Analysis and Compensation of Non-linear Signal Distortion in Optical OFDM

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***Abstract*— This contribution analyzes non-linear signal distortion in optical OFDM transmission channels. A "weakly non-linear" system approach is used, which treats non-linear signal distortion as additive noise. We review a technique for the separation of observed noise variances into several contributions. Furthermore scenarios are identified for which the compensation of non-linear signal distortion is feasable.**

I. Introduction

FUTURE optical long-haul transmission systems will have to operate at high spectral efficiency due to the increasing bit-rate demand. Orthogonal frequency division multiplexing (OFDM) along with coherent detection is well suited for such systems, as it allows for dense wavelength multiplexing (WDM). Furthermore the modulation alphabet for individual OFDM sub-carriers can be scaled easily and thus spectral efficiency can be adapted to given channel conditions.

The linear dispersive effects of the optical fiber channel exhibit all-pass characteristic; thus, they can be compensated without impairment. The transmission performance is limited by amplified spontaneous emission (ASE) noise and non-linear fiber effects. The most important one is the Kerr effect, which describes the influence of the optical signal power on the refractive index of the waveguide. As a consequence, the Kerr effect is responsible for power dependent phase modulation ("self phase modulation", SPM). Furthermore non-linear crosstalk between WDM channels can be observed ("cross phase modulation", XPM).

For the purpose of characterizing the non-linear signal distortion, a "weakly non-linear" system approach is utilized. We assume that the system's characteristic is dominated by a linear transfer function; non-linear signal impairment is modeled as an additive noise-like distortion. The total observed noise power is the sum of ASE noise power and the mentioned contributions due to SPM and non-linear crosstalk. We review a monitoring approach [1], which allows for separation of the noise power contributions. Simulations are carried out in order to identify scenarios in which the different kinds of distortion are dominating.

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For the case of small numbers of WDM channels distortion due to SPM is dominating. A couple of approaches for SPM compensation have been proposed [2], [3], [4], which show different performance, but also different computational complexity. Their effectiveness can be evaluated through our tool of noise power separation.

Finally, we briefly address the XPM limited case, where the compensation of the non-linear distortion becomes even more challenging. Multiple WDM-channels have to be detected and processed jointly.

II. Analysis of Kerr effect induced signal distortion

The most important source of non-linear distortion in silica fibers is the Kerr effect. It describes the observation that the refractive index of the waveguide is influenced by the power of the optical signal. Hence, there is an interaction between the transmitted signal and the channel: The variation of the refractive index causes power dependent phase modulation of the signal. The propagation of the optical signal in the fiber can be described by the non-linear Schrödinger equation [5]

$$\frac{\partial A(z,T)}{\partial z} + \frac{\alpha}{2}A(z,T) + \frac{j}{2}\beta_2\frac{\partial^2 A(z,T)}{\partial T^2} - \\ -\frac{1}{6}\beta_3\frac{\partial^3 A(z,T)}{\partial T^3} = j\gamma\cdot|A(z,T)|^2 A(z,T). \tag{1}$$

This partial differential equation describes the complex envelope $A(z,T)$ of the optical signal at the position z. T is a modified time variable which is valid in a coordinate system moving with the signal at group velocity. The parameters α, β_2, and β_3 denote attenuation and chromatic dispersion coefficients. Finally, the parameter γ accounts for signal distortion caused by the Kerr effect. It is determined by the fiber material and waveguide structure. No analytical solution is known for this differential equation. For simulations numerical evaluation is utilized, e.g. the split-step Fourier method [5].

As mentioned above the transmission channel is assumed to be dominated by a linear transfer function. Non-linear signal distortion shall be regarded as an additive noise contribution ("weakly non-linear system"). In order to characterize the system and the additive noise, at first pilot symbols are transmitted. After estimation of the linear 2×2-transfer function $\mathbf{H}(d)$ further symbols known to the receiver are transmitted, i.e. $X_1(d)$ and $X_2(d)$ in both orthogonal polarizations. Then the samples of additive distortion on sub-carrier d can be determined

$$\begin{bmatrix} n_1(d) \\ n_2(d) \end{bmatrix} = \begin{bmatrix} Y_1(d) \\ Y_2(d) \end{bmatrix} - \mathbf{H}(d) \cdot \begin{bmatrix} X_1(d) \\ X_2(d) \end{bmatrix}. \quad (2)$$

Next the inverse SNR for both states of polarization ($i \in \{1, 2\}$) is evaluated

$$\frac{N_i}{S_i} = \frac{\sum_{d=1}^{Q} |n_i(d)|^2}{\sum_{d=1}^{Q} |H_{i1}(d)X_1(d) + H_{i2}(d)X_2(d)|^2}. \quad (3)$$

Numerous simulations have been carried out with different optical channel settings, signal powers, WDM parameters, etc. These are the most important observations [1]:

- Low optical input power: N/S[1] decays by 1 dB per 1 dB input power increment (as expected for a linear system with AWGN).
- High optical power: N/S increases by 2 dB per 1 dB power increment (Kerr effect causes self phase modulation).
- Optical powers of WDM neighbors: N/S increases by 2 dB per 1 dB power increment of neighboring WDM channels.

The observed interrelations allow for setting up equations which let us determine individual noise contributions which add up to the value found in (3). In a first step we estimate

$$m_0 = \frac{N_{\text{ASE}} + N_{\text{SPM}} + N_{\text{XPM}}}{S}. \quad (4)$$

[1]The index i for distinguishing between orthogonal states of polarization is omitted in the following.

This is the sum of three noise power contributions at the investigated point of operation. Then the optical input power of the observed WDM channel is increased by $10 \log_{10}(k)$ dB which results in a second estimate

$$m_1 = \frac{N_{\text{ASE}}}{kS} + \frac{k^2 N_{\text{SPM}}}{S} + \frac{N_{\text{XPM}}}{S} \quad (5)$$

according to above mentioned observations. For the separation of the distorting powers due to XPM and SPM a third measurement is necessary. The power of channel 1 is reduced to its original value; the powers of the co-propagating channels are increased by $10 \log_{10}(k)$ dB. This operation yields

$$m_2 = \frac{N_{\text{ASE}} + N_{\text{SPM}}}{S} + \frac{k^2 N_{\text{XPM}}}{S}. \quad (6)$$

From m_2 and m_0 we can derive

$$\frac{N_{\text{XPM}}}{S} = \frac{m_2 - m_0}{k^2 - 1}. \quad (7)$$

This result along with measurements 1 and 2 let us determine

$$\frac{N_{\text{SPM}}}{S} = \frac{-km_0 + k(k+1)m_1 - m_2}{(k+1)(k^3 - 1)}. \quad (8)$$

Finally, these results are subtracted from m_0 in order to obtain the ASE noise power

$$\frac{N_{\text{ASE}}}{S} = m_0 - \frac{N_{\text{SPM}}}{S} - \frac{N_{\text{XPM}}}{S}. \quad (9)$$

III. Simulations

The measurement technique for the separation of noise power contributions is now applied in different WDM scenarios. The simulated optical OFDM transmission setup exhibits a gross bit-rate of 70 Gbit/s per WDM channel. In each polarization 192 sub-carriers are modulated using a 16QAM alphabet. After discrete Fourier transform a cyclic extension of 1/12 of the original symbol duration is applied.

The transmission channel consisted of 2 to 12 spans of standard single mode fiber (80 km each). Further fiber parameters are 0.2 dB/km attenuation, 17 ps/nm/km chromatic dispersion. The non-linear coefficient describing the Kerr effect was set to $\gamma = 1.33/\text{W/km}$. No optical chromatic dispersion compensation is utilized; the OFDM parameters allow for inter-symbol-interference-free transmission over more than 1000 km standard fiber. Optical amplification is done after each span by a fiber amplifier (noise figure 4 dB). The behaviour of AD-/DA-conversion is accounted for by 8 bit quantization and a clipping ratio of 10 dB. The simulation chain also

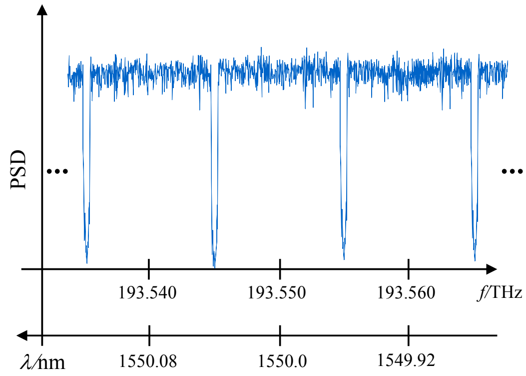

Fig. 1. Dense wavelength division multiplexing.

comprises laser phase-noise, which is modeled as a Wiener process [6], i.e. the summation over Gaussian phase increments. The local laser at the receiver is assumed to have 100 kHz line-width. Based on these parameters the OFDM transmitters generate signals with 9.48 GHz two-sided bandwidth. Simulations have been conducted for different optical powers and numbers of WDM channels (WDM frequency grid: 10 GHz, c.f. Figure 1). Figure 2 depicts noise power contributions for the case of three WDM channels; the per-channel optical power was set to -6 dBm. The separation method according to (4)-(9) was carried out with $10\log_{10}(k) = 2$ dB. The diagram shows the relative noise powers of the ASE-noise contribution, distortion due to SPM and non-linear cross-talk in logarithmic scale versus distance. For the given optical power configuration self phase modulation is the limiting kind of distortion for transmission distances larger than 6 fiber spans. The simulation results show that non-linear cross-talk from the two neighboring WDM channels is approximately 5 dB below ASE-noise

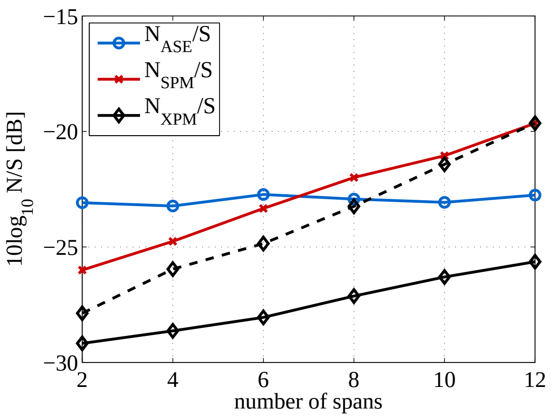

Fig. 2. Separated noise powers versus distance; launch power per channel: -6 dBm, 3 WDM channels; dashed line: Non-linear crosstalk for the setup with 9 WDM channels.

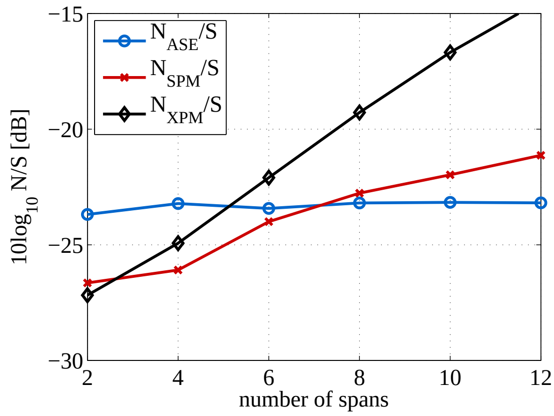

Fig. 3. Separated noise powers versus distance; launch power per channel: -6 dBm, 15 WDM channels.

and SPM-induced distortion. Hence, the total SNR of the observed WDM channel can be optimized by adjusting the power of the investigated channel. Moreover, one can observe that the estimated ASE-noise contribution is approximately constant but, in theory, it should be increasing for longer distances. One reason for this result is, that the separation approach does not consider further sources of distortion, like quantization noise, clipping, and phase-noise. These kinds of distortion become significant for short distances [7] and are independent from the optical signal power. Therefore N_{ASE}/S is slightly overestimated for a low number of fiber spans.

Next, the number of WDM channels was increased. An odd number of channels was chosen and we assume that the observed channel is located at the center of the used spectrum, i.e. there is an equal number of neighboring channels at lower and higher carrier frequencies. The dashed line in Figures 2 along with the diagram in Figure 3 summarize the noise power contributions for 9 channels and for a 15 channels scenario. Obviously, now distortion due to XPM becomes significant. In the latter case it distinctly dominates the overall distortion. For a transmission distance beyond 5 spans one should lower the launch powers of all WDM channels.

These investigations have shown that even moderate numbers of WDM channels lead to XPM-limited transmission. On the other hand if there are just a few wavelengths (below 9 – for our signal and WDM parameters) the SNR is limited by ASE-noise and SPM. As the latter is a function of the signal (however non-linear; an analytical function is not known), the weakly non-linear system approach seems too pessimistic, as – to some extent – distortion due to SPM can be compensated.

IV. Compensation of non-linear distortion

A. Backpropagation

Linear as well as non-linear signal distortion in silica fibers develop as described by (1). The fiber model exhibits the inverted behaviour when the parameters α, β_i and γ are multiplied with -1. Thus, the numerical evaluation of the non-linear Schrödinger equation with inverted parameters is applicable for compensation of all mentioned fiber effects. This approach has been investigated for polarization multiplexing systems (single-carrier as well as multi-carrier) in [4]. The authors show that non-linear distortion can be reduced, which allows for higher optical signal powers. Overall, the SNR could be increased by 2 to 6 dB (no WDM) in dispersion unmanaged fiber links of 2000 km length. The method is highly effective but requires very high computational complexity. In practice, it can not yet be implemented, but it can serve as a benchmark for sub-optimum methods.

B. Non-linear phase modulation

A sub-optimum technique for SPM compensation is based on non-linear phase modulation and was applied along with OFDM in [2], [3]. The starting point is the assumption of a dispersion free fiber channel. Then the non-linear Schrödinger equation (1) reduces to

$$\frac{\partial A(z,T)}{\partial z} + \frac{\alpha}{2} A(z,T) = j\gamma \cdot |A(z,T)|^2 A(z,T) \qquad (10)$$

In that case a solution for the differential equation is known:

$$A(z,T) = A(0,T) \cdot e^{-\frac{\alpha}{2}z} \cdot e^{j\Phi_{\mathrm{NL}}(z,T)} \qquad (11)$$

with

$$\Phi_{\mathrm{NL}}(z,T) = |A(0,T)|^2 \cdot \gamma \cdot \frac{1-e^{-\alpha z}}{\alpha}. \qquad (12)$$

This solution indicates that the influence of the Kerr effect can be compensated by phase-shifting the signal by the respective inverse phase. This operation can be carried out by digital signal processing implemented at the transmitter (“pre-distortion”) and/or at the receiver. A possible way to implement this method in an OFDM receiver is scetched in Figure 4. The time-domain samples $r[k]$ are phase-shifted (after removing the cyclic prefix) by

$$\Phi_{\mathrm{RX}}[k] = |r[k]|^2 \cdot \frac{P_{\mathrm{opt}}}{E\{|r[k]|^2\}} \cdot \gamma \cdot s \cdot L_{\mathrm{eff,RX}}, \qquad (13)$$

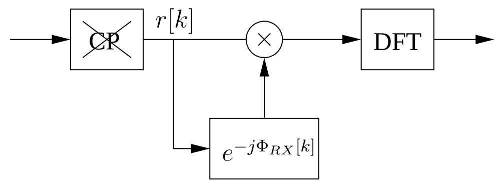

Fig. 4. Non-linear phase-shifting at the OFDM receiver.

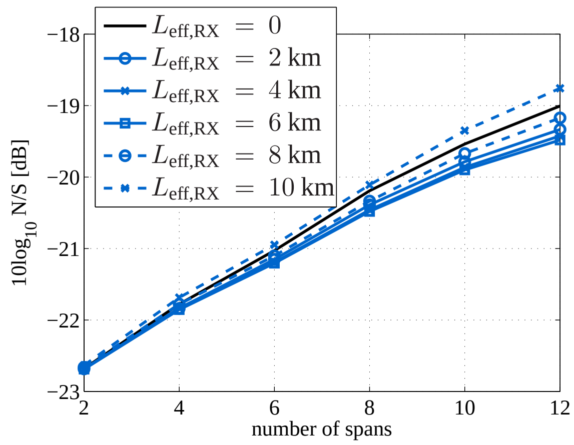

Fig. 5. Estimated N/S versus transmission distance for different settings of the non-linear phase modulator.

with s denoting the number of fiber spans. The squared amplitudes are normalized to the instantaneous optical launch power. As the realistic fiber channel exhibits chromatic dispersion the initial assumption for this method is not valid, but it achieves a compensating effect when the parameter $L_{\mathrm{eff,RX}}$ is chosen properly. Estimated inverse SNRs for values $L_{\mathrm{eff,RX}} = 0 \dots 10$ km are depicted in Figure 5. It shows that SPM compensation is effective for long transmission distances. The optimum $L_{\mathrm{eff,RX}}$ for the given scenario amounts to 6km. There is a SNR gain of approximately 0.5 dB for a transmission distance of 12 spans.

In a next step the non-linear phase modulation is combined with the above discussed method for the separation of sources of distortion. Figures 6 and 7 summarize the simulation results for the 3 and 9 WDM channel scenario, respectively. Dashed lines repeat the curves for the uncompensated case, whereas the solid lines correspond to the system with non-linear phase modulation ($L_{\mathrm{eff,RX}} = 6$ km). One can observe, that N_{SPM}/S can be reduced. The compensating effect increases with a larger number of fiber spans. Contributions by ASE-noise and non-linear crosstalk stay approximately the same. Slight differences may be caused by interaction between SPM compensation and the separation technique. The extraction of the individual contributions re-

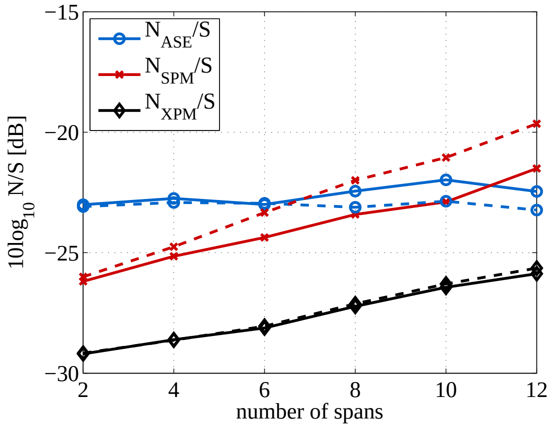

Fig. 6. Separated noise powers versus distance; 3 WDM channels (-6 dBm); dashed lines: no SPM compensation; solid lines: with SPMC, $L_{\mathrm{eff,RX}} = 6$ km.

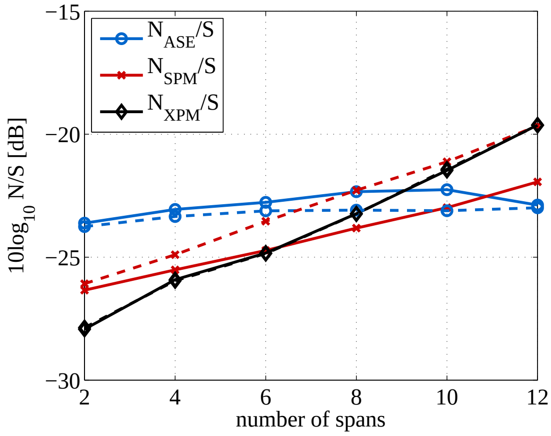

Fig. 7. Separated noise powers versus distance; 9 WDM channels (-6 dBm); dashed lines: no SPM compensation; solid lines: with SPMC, $L_{\mathrm{eff,RX}} = 6$ km.

quires variation of the transmit power, which influences the non-linear phase modulation. This interrelation could be a reason why N_{ASE}/S is estimated slightly larger than in the uncompensated case. For both setups – 3 and 9 WDM channels – there is a similar reduction of the SPM induced distortion, but solely for a low channel count the total SNR is increased.

C. Compensation of XPM

For XPM limited systems the discussed compensation approaches would have to be extended. Back-propagation also works in the multi-channel case [4], but would require joint detection and processing of multiple wavelengths. Of course, this is accompanied by a drastic increase of complexity. As a sub-optimum approach, one can also think of an adaptation of the phase modulation technique. The instantaneous optical power would have to be determined before wavelength demultiplexing [8].

V. Conclusions

In this contribution non-linear signal distortion caused by the Kerr effect has been modeled as an additive noise-like distortion. A method has been presented which allows for separation of observed noise variances into three contributions: ASE-noise, distortion due to self phase modulation and non-linear crosstalk between WDM channels. If there are just few WDM channels the effect of non-linear cross-talk is not dominating and distortion due to self phase modulation may be reduced to some extent by non-linear phase modulation. For future systems high spectral efficiency is desired and therefore they will probably be XPM limited. A sensible approach for the system design would be the usage of the presented separation method for the optimization of optical powers along with omitting SPM/XPM compensation. Instead forward error correction schemes should be defined and optimized on the basis of the observed noise-like distortion, e.g. like in [9].

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