Jr. F. R. Magee. Adaptive Channel Memory Truncation for Maximum–Likelihood Sequence Estimation. *Bell Syst. Tech. Journal*, 52:1541–1562, November 1973.

[8] C. A. Belifiore and J. H. Park. Decision–feedback equalization. *Proceedings of the IEEE*, 67:1143–1156, August 1979.

[9] A. Furuskär, S. Mazur, F. Müller, and H. Olofsson. EDGE: Enhanced Data Rates for GSM and TDMA/136 Evolution. *IEEE Personal Communications Magazine*, pages 56–66, June 1999.

[10] A. Edelman and E. Kostlan. How Many Zeros of a Random Polynomial are Real. *Bulletin of the American Mathematical Society*, 32:1–37, January 1995.

[11] V. S. Vladimirov. *Methods of the Theory of Functions of Many Complex Variables.* M.I.T. Press, Cambridge, Massachusetts, 1964.

[12] R. Schober and W.H. Gerstacker. On the Distribution of Zeros of Mobile Channels with Applications to GSM/EDGE. *Submitted to IEEE Journal on Selected Areas in Communications*, October 1999.

[13] W. Gerstacker and J. Huber. Improved Equalization for GSM Mobile Communications. In *Proceedings of ICT'96*, Istanbul, April 1996.

[14] M. Schmidt and G.P. Fettweis. Fractionally–Spaced Prefiltering for Reduced State Equalization. In *Proceedings of the IEEE Global Telecommunication Conference (GLOBECOM'99)*, pages 2291–2295, Rio de Janeiro, 1999.

[15] W.H. Gerstacker, F. Obernosterer, R. Meyer, and J.B. Huber. An Efficient Method for Prefilter Computation for Reduced–State Equalization. *Proceedings of PIMRC'2000*, September 2000.

[16] L. Arnold. Über die Nullstellenverteilung zufälliger Polynome (*in German). Math. Zeitschrift*, 92:12–18, 1966.

[17] *GSM Recommendation 05.05: "Propagation Conditions", Vers. 5.3.0, Release 1996.*