Active Constellation Extension for Increasing the Capacity of Clipped OFDM

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***Abstract*— The paper studies the method active constellation extension (ACE) for increasing the channel capacity of orthogonal frequency-division multiplexing (OFDM) with a nonlinear power amplifier at the transmitter. It turns out that ACE is able to optimize the capacity but at the expense of an increased out-of-band radiation. In order to solve this issue, ACE is combined with the strategy clipping and filtering. With this combination it is possible to increase the channel capacity while reducing the out-of-band radiation.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) [1] in the presence of a nonlinear power amplifier is considered. Recently, a very tight upper bound on the channel capacity, assuming 4-QAM modulation, has been derived in [2]. An observation during the derivation has been that the channel capacity is proportional to the power of the clipped transmit signal. Hence, in order to increase the channel capacity, the strategy of the transmitter should be to maximize the power of the clipped transmit signal.

In this paper we consider the algorithm *active constellation extension (ACE)*, originally proposed in [3] as a *peak-to-average power ratio (PAR)* reduction scheme, in order to maximize the power of the clipped signal. The paper is organized as follows. Section II gives an overview on the considered OFDM system model; Section III describes the adaptation of the original ACE algorithm in order to maximize the power at the output of the nonlinearity, and in Section IV this algorithm is further extended to control the out-of-band radiation; conclusions are drawn in Section V.

II. OFDM System Model

We consider an OFDM transmitter together with a soft limiter (unit gain) as power amplifier model. The whole system is considered as a discrete-time model. A block diagram of this system model is depicted in Fig. 1.

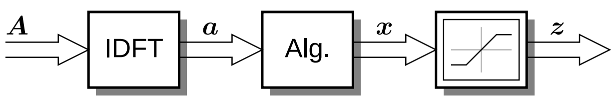

Fig. 1. Block diagram of the OFDM transmitter.

The frequency-domain OFDM frame is denoted as a vector $\boldsymbol{A}_{\mathrm{act}}$ of length D, containing 4-QAM modulated information carrying symbols at each (active) subcarrier. This OFDM frame is transformed into time domain via the energy preserving *inverse discrete Fourier transform (IDFT)* [4]. In order to reflect more closely the properties of the continuous-time signal, I-times oversampling is applied. Moreover, we consider a rectangular pulse shaping filter in frequency-domain. Hence, the time-domain OFDM frame (of length ID) $\boldsymbol{a} = [a_k]$, $k = 0, \ldots, ID-1$, reads

$$a_k = \frac{1}{\sqrt{D}} \sum_{d=0}^{D-1} A_d \mathrm{e}^{\mathrm{j}2\pi \frac{kd}{ID}} \,. \qquad (1)$$

The transformation from frequency into time domain and its inverse, both with transformation length ID, are subsequently abbreviated as

$$\boldsymbol{a} = \mathrm{IDFT}_{ID}\{\boldsymbol{A}\} \quad \text{and} \quad \boldsymbol{A} = \mathrm{DFT}_{ID}\{\boldsymbol{a}\} \,, \qquad (2)$$

where $\boldsymbol{A}$ is the zero-padded version of $\boldsymbol{A}_{\mathrm{act}}$.

The variance of the time-domain samples is denoted as $\sigma_a^2 \stackrel{\mathrm{def}}{=} \mathrm{E}\{|a_k|^2\}$. For convenience, we do not consider a cyclic prefix [1] or modulation to radio frequency, subsequently.

Due to the central limit theorem, the samples of the time-domain signal are approximately Gaussian distributed and therefore prone to the clipping caused by the nonlinear power amplifier.

In order to optimize the performance of the OFDM system, the time-domain OFDM frame is processed by some algorithm. Possible algorithms are discussed in Sections III and IV. As reference, we consider no application of an algorithm to which we refer as "original approach". The processed signal is further denoted as $\boldsymbol{x}$.

The time-domain transmit signal $\boldsymbol{x} = [x_k]$, $k = 0, \ldots, ID-1$, is now processed by a non-linear power amplifier. As power amplifier model we consider the soft limiter with clipping level G. Hence, the clipped signal is given by

$$\boldsymbol{z} = [z_k] \stackrel{\mathrm{def}}{=} g(\boldsymbol{x}) \qquad (3)$$

with

$$g(x) \stackrel{\mathrm{def}}{=} \begin{cases} x & , \; |x| < G \\ G \cdot x/|x| & , \; \text{else} \end{cases} \,. \qquad (4)$$

According to this definition of the nonlinear power amplifier model, the maximum output power of clipped OFDM is given by

$$P_{\max} = \min\{G^2, \sigma_a^2\} \,. \qquad (5)$$

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III. Active Constellation Extension

A. Fundamental Idea of ACE

In [3], Krongold and Jones presented a novel idea to reduce the PAR in OFDM systems, which was named *active constellation extension*.

With this approach, the position of outer points of the signal constellation in each of the D carriers are algorithmically adjusted to minimize the PAR of the particular OFDM frame. Thereby, the signal points may be moved within allowed regions, specific for each constellation. The regions are defined in a way such that the minimum distance between signal points is never decreased. Fig. 2 shows the feasible extension regions (gray) for 4-QAM modulation. Noteworthy, in ACE the energy of the unclipped OFDM frame is always increased. However, due to the choice of the extension region, the (uncoded) error performance does not decrease. Moreover, ACE has the advantage that in order to recover the data at the receiver, no side information is required, and therefore no loss in data rate occurs.

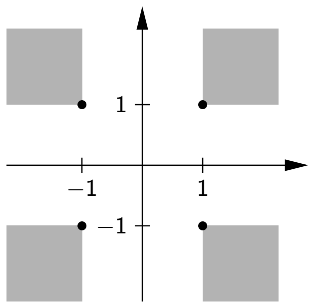

Fig. 2. Feasible extension regions (gray) of 4-QAM when using ACE.

Given the frequency-domain OFDM frame $\boldsymbol{A}_{\text{act}}$, the extended OFDM frame is given by

$$\boldsymbol{X} = \boldsymbol{A} + \boldsymbol{C}\,, \tag{6}$$

where the extension vector $\boldsymbol{C}_{\text{act}}$ must fulfil the so-called ACE constraints, i.e., it has to ensure that the signal points of the extended vector $\boldsymbol{X}$ lie within the feasible extension region of ACE (cf. Fig. 2) and has to be zero within the nonactive subcarriers. Due to the linearity property of the Fourier transform [4] it is possible to write (6) in time domain, i.e.,

$$\boldsymbol{x} = \text{IDFT}_{ID}\left\{\boldsymbol{A} + \boldsymbol{C}\right\} = \boldsymbol{a} + \boldsymbol{c}\,, \tag{7}$$

where $\boldsymbol{c}$ is the time-domain representation of the extension vector $\boldsymbol{C}$.

In [3] an algorithm has been proposed, namely the *gradient-project method*, which finds a suited extension vector $\boldsymbol{c}$ which optimizes the PAR of the transmit signal.

B. Interpretation

As mentioned above, our aim is to increase the power of the transmit signal $\boldsymbol{z}$ after the clipping. The clipping process removes power from the original signal. In particular, this means that the QAM signal points are (on average) shifted to the origin. At this point, the strategy of ACE seems helpful as it works again this effect (the points are shifted further away from the origin). In the following we adapt the gradient-project method from [3] in order to overcome the issue of a reduced transmit power after the clipping.

function $\boldsymbol{x} \leftarrow \texttt{ACE}(\boldsymbol{a},\, N_{\text{ACE}},\, \varepsilon)$
01 $\boldsymbol{x} \leftarrow \boldsymbol{a}$
02 **for** $1, \ldots, N_{\text{ACE}}$
03 $\quad \boldsymbol{c} \leftarrow g(\boldsymbol{x}) - \boldsymbol{x}$
04 $\quad \boldsymbol{C} \leftarrow \text{DFT}_{ID}\left\{\boldsymbol{c}\right\}$
05 $\quad C_d \leftarrow 0,\ d = D, \ldots, ID-1$
06 $\quad C_d \leftarrow 0$, for all $d = 0, \ldots, D-1$, which do not fulfil the ACE constraints
07 $\quad \boldsymbol{c} \leftarrow \text{IDFT}_{ID}\left\{\boldsymbol{C}\right\}$
08 $\quad$ optimization of the step size μ
09 $\quad$ **if** $\mu < \varepsilon$, **break**, **end**
10 $\quad \boldsymbol{x} \leftarrow \boldsymbol{x} + \mu \cdot \boldsymbol{c}$
11 **end**

Fig. 3. Pseudocode of active constellation extension.

C. Gradient-Project Method

A pseudocode description of ACE via the gradient-project method is given in Fig. 3. The parameters of this function are the original (oversampled) time-domain OFDM frame $\boldsymbol{a}$, the maximum number N_{ACE} of iterations, and a positive threshold ε; the return value is the optimized time-domain OFDM frame $\boldsymbol{x}$.

One iteration of this algorithm works as follows. First, the difference between the clipped and the unclipped time-domain OFDM frame is determined (line 03). Next, it has to be ensured that this difference fulfills the ACE constraints (lines 04 to 07), which leads to the extension vector $\boldsymbol{c}$. The update of the time-domain OFDM frame $\boldsymbol{x}$ for the next iteration (line 10) follows (7), but the extension vector is further scaled by the scalar step size μ, which can be optimized with respect to the actual optimization problem. For instance, in [3] the step size μ is chosen in order to minimize the PAR.

In the present case, our aim is to achieve the maximum possible power (5) of the clipped signal. Hence, a suitable choice of μ is given when the power of clipped signal is as close as possible (ideally equal) to $P_{\max}$. Accordingly, the optimization of the step size μ is given by

$$\mu = \underset{\forall \tilde{\mu} \geq 0}{\operatorname{argmin}} \left\{ \left| P_{\max} - \| g(\boldsymbol{x} + \tilde{\mu}\boldsymbol{c}) \|^2 \right| \right\}\,. \tag{8}$$

The algorithm terminates either if the maximum number N_{ACE} of iterations has been completed or if the step size μ falls below a given certain threshold ε, a small positive number.

Especially for small numbers D of subcarriers it might happen, that the feasible extension vector $\boldsymbol{C}$ (after fulfilling the ACE constraints) is the zero vector. In order

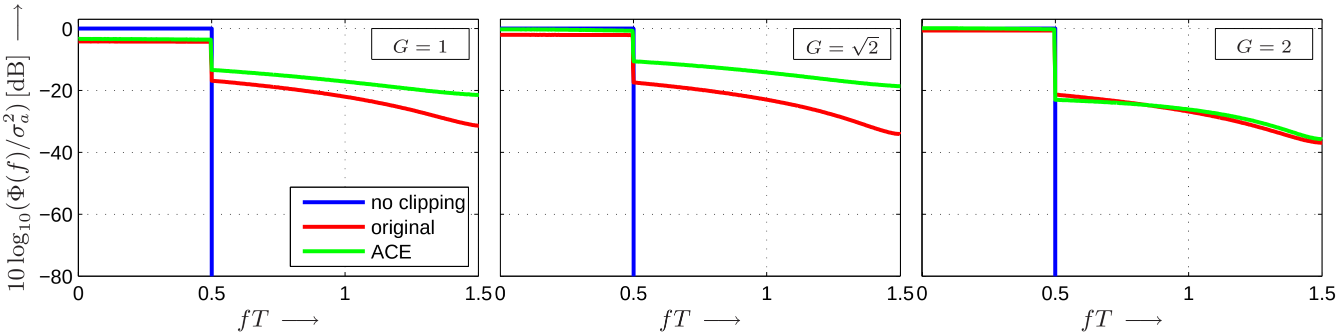

Fig. 5. Normalized average power spectral density of the original clipped signal (red) and after application of ACE with $N_{\mathrm{ACE}} = 10$ iterations (green) for clipping levels (left to right) $G = 1$, $\sqrt{2}$, and 2; the rectangular pulse shaping filter (blue) serves as reference; 4-QAM, $D = 256$, $I = 4$.

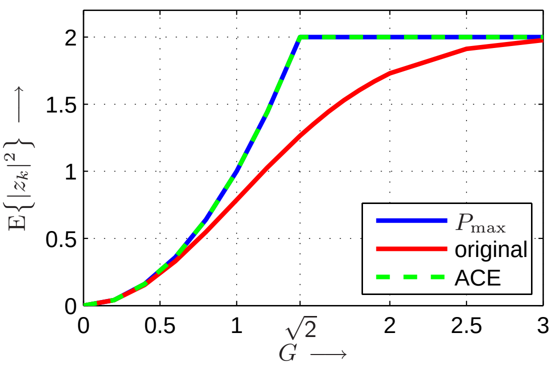

Fig. 4. Average power of the clipped signal samples z_k of the original signal (red) and after application of ACE with $N_{\mathrm{ACE}} = 10$ iterations (green); the maximum possible power $P_{\max}$ is depicted blue; 4-QAM, $D = 256$, $I = 4$.

function $\boldsymbol{x} \leftarrow \texttt{C\&F}\,(\boldsymbol{a},\, N_{\mathrm{C\&F}})$

```
01  x ← a
02  for 1, ..., N_C&F
03      x   ← g(x)
04      X   ← DFT_ID {x}
05      X_d ← 0, d = D, ..., ID − 1
06      x   ← IDFT_ID {X}
07  end
```

Fig. 6. Pseudocode of repeated clipping and filtering.

to help the algorithm to escape from this situation it is convenient to choose an arbitrary vector fulfilling the ACE constraints.

D. Numerical Results and Discussion

Fig. 4 shows the power of the clipped signal over different clipping levels G. The power of the original clipped signal, i.e., where no ACE is applied, serves as reference. It can be recognized that it is possible to achieve the maximum possible output power $P_{\max}$ when applying ACE.

So far, the aim of the ACE algorithm has been to maximize the power of the clipped signal in order to maximize the capacity. Fig. 5 shows the average power spectral density $\Phi(f)$ of the resulting signals over the normalized frequency fT, where T is the duration of the time-domain samples (chip duration), for clipping levels $G = 1$, $\sqrt{2}$, and 2 (from left to right). The power spectral density when no clipping is present serves as reference. From these plots it can be seen that the optimization via ACE increases the power of the clipped signal not only within the used bandwidth but also in the out-of-band region. The generated out-of-band radiation is very undesirable as it disturbs adjacent transmission channels. Hence, our next aim is to increase the power in the used bandwidth while keeping the out-of-band radiation as low as possible.

IV. Active Constellation Extension with Clipping and Filtering

A. Review and Discussion of Clipping and Filtering

In [5] *(repeated) clipping and filtering (C&F)* has been introduced as PAR reduction scheme. Fig. 6 shows the pseudocode of this algorithm. Input parameters are the oversampled OFDM frame in time domain and the number $N_{\mathrm{C\&F}}$ of iterations. Return value is an optimized version of the time-domain OFDM frame with reduced out-of-band power. First, the signal is clipped (line `03`) and second, the signal is filtered (lines `04` to `06`). This procedure is repeated $N_{\mathrm{C\&F}}$ times in order to achieve further gains in out-of-band power reduction.

Looking at the frequency-domain signal after the application of `C&F()`, it turns out that the signal points jitter around the original QAM points. Thereby, the out-of-band power is reduced but also the average power of the clipped signal. Hence, this scheme works against our ACE approach of Sec. III-C. However, a joint application of these two approaches seems reasonable in order to achieve both, maximized power of the clipped signal and a reduction of the out-of-band power.

B. Combination of ACE with C&F

The combination of ACE with C&F is defined as the new function

$$\boldsymbol{x} \leftarrow \texttt{ACE\_C\&F}\,(\boldsymbol{a},\, N_{\mathrm{ACE}},\, \varepsilon,\, N_{\mathrm{C\&F}})\,.$$

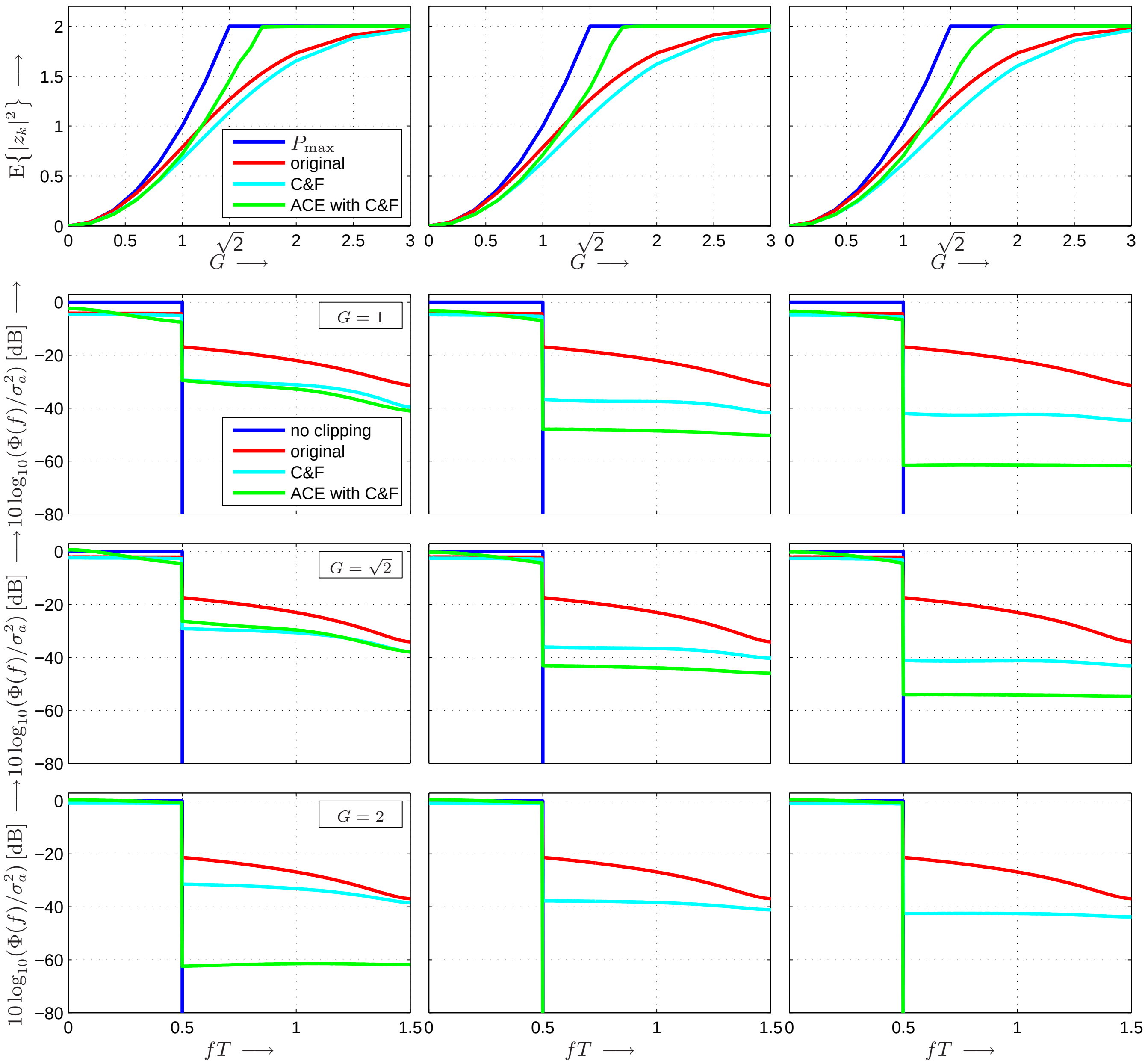

Fig. 7. Top: average power of the clipped signal samples z_k of the original clipped signal (red), after application of pure C&F (light blue), and after application of ACE with $N_{\mathrm{ACE}} = 10$ iterations (green); the maximum possible power $P_{\max}$ is depicted blue; bottom: normalized average power spectral density of the original clipped signal (red), after application of pure C&F, and after after application of ACE with $N_{\mathrm{ACE}} = 10$ iterations (green) for clipping levels (top to bottom) $G = 1$, $\sqrt{2}$, and 2; the rectangular pulse shaping filter (blue) serves as reference; columns from left to right: $N_{\mathrm{C\&F}} = 1$, 2, and 3; 4-QAM, $D = 256$, $I = 4$.

This function is identical to the one of Fig. 3 but with the difference that after line `09` the algorithm `C&F`() (cf. Fig. 6) is called, which leads to the additional line:

$$\boldsymbol{x} \leftarrow \mathtt{C\&F}\left(\boldsymbol{x},\, N_{\mathrm{C\&F}}\right).$$

C. Numerical Results

Fig. 7 shows the power of the clipped signal samples (top row) and the normalized average power spectral density (bottom) for (columns from left to right) $N_{\mathrm{C\&F}} = 1$, 2, and 3. The power spectral densities are for clipping levels (from top to bottom) $G = 1$, $\sqrt{2}$, and 2.

In contrast to pure ACE, the combination of ACE with C&F is not able to achieve the maximum possible output power $P_{\max}$. However, now it is possible to reduce the out-of-band radiation significantly compared to the original signal. This effect becomes stronger for larger numbers $N_{\mathrm{C\&F}}$ of repetitions of the clipping and filtering. Noteworthy, increasing $N_{\mathrm{C\&F}}$ has almost no effect on the achievable power of the clipped signal.

So far, the power of the clipped original signal (red curve in Figs. 4 and 7) served as reference. However, when applying the combination of ACE with C&F, this reference is unfair because it is not affected by the

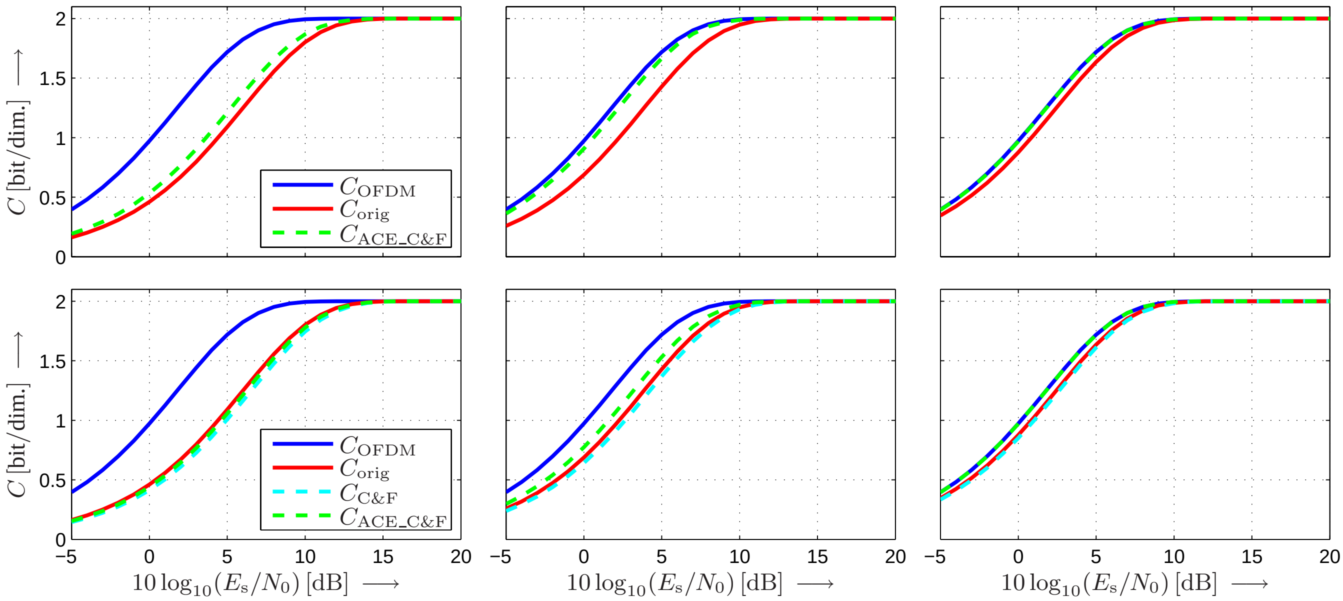

Fig. 8. Upper bounds of the capacity (in bits per complex dimension) over the signal-to-noise ratio (E_s/N_0, with E_s energy per symbol and N_0 one-side noise power density) determined according to [2, Eq. (23)] after the optimization with ACE in combination with C&F (green) for $N_{\mathrm{C\&F}} = 0$ (top row) and $N_{\mathrm{C\&F}} = 1$ (bottom row); as reference serves the capacity without optimization (red), the one with pure C&F using the same number $N_{\mathrm{C\&F}}$ of iterations (light blue), and the one of unclipped OFDM (blue); clipping levels (columns from left to right) $G = 1$, $\sqrt{2}$, and 2; 4-QAM, $D = 256$, $I = 4$.

application of clipping and filtering. To this end, the power of the clipped signal, which is only influenced by pure clipping and filtering (i.e., algorithm $\mathtt{C\&F}()$) employing the same number $N_{\mathrm{C\&F}}$ of iterations, is also depicted in the plots of Fig. 7 (light blue curves). It can be seen, that the power of the clipped optimized signal (i.e., after application of $\mathtt{ACE\_C\&F}()$) is always greater than this reference curve.

Up to now, we considered exclusively the average power of the clipped signal and its power spectral density as performance measure. Our initial aim has been to maximize the channel capacity when transmitting over an AWGN channel with nonlinear power amplifier. Fig. 8 shows upper bounds of the channel capacity according to [2, Eq. (23)]. These upper bounds of the respective capacities have been determined by averaging over $100,000$ out of the $4^D = 4^{256} \approx 1.3 \cdot 10^{154}$ different OFDM frames. The top row shows the capacity when applying pure ACE (i.e., $\mathtt{ACE\_C\&F}()$ with $N_{\mathrm{C\&F}} = 0$), which outperforms the capacity of the original approach. For large clipping levels ($G \geq \sqrt{2}$), the capacity of unclipped OFDM is achievable. Combining C&F with the ACE algorithm leads to slightly lower capacities. However, an improvement compared to the fair reference "pure C&F" is always possible.

V. Conclusions

In this paper, we have adapted the method active constellation extension from [3] in order to maximize the power of the transmit signal of an OFDM system at the output of a nonlinear power amplifier. This step, in turn, increases the channel capacity of the system [2]. It can be shown, that it is possible to achieve the maximum possible transmit power with this algorithm. However, the out-of-band radiation of the system is increased. In order to solve this issue, the algorithm clipping and filtering [5] has been combined with ACE. With this combination the out-of-band radiation can be reduced significantly. Even if it is also possible to increase the power of the clipped signal compared to the reference it is not completely possible to achieve the maximum possible transmit power.

Finally it has to be noted that we restricted our considerations on rectangular (in frequency-domain) pulse shaping filters. However, it turns out that applying more realistic pulse shaping filters into the algorithms discussed in this paper leads to almost the same numerical results.

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