An Efficient Method for Prefilter Computation for Reduced–State Equalization

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ABSTRACT

In advanced TDMA mobile communications systems, reduced–state equalization algorithms have to be employed because high–level modulation is used in order to improve spectral efficiency. Such equalizers have only high performance, if the overall discrete–time system to be equalized is minimum–phase. Therefore, in general, a discrete–time prefilter has to be inserted in front of equalization. In literature, several approaches have been proposed for computation of a suitable FIR or IIR prefilter. In this paper, we present an approach for FIR prefilter computation, which is quite robust and requires an only moderate computational complexity. The prefilter consists of the cascade of a channel–matched filter and a prediction–error filter, which can be calculated via the Levinson–Durbin algorithm. Simulation results are given, which demonstrate that the performance of the proposed approach is essentially equivalent to the case of reduced–state equalization combined with ideal allpass prefiltering.

1. INTRODUCTION

In high–rate digital transmission over dispersive channels, the received signal is impaired by intersymbol interference (ISI) and additive noise. E.g., in time–division multiple access (TDMA) mobile communications, ISI is caused by phenomena like multipath propagation and fading. Equalization at the receiver side is necessary in order to obtain reliable estimates of the transmitted symbols. It is well known, that the optimum equalization algorithm is given by maximum–likelihood sequence detection (MLSD) [1], which can be realized in a recursive manner using the Viterbi algorithm (VA). However, for channels with large delay spread, i.e., long impulse responses, and/or transmission using nonbinary signal alphabets, a very high complexity may result for the VA, and suboptimum schemes have to be considered for a practical implementation. Potential candidates are e.g. decision–feedback equalization (DFE), delayed decision–feedback sequence estimation (DDFSE) [2], reduced–state sequence estimation (RSSE) [3], the M–algorithm [4], and various members of the family of sequential decoding algorithms like the Fano algorithm or the stack algorithm [4]. Because of their favourable tradeoff between performance and complexity and their high regularity, RSSE and DDFSE are often preferred.

For any suboptimum trellis–based equalizer, a minimum–phase discrete–time overall impulse response is essential for high performance, cf. e.g. [2, 3]. In general, this necessitates the introduction of a discrete–time prefilter in front of the equalizer, which transforms the channel impulse response into its minimum–phase equivalent. In TDMA mobile communications, the computation of this filter poses serious problems, because data is usually organized in bursts, each one containing a training sequence for estimation of the channel impulse response, which however is in most cases too short for the application of recursive adaptation algorithms like the least–mean–squares (LMS) or the recursive least–squares (RLS) algorithm for adjustment of the prefilter coefficients. Therefore, a closed–form calculation of the prefilter coefficients using the result of channel estimation seems to be necessary. For this task, several approaches have been proposed in literature. In this paper, an algorithm is presented, cf. also [5], which enables a computation of the prefilter with less complexity than with state–of–the–art algorithms, and seems to be highly suited for implementation in third generation (3G) TDMA mobile communication receivers.

In Section 2, the transmission model is given. Section 3 presents a review of state–of–the–art schemes for prefilter computation, whereas the proposed algorithm is described in Section 4. The simulation results of Section 5 demonstrate, that the performance of a scheme with an ideal allpass prefilter, which is the theoretically optimum prefilter, can be closely approached with the proposed scheme.

2. SYSTEM MODEL

Fig. 1 shows the discrete–time equivalent baseband model of a transmission with pulse amplitude modulation (PAM) over a dispersive channel, producing ISI. The PAM symbols $a[k]$, drawn from an M–ary symbol alphabet, are either real– or complex–valued; this includes e.g. the cases of ASK (amplitude–shift keying) and PSK (phase–shift keying). The received signal, sampled at times kT (T: symbol interval), is given by

$$r[k] = \sum_{\kappa=0}^{q_h} h[\kappa]\, a[k-\kappa] + n[k], \qquad (1)$$

where $h[k]$ denotes the discrete–time overall impulse response of the cascade of continuous–time transmit filter, channel, and receiver input filter. $h[k]$ is assumed to be causal and of finite order q_h. The corresponding FIR transfer function $H(z)$ is given by

$$H(z) = \sum_{k=0}^{q_h} h[k]\, z^{-k}. \tag{2}$$

For the continuous–time receiver input filter, a square–

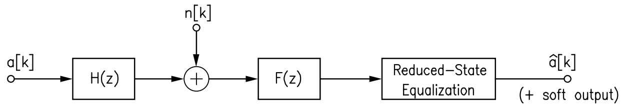

Figure 1: Discrete–time model of transmission.

root Nyquist frequency response is assumed, resulting in discrete–time white Gaussian noise $n[k]$. In the receiver, a reduced–state trellis–based equalization algorithm is employed, e.g. reduced–state sequence estimation (RSSE) [3] or delayed decision–feedback sequence estimation (DDFSE) [2], delivering estimated data $\hat{a}[k]$. For DDFSE, only the first part of order $p \le q_h$ of the overall impulse response is used for definition of trellis states in the VA. Hence, the number of states is reduced from $Z = M^{q_h}$ for MLSD [1] to $Z = M^p$. For metric calculations, a path register is assigned to each state. In RSSE, additional set partitioning is applied, which allows an even finer tradeoff between performance and complexity. If additional soft output is required for subsequent channel decoding, modified variants of both algorithms may be applied. In the following, the design of the prefilter $F(z)$ in front of reduced–state equalization is addressed.

3. PROBLEM STATEMENT AND PREVIOUS APPROACHES

For high performance of reduced–state equalization, a minimum–phase discrete–time overall impulse response is essential [2, 3]. However, in general, $H(z)$ is a mixed–phase transfer function. Therefore, a prefilter $F(z)$ is required, which transforms the impulse response (at least approximately) into its minimum–phase equivalent. The ideal prefilter, generating exactly the minimum–phase equivalent of $H(z)$, is denoted by $A(z)$. Thus,

$$H_{\min}(z) = A(z) \cdot H(z) = \sum_{k=0}^{q_h} h_{\min}[k]\, z^{-k}, \tag{3}$$

with

$$|H_{\min}\left(e^{j2\pi fT}\right)|^2 = |H\left(e^{j2\pi fT}\right)|^2, \tag{4}$$

or equivalently

$$H_{\min}(z) \cdot H^*_{\min}(1/z^*) = H(z) \cdot H^*(1/z^*), \tag{5}$$

where $H_{\min}(z)$ has roots only inside and on the unit circle. It should be noted, that for an FIR transfer function $H(z)$, $H_{\min}(z)$ is also FIR with the same order as $H(z)$. $A(z)$ is an IIR allpass transfer function, given by

$$A(z) = \frac{H_{\min}(z)}{H(z)}. \tag{6}$$

One category of approaches for prefilter calculation is characterized by initially determining $H_{\min}(z)$ corresponding to the given $H(z)$, which can be assumed to be known from channel estimation. $H_{\min}(z)$ can be calculated via *spectral factorization* of $H(z) \cdot H^*(1/z^*)$, applying well–known signal processing algorithms which use the *cepstrum* of the channel autocorrelation sequence, cf. e.g. [6]. Alternatively, $H_{\min}(z)$ may be determined via root finding. The roots of $H(z)$ inside or on the unit circle are retained for $H_{\min}(z)$, and each root z_0 outside the unit circle is replaced by $1/z_0^*$. A third method calculates $H_{\min}(z)$ as a prediction–error filter for a stochastic process with power spectral density $\frac{1}{|H(e^{j2\pi fT})|^2}$ [7].

After calculation of $H_{\min}(z)$, $A(z)$ is known from Eq. (6). However, the direct realization of $A(z)$ with a recursive filter would exhibit a nonstable behaviour. One solution to this problem consists in viewing $A(z)$ as a transfer function corresponding to a noncausal and stable impulse response. This impulse response can be determined from $A(z)$, then is truncated to a sufficient finite order q_f and realized (with delay q_f) with a causal FIR filter. However, it turns out, that for this approach, a quite high numerical sensitivity results, which necessitates high filter orders in order to avoid degradation of reduced–state equalization [8].

In a different solution, these problems are avoided by recursive filtering in *reversed time* [9]. Here, the receiver first waits until reception of a burst is complete. Then, the stored received samples are time–reversed and fed through a stable recursive filter with transfer function $A(1/z)$. The desired allpass–filtered sequence can be obtained by time–reversal of the output sequence of $A(1/z)$.

A related approach first transforms the channel impulse response into its *maximum–phase* equivalent by applying an allpass filter with transfer function

$$\tilde{A}(z) = \frac{H_{\max}(z)}{H(z)}, \tag{7}$$

where the maximum–phase equivalent of $H(z)$ is given by

$$H_{\max}(z) = z^{-q_h}\, H^*_{\min}(1/z^*). \tag{8}$$

$\tilde{A}(z)$ can be realized with a causal and stable recursive filter. After prefiltering, reduced–state equalization is applied in negative time direction (*backward decoding*) [10, 11]. The resulting performance is equivalent to that of conventional reduced–state equalization of the minimum–phase–filtered received signal.

For all approaches described above, the minimum–phase transfer function $H_{\min}(z)$ has to be determined. In practical applications, this causes some problems, because computational complexity of spectral factorization is quite high; $\log(\cdot)$– and $\exp(\cdot)$–operations are required [6], which are not well suited for an implementation with digital signal processors. On the other hand, root finding is also often not desired for an implementation, because the corresponding algorithms are not regular enough, and the prediction–error approach described in [7] may require a discrete

Fourier transform (DFT) of very high order for computation of the autocorrelation sequence corresponding to $\frac{1}{|H(e^{j2\pi fT})|^2}$.

In order to avoid the drawbacks of this first category of approaches, it has been proposed by several authors, to apply the FIR feedforward filter of a minimum mean–squared error decision–feedback equalizer (MMSE–DFE) for prefiltering [12, 13, 14]. If the design parameters of the MMSE–DFE are chosen properly, a reasonable FIR approximation of the desired allpass transfer function can be expected [12]. However, a direct calculation of the MMSE–DFE solution as described e.g. in [12] turns out to be quite complex, because a matrix inversion is required. For reduction of computational complexity from $\mathcal{O}(q_f^3)$, where q_f denotes the prefilter order, to $\mathcal{O}((q_f+q_h)^2)$, an alternative algorithm, calculating the optimum feedforward filter via *fast Cholesky factorization*, has been proposed in [15]. Furthermore, improved versions of this algorithm have been derived [13, 14], which are more regular and modular and avoid square–root operations.

However, it has been shown in [13], that the MMSE–DFE approach is not robust to a mismatch of design parameters in certain cases. The solution might be quite sensitive to the selection of the noise variance, which has to be considered as a free parameter for filter design. Robustness can be increased by fractionally–spaced prefiltering [13], but this increases complexity as well.

In the next section, a quite robust algorithm for FIR prefilter computation is presented, cf. also [5], which has less complexity than previously proposed schemes.

4. PROPOSED METHOD FOR PREFILTER COMPUTATION

For computation of an FIR prefilter, we make use of the basic identity Eq. (5), which is equivalent to

$$\frac{H_{\min}(z)}{H(z)} = \frac{H^*(1/z^*)}{H^*_{\min}(1/z^*)}. \tag{9}$$

Comparing Eq. (9) to Eq. (6), it is obvious, that the allpass transfer function for transformation of the channel into its minimum–phase equivalent also equals

$$A(z) = \frac{H^*(1/z^*)}{H^*_{\min}(1/z^*)}, \tag{10}$$

or

$$A(z) = A_1(z) \cdot A_2(z), \tag{11}$$

with

$$A_1(z) = H^*(1/z^*), \tag{12}$$

$$A_2(z) = \frac{1}{H^*_{\min}(1/z^*)}. \tag{13}$$

Hence, $A(z)$ can be represented as a cascade of two filters $A_1(z)$ and $A_2(z)$. For a given nonrecursive (FIR) channel, the impulse response of the filter $A_1(z)$, which then is nonrecursive as well, is given by the complex–conjugated and time–reversed channel impulse response, i.e., $A_1(z)$ is a matched filter, matched to the discrete–time channel impulse response. Thus, the remaining problem consists of the approximation of the allpole transfer function $A_2(z)$ by an FIR filter $F_2(z)$,

$$F_2(z) \approx C\,\frac{1}{H^*_{\min}(1/z^*)}, \tag{14}$$

with an arbitrary constant $C \neq 0$. An equivalent problem is the determination of an FIR filter $G(z)$ according to

$$G(z) \approx C^*\,\frac{1}{H_{\min}(z)}. \tag{15}$$

Then, $F_2(z)$ results from

$$F_2(z) = G^*(1/z^*). \tag{16}$$

Now, for the transfer function $G(z)$, we propose the choice

$$G(z) = 1 - P(z), \tag{17}$$

where

$$1 - P(z) = 1 - \sum_{k=1}^{q_p} p[k]\, z^{-k} \tag{18}$$

is a prediction–error filter of order q_p for a stochastic process with power spectral density $\sim |H\left(e^{j2\pi fT}\right)|^2$. The prediction filter $P(z)$ is designed for minimization of the output power of the error filter. The optimum coefficients for this criterion are given by the solution of the *Yule–Walker* equations [16]

$$\boldsymbol{\Phi}\,\boldsymbol{p} = \boldsymbol{\varphi}, \tag{19}$$

with

$$\boldsymbol{\Phi} = \begin{bmatrix} \varphi[0] & \varphi[-1] & \dots & \varphi[-(q_p-1)] \\ \varphi[1] & \varphi[0] & \dots & \varphi[-(q_p-2] \\ \vdots & \vdots & \ddots & \vdots \\ \varphi[q_p-1] & \varphi[q_p-2] & \dots & \varphi[0] \end{bmatrix}, \tag{20}$$

$$\boldsymbol{p} = [\,p[1] \quad p[2] \quad \dots \quad p[q_p]\,]^T, \tag{21}$$

$$\boldsymbol{\varphi} = [\,\varphi[1] \quad \varphi[2] \quad \dots \quad \varphi[q_p]\,]^T, \tag{22}$$

$$\varphi[k] = h[k] * h^*[-k], \tag{23}$$

where $(\cdot)^T$ and $*$ denote transposition and convolution, respectively.

The error filter $1-P(z)$ is always minimum–phase [16]; in addition to that, for the limit case $q_p \to \infty$, white output noise results, i.e.,

$$|1 - P\left(e^{j2\pi fT}\right)|^2\,|H\left(e^{j2\pi fT}\right)|^2 = \text{const.}, \tag{24}$$

or

$$(1-P(z))\,(1-P^*(1/z^*))\,H(z)\,H^*(1/z^*) = \text{const.} \tag{25}$$

$$\Rightarrow (1-P(z))\,(1-P^*(1/z^*)) = \frac{\text{const.}}{H_{\min}(z)\,H^*_{\min}(1/z^*)}, \tag{26}$$

where Eq. (5) has been used. Because not only $1 - P(z)$, but also $1/H_{\min}(z)$ is causal and minimum–phase (this holds because $H_{\min}(z)$ is causal, stable, and minimum–phase), it can be concluded from Eq. (26) that for infinite filter order $q_p \to \infty$

$$G(z) = 1 - P(z) = \frac{C^*}{H_{\min}(z)} \tag{27}$$

is valid, i.e., (15) holds with equality. For $|z| \to \infty$, Eq. (27) yields $1 = C^*/h_{\min}[0]$, i.e., $C^* = h_{\min}[0]$.

For finite filter orders q_p, which are not too low, a reasonable FIR approximation of the desired transfer function can be expected.

Finally, allowing an additional delay, which is necessary for a causal realization, we obtain for the overall transfer function of the FIR prefilter

$$\begin{aligned} F(z) &= z^{-q_h}\, A_1(z)\, z^{-q_p}\, F_2(z) \\ &= z^{-(q_h+q_p)}\, H^*(1/z^*)\,(1 - P^*(1/z^*)). \end{aligned} \tag{28}$$

It should be noted, that $F_2(z) = 1 - P^*(1/z^*)$ is an optimum *backward* prediction–error filter for a stochastic process with power spectral density $\sim |H\left(e^{j2\pi fT}\right)|^2$ [17].

The computational complexity of the proposed method is quite moderate, because Eq. (19) can be solved via the Levinson algorithm, cf. [16], which can be applied to Eq. (19) for arbitrary rhs, or via the Levinson–Durbin (LD) algorithm [16], which utilizes the special rhs of the Yule–Walker equations for further reduction of computational complexity. The LD algorithm requires $q_p^2 + \mathcal{O}(q_p)$ operations and is approximately twice as efficient as the Levinson algorithm. It calculates a predictor with a desired order q_p recursively via determination of all predictors of order $< q_p$. The numerical stability of the LD algorithm is sufficient for a practical implementation in most cases [16]. The total number of required operations for the proposed approach for prefilter computation is $q_p^2 + \mathcal{O}(q_p) + \mathcal{O}(q_h^2)$. The complexity of the algorithm of [15] is also quadratic in the filter orders q_h and q_f, but the exact expression for the number of required operations in the form $c_1\, q_f^2 + c_2\, q_f\, q_h + \ldots$ exhibits quite large constants c_i, cf. [15], in contrast to the proposed approach, resulting in a significantly higher complexity for filter orders used in practical applications.

For transmission formats using midamble sequen ces for channel estimation, two separate equalization processes, starting from the midamble and proceeding in positive and negative time direction, respectively, are often applied. For reduced–state equalization in negative time direction, a maximum–phase overall impulse response is required, cf. Section 3. This can be guaranteed by a prefilter $\tilde{F}(z)$, which again is composed of two components depending only on $H(z)$ and $1 - P(z)$, respectively:

$$\tilde{F}(z) = z^{-q_h}\, H^*(1/z^*)\,(1 - P(z)). \tag{29}$$

5. NUMERICAL RESULTS

In a first example, a real–valued impulse response $h[k]$ of order $q_h = 4$ is selected, which may be viewed as a snapshot of an equalizer test channel with constant power delay profile, see Fig. 2 (channel 1). For the predictor, an order of $q_p = 20$ is chosen. The resulting overall impulse response corresponding to $F(z)\,H(z)$ is also shown in Fig. 2. According to Fig. 3, channel zeros with magnitude larger than one are reflected on the unit circle. It should be mentioned, that the first $q_h + q_p$ coefficients of $F(z)\,H(z)$ would ideally equal zero, which is due to the causal realization of the prefilter. A subsequent equalizer should take into account only the last $q_h + 1$ coefficients; only this last part was taken for calculation of the zeros of the transformed impulse response.

In a second example, we consider a complex–valued impulse response of order $q_h = 4$, whose real and imaginary part is shown in Fig. 4 (channel 2). The predictor order is selected to $q_p = 15$. In Fig. 5, the magnitude of the original and of the transformed impulse response is shown. Fig. 6 shows the magnitude of the frequency response of the original system $H(z)$ and of the transformed system $F(z)\,H(z)$. Obviously, it is only slightly changed by prefiltering. Thus, the noise essentially remains white after prefiltering, and the Euclidean distance still is the optimum metric for trellis–based equalization.

Next, we consider reduced–state equalization of channel 1 and channel 2, using RSSE with $Z = 2$ states. Fig. 7 shows simulated bit error rates (BERs) versus E_b/N_0 (E_b: received energy per bit; N_0: noise power spectral density) for transmission with 8PSK, using an ideal allpass for prefiltering and an FIR prefilter with order $q_f = 28$ (channel 1) and $q_f = 23$ (channel 2), respectively, which has been calculated according to the proposed method. Perfect knowledge of the channel impulse response has been assumed in each case. For BER $\geq 10^{-2}$, prefiltering using the proposed method causes an only slight loss compared to ideal allpass prefiltering. For high E_b/N_0, the results differ because of the small precursor ISI produced by FIR prefiltering, cf. Figs. 2 and 5. An improvement could by obtained by increasing the prefilter order. However, it should be noted, that for mobile communications with additional channel coding, mainly the BER region BER $\geq 10^{-3}$ is of interest at the equalizer output.

Finally, we consider equalization for the EDGE (Enhanced Data Rates for GSM Evolution) system [18], which also employs 8PSK modulation. In [19, 20], it has been shown that reduced–state equalization with only a few states is highly suited for EDGE, if an additional minimum–phase transformation is applied. Without prefilter, however, a severe performance degradation occurs.

In the receiver, first, the channel impulse response is estimated from the training sequence for each burst. A fixed prefilter is then designed for each burst using the result of channel estimation. This is justified for channels with slow to moderate time variations.

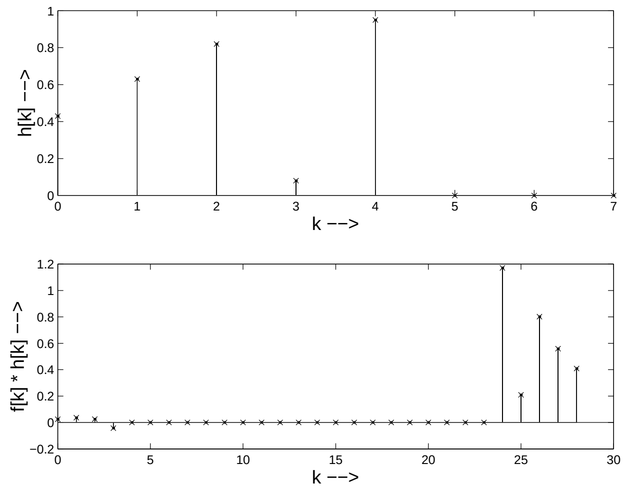

Figure 2: Impulse response of channel 1 and transformed impulse response.

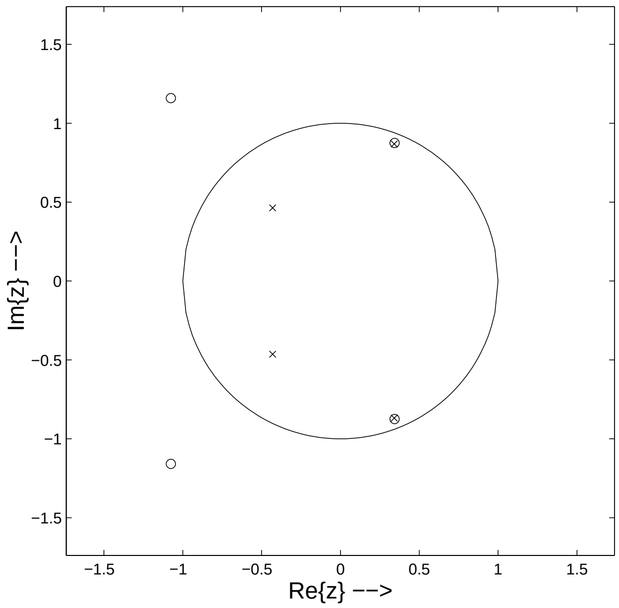

Figure 3: Zeros of $H(z)$ ('o') and zeros of overall transfer function including prefilter ('x') (channel 1).

Fig. 8 shows BER for the power delay profile EQ50 [21] for RSSE ($Z = 2$) with prefiltering using FIR filters with different orders. Obviously, a good performance can be obtained, if the filter order is selected to ≈ 20 or higher. In contrast to that, for $q_f = 15$, precursor ISI seems to play a significant role.

6. CONCLUSIONS

A simple method for calculation of an FIR prefilter for reduced–state equalization has been presented. Less computational complexity is required than for previously proposed algorithms. The prefilter consists of two stages, a matched filter, which can be directly obtained from the (estimated) channel impulse response, and a prediction–error filter, which can be calculated via Yule–Walker equations. In contrast to the MMSE–DFE approach, where a proper selection of the virtual noise variance is essential, no additional design parameters besides the filter order have to be

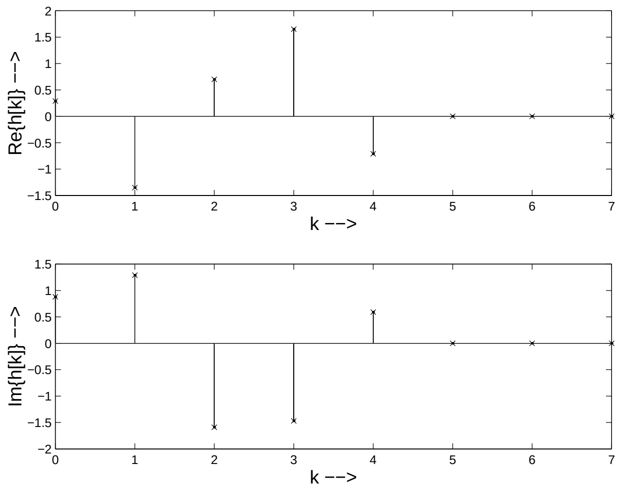

Figure 4: Real and imaginary part of impulse response of channel 2.

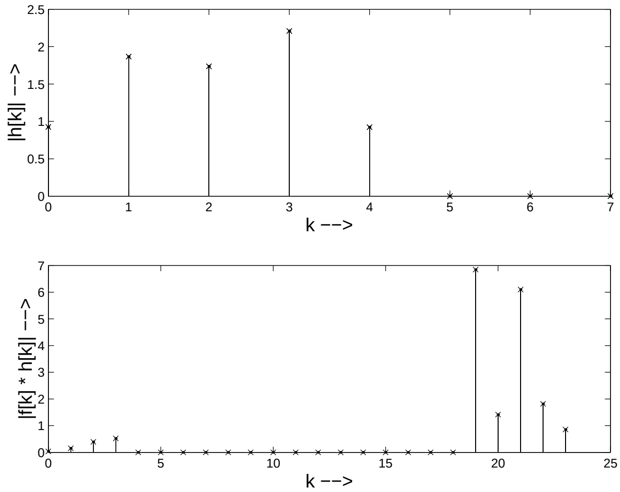

Figure 5: Magnitude of original and transformed impulse response (channel 2).

optimized, and the solution is quite robust.

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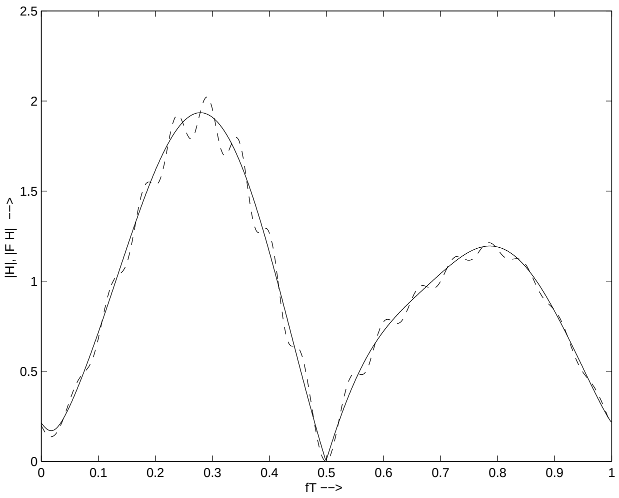

Figure 6: $|H\left(\mathrm{e}^{j2\pi fT}\right)|$ (solid line) and $|F\left(\mathrm{e}^{j2\pi fT}\right) \cdot H\left(\mathrm{e}^{j2\pi fT}\right)|$ (dashed line) (channel 2).

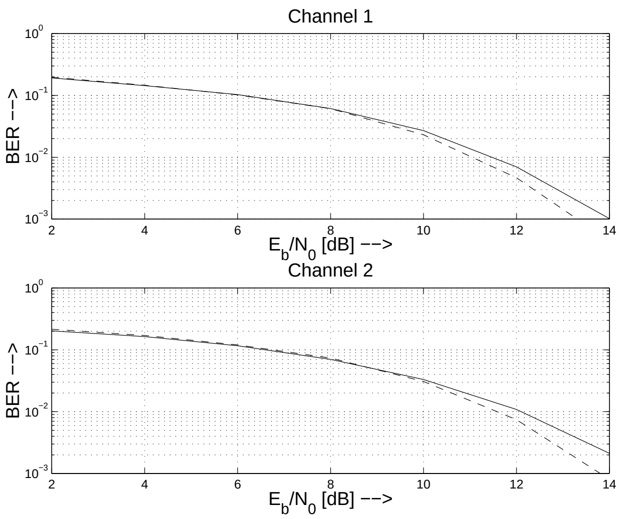

Figure 7: BER vs. E_b/N_0 for RSSE ($Z = 2$) with proposed prefiltering method (solid lines) and allpass prefiltering (dashed lines). Top: channel 1, bottom: channel 2.

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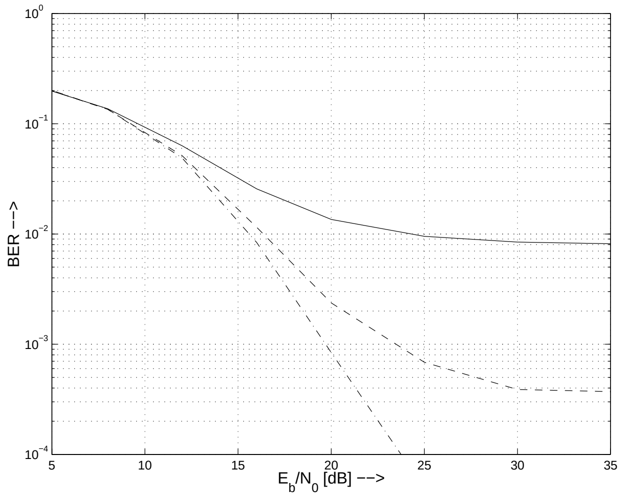

Figure 8: BER at equalizer output vs. E_b/N_0 for EDGE transmission over EQ50 channel. Equalization: RSSE ($Z = 2$) with proposed prefiltering method using filters with different orders. Solid line: $q_f = 15$, dashed line: $q_f = 22$, dash–dotted line: $q_f = 32$.

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