Aspects of LDPC Code Application in High-Speed Coherent Optical OFDM Systems

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***Abstract*— Nowadays one of the discussed solutions for optical transmission systems beyond 100 Gb/s (N x 100 Gb/s) is coherent optical orthogonal frequency division multiplexing (CO-OFDM). A soft-FEC low-density parity-check (LDPC) code is one of the promising candidates that could improve the receiver sensitivity. In this work the efficiency and practical implementation of soft-FEC LDPC codes with 7% and 16% overheads (OH) in coherent optical OFDM systems is discussed.**

I. INTRODUCTION

High spectral efficiency and robustness against fiber dispersion make OFDM attractive for high-speed transmission. Combining optical OFDM with multi-level modulation could further boost spectral efficiency. On the other hand the larger the number of levels of a modulation format the worse the signal sensitivity compared to that of the single-carrier transmission. One of the possible solutions of the arising problem is to apply FEC. One of the requirements for FEC in systems beyond 100 Gb/s is to achieve a net coding gain (NCG) larger than 9.6 dB at post-FEC bit error ratio (BER) of 10^{-13} [8]. This value might also serve as benchmark if we are looking at future applications beyond 100 Gb/s

Recently it was shown that the concatenation of LDPC with low-overhead hard-decision Reed-Solomon (RS) code serving as inner and outer code, respectively, could be used, as RS codes are able to effectively eliminate an error-floor and correct burst-errors after LDPC decoder [5, 8].

In high-speed systems a severe problem that draws attention is circuit complexity. LDPC codes issue a trade-off between efficiency and implementation complexity. We analyze several solutions of the realization complexity problem, namely novel encoding and decoding algorithms as well as the construction of irregular repeat-accumulate (IRA) LDPC codes.

The organization of the paper is as follows. In Section 2 two system models are presented and are used to assess the performance of the code itself and the transmission system operating with FEC, respectively. In Section 3 we investigate the main features of 7% and 16% OH standard irregular and IRA- LDPC codes. We analyze the performance of the codes alone as well as in combination with the 3.8% OH RS code and show the corresponding simulation results. The implementation complexity of these codes is also discussed. In Section 4 the simulated performance is compared with OFDM offline lab experiments by looking at one 20 Gb/s OFDM slice of an up to 250 Gbit/s super channel. Finally in Section 5 the conclusions are drawn.

II. SYSTEM MODEL

A. AWGN SIMULATION SETUP

In order to assess the basic performance of the isolated code a simple system is used. A signal is modulated using binary phase shift keying (BPSK) and the BER is calculated for the additive white Gaussian noise (AWGN) channel by using Monte-Carlo (MC) techniques. We refer to this system model by AWGN simulation setup.

Due to the fact that sequences of length 10^{13} cannot be simulated BER values after the LDPC decoder were analytically extrapolated by using the formula to calculate the BER after an RS decoder given the BER before it. This formula serves only as rough extrapolation means for our simulations and measurements since it assumes that errors are independent from each other and incorrect decoding is not probable [13]. Owing to the bit deinterleaver the LDPC decoder output bits can be assumed to be statistically independent.

B. OFDM

B.1 Simulation Setup

In our polarization multiplexed OFDM scheme 213248 bits are transmitted per one polarization over an AWGN channel (Fig. 1). With a sampling rate of 10 Gsample/s the resulting bit rate on a single polarization is about 10 Gb/s [10, 11, 12].

For numerical study of a CO-OFDM system with the soft-FEC LDPC code the OFDM simulation model was extended by an LDPC encoder and an LDPC soft-decoder with decision unit for RS decoding (Fig.1).

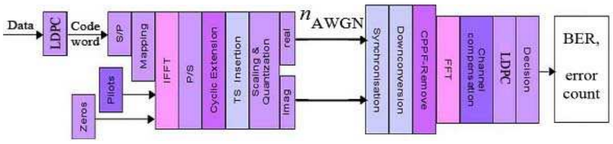

Fig. 1. OFDM simulation setup. Offline processing.

B.2 Experimental Setup

The OFDM system is organized in such a way that OFDM signals' generation and the signal processing at the receiver side are done off-line. I- and Q-components of an OFDM signal are calculated in MATLAB and then generated by an arbitrary waveform generator (AWG) with sampling speed of 10 Gsample/s. They drive an optical I-/Q-modulator with a nested Mach-Zehnder modulator (MZM) structure and followed by a polarization multiplexer emulator (Fig. 2).

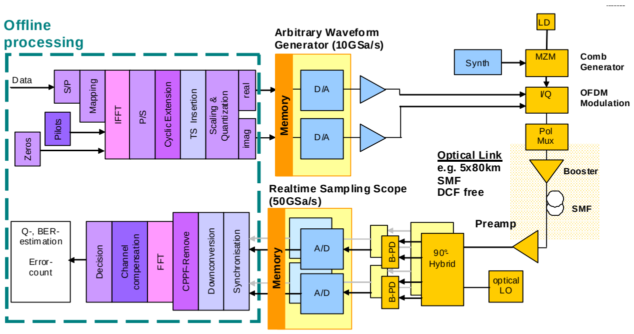

Fig. 2. OFDM experimental setup.

One continuous waveform laser is used. The successive MZM-based comb generator produces 15 lines with 5.96 GHz spacing. The lines, which form 15 sub-bands after modulation, are separated into 'odd' and 'even' paths each of which is OFDM modulated in an independent I/Q-modulator. Afterwards 15 OFDM sub-bands are combined again and put into a polarization division multiplex emulator that forms one super-channel of about 150 Gb/s in each polarization.

We use a dual polarization diversity coherent receiver. One OFDM sub-band is selected by a local oscillator (LO). Two 90°-hybrids aligned to orthogonal polarization directions are followed by 4 balanced receiver frontends that convert the incoming optical signals into the electrical domain. The 4 output signals are sampled by a real time sampling oscilloscope at 50 Gsample/s. The resulting data is then processed on a PC [10, 11, 12].

III. SOFT-FEC

We concentrate on irregular LDPC codes since they perform closer to the channel capacity. Although powerful, LDPC codes tend to have an error-floor that turns FEC to unusable and hence has to be removed for BER of at least 10^{-13}. An outer RS code could effectively eliminate error-floors and correct burst-errors that were observed in simulations and experiments [5].

In [8] an 16% OH LDPC(9252,7967) code was proposed. Its error-floor appearing at post-FEC BER of 10^{-9} is eliminated by the outer 3.8% OH RS(992,956) code. Analytically estimated, the post-RS BER is already lower than 10^{-13}, when input BER is $2.7{\cdot}10^{-4}$. Simulation results based on the AWGN system model showed that the concatenation of the investigated LDPC and RS codes with the total redundancy of 20.5% has no error-floor until a post-FEC BER of at least 10^{-13} [8]. We investigate a similar concatenated scheme with LDPC codes, constructed by using the greedy permutation algorithm, and RS codes. It was analytically estimated that if the maximum number of iterations at the LDPC decoder is set to 16 then the NCG is about 10 dB (Fig. 3).

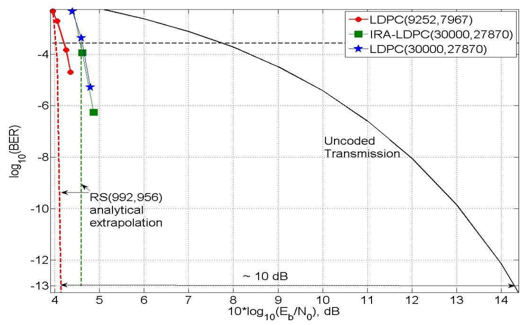

Fig. 3. AWGN channel model. Uncoded transmission (solid rightmost line), 20.5% OH LDPC+RS analytical extrapolation (left dashed line). Maximum number of LDPC iterations: 16. The two-side arrow indicates NCG of about 10 dB at BER of 10^{-13}. 7% OH standard irregular LDPC and IRA-LDPC + RS analytical extrapolation (right dashed line). Maximum number of LDPC iterations: 100. Horizontal dashed line shows RS input BER for output BER of 10^{-13}.

3.1 IRA-LDPC

IRA-LDPC codes, whose first constituent repetition code is irregular, operate very close to a theoretical limit and allow high code rates [2]. The distance from the Shannon limit of the investigated IRA-LDPC(30000,27870) with 7% OH is 0.3529 dB, and it is 0.2495 dB of IRA-LDPC(9252,7967) with 16% OH.

Fig. 3 and 4 show performance results of the 7% and 16 % OH standard irregular and designed IRA-LDPC codes in conjunction with AWGN and with OFDM channel models. At the decoder the maximum number of iterations was set to 100 and then to 16. It was shown that on average there is no degradation in the system performance if the number of iterations is reduced from 100 to 16. This decrease involves the reduction of the decoder throughput and/or latency. The standard irregular and designed IRA-LDPC codes perform indeed comparable. At BER of 10^{-13} NCG estimated by means of RS analytical extrapolation is 9.6 and 10 dB for IRA-LDPC and standard irregular LDPC codes with 7% and 16 % OH, respectively (Fig. 3, 4).

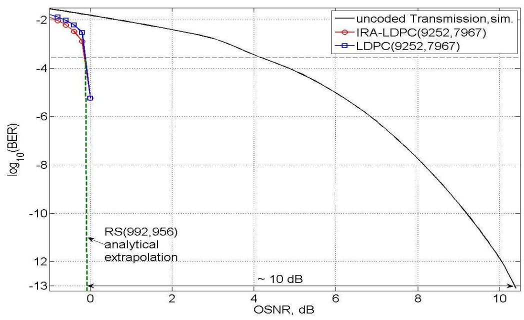

Fig. 4. OFDM channel model. 16% OH IRA-LDPC and standard irregular LDPC. Maximum number of LDPC iterations: 100. Horizontal line shows RS input BER for the output BER of 10^{-13}.

3.2 IMPLEMENTATION COMPLEXITY

A.1 LDPC codes

The conventional LDPC encoding implies two steps. Firstly, an unknown generator matrix (GM) G is derived from a parity-check matrix (PCM) H by using the Gaussian elimination algorithm at a cost of order n^3, where n is the block length. Secondly, an information word is encoded by its multiplication with G. As G is not sparse the encoding complexity has a cost of order n^2 and dramatically increases when n grows. One of the techniques used for efficient LDPC encoding is “Approximate Linear Triangulation” (ALT) of the PCM [6]. On the one hand, ALT implies only row and column permutations preserving the sparseness of the matrix and reducing the LDPC encoding complexity to $O(n+g^2)$, where a gap g is much smaller than n. The greedy permutation algorithm presented in [3] and based on ALT technique minimizes g. The PCM transformation to the ALT form has the computational cost of order n^3. However, it could be done once offline and is to be not critical. On the other hand, for a PCM with a fixed column weight a larger girth always involves a larger gap inducing a tradeoff between encoding complexity defined by the gap and decoding performance defined by the girth [3]. One of the possibilities to increase girth is to expand the redundancy of a code. As a result the 16% OH LDPC code has a larger gap than that with 7% OH (Tab. 1).

Tab. 1. The gap of two different codes created by using the greedy permutation algorithm and whose performance is analyzed in this work.

PCM $H(m, n)$	OH, %	Column weight	Girth	Gap
H(1285,9252)	16.1	3	>6	13
H(2129,30000)	7	3	>6	8

The total decoding complexity of LDPC codes using the iterative decoding belief propagation (BP) algorithm is linear in block length and defined as a product of block length, number of iterations and operations per bit per iteration [10]. But at first, the realization of the BP algorithm requires a huge amount of memory as all log-likelihood ratios (LLR) passed from variable to check nodes have to be saved. Secondly, in order to calculate a nonlinear phi-function a huge number of operations has to be done. Therefore in optical communications the hardware (HW) implementation of the BP decoder is quite complicated [7]. To reduce complexity a min-sum decoder could be used. It needs less calculations and hence provides smaller circuit size, but looses in performance [4, 5]. The cyclic approximated delta-minimum algorithm was presented in [5, 7], which enables a reduced circuit size and does not cost in performance. The delta-minimum algorithm implies the storage of only three most significant LLRs and approximates the complex phi-function [7]. As a result the amount of memory become 5 and the number of operations 10 times smaller than that of BP algorithm [8]. In [5] it was also demonstrated that by using the cyclic approximated delta-minimum algorithm it is possible to implement 20.5% OH LDPC+RS decoder for 100 Gb/s transport systems and to reach a NCG of 9.0 dB at the output BER of 10^{-13}.

A.2 RA codes

RA codes could be viewed as a serial concatenation of repetition and convolutional codes with an interleaver in between, where the convolutional code plays the role of an accumulator. According to the check equations, specific numbers of bits are summed up modulo 2, forming the input sequence of a binary accumulator that produces the parity bit sequence (Fig. 5) [2].

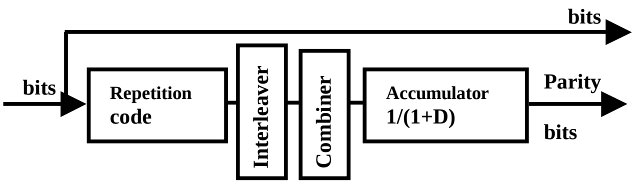

Fig. 5. Systematic encoding of IRA codes

The accumulator corresponds to columns with weight two in the PCM. The repetition code and the interleaver determine weights and structure of the remaining columns, respectively. The complexity of the repetition encoder together with an interleaver is linear. The complexity of the accumulator is negligible. Consequently, RA codes possess linear encoding complexity.

RA codes are a class of LDPC codes, whose encoding is powerful and has low complexity in comparison to turbo-codes [2]. Due to their Tanner graph presentation BP algorithm is well suited to decode IRA codes. Considering the aspects of HW implementation discussed in Section A.1 it is proposed to replace the BP by the cyclic approximated delta-minimum algorithm.

A.3 RS codes

In [9] it was shown that for high-rate RS codes the syndrome-based decoder architecture is preferred because less hardware is required and higher throughput and shorter latency are achievable. The decoder complexity is proportional to the code word length n (Tab. 2). Here $t = (N-m)/2$ is the symbol error correcting capability of the code, and m is the number of bits per symbol [9].

Tab. 2. Syndrome-based decoding. RS decoder complexity.

/	Multiplier	Adder	Register	Mux's	Latency	Throughput^{-1}
$\sum$	$7t$+4	$7t$+2	N+53t+15	19t+8	N+21t	12t

IV. Experimental Results

First, we analyzed the performance of the uncoded signal modulated by QPSK and transmitted over the OFDM channel. The optical signal-to-noise ratio (OSNR) difference between simulation and experimental results in the case of the uncoded transmission and transmission with soft-FEC LDPC is about 3.5 dB and is referred to real system impairments. At BER of 10^{-13} NCG estimated by means of RS analytical extrapolation is at least 10 dB (Fig. 6).

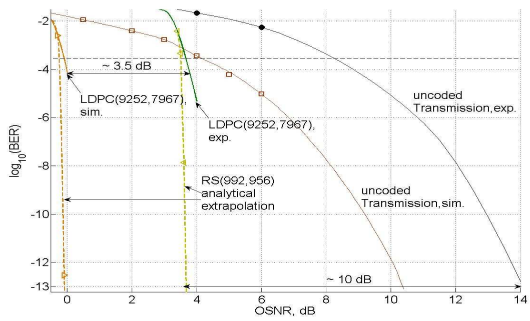

Fig. 6. Comparison of simulation (solid curves marked with right-pointing triangles and squares) and experimental results (solid curves marked with left -pointing triangles and circles) for OFDM channel model with (left curves) and without (right curves) soft-FEC. Maximum number of LDPC iterations: 10. Horizontal dashed line shows RS input BER for the output BER of 10^{-13}.

Along with QPSK the performance of the signal modulated by 8-QAM was analyzed. Both simulations and measurements confirmed the analytically expected 3.5 dB OSNR-offset between QPSK and 8-QAM transmission (Fig. 7). The OSNR difference between the simulation and experimental results for transmission of the 8-QAM modulated signal over the OFDM channel model do not overcome the maximal possible 4.5 dB. Due to real system impairments the transmission of the uncoded 8-QAM signal over the OFDM channel shows an error-floor of approximately 10^{-3} (Fig.7).

We also performed the OFDM offline lab experiment for the 7% OH IRA-LDPC code. Opposite to the standard LDPC, in the case of IRA-LDPC codes only few words were transmitted. The maximal OSNR-offset between simulation and experimental results reaches the expected value of 3.5 dB. Hence at BER of 10^{-13} NCG estimated by means of applying the outer 3.8% OH RS code is at least 9.6 dB (Fig. 8).

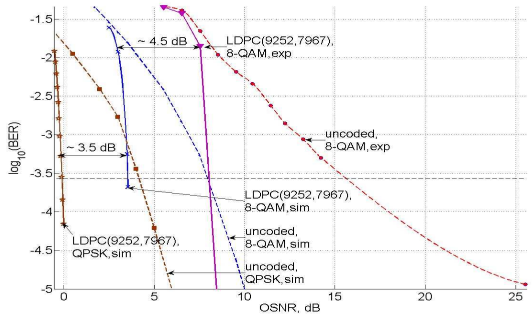

Fig. 7. Comparison of simulation and experimental (solid and dashed right curves) results for QPSK (solid and dashed left curves) and 8-QAM over OFDM channel model with (solid curves) and without (dashed curves) soft-FEC. Maximum number of LDPC iterations: 10.

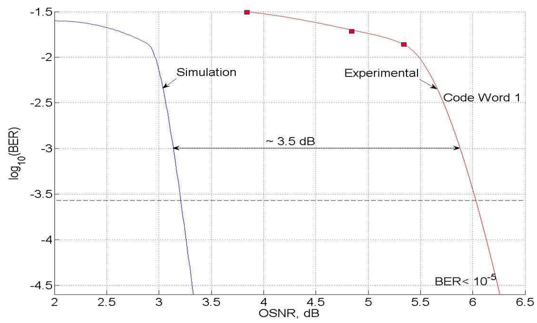

Fig. 8. 7% OH IRA-LDPC code is transmitted over the OFDM channel model. Simulation and experimental results for different IRA-LDPC code words reveal the maximal OSNR difference of 3.5 dB. Maximum number of IRA-LDPC iterations: 100. Horizontal line shows RS input BER for the output BER of 10^{-13}.

Simulation and experimental results show the similar performance of the 7% OH IRA-LDPC code and LDPC code. Moreover simulations reveal a similar performance of the 16% OH IRA-LDPC and LDPC codes. Therefore we can conclude that the performance of the 16% OH codes is also comparable in real systems.

V. Conclusion

As candidates for soft-FEC 7% and 16% OH standard irregular LDPC and IRA-LDPC concatenated with 3.8% OH RS codes were analyzed in this work. Simulations and experiments done with the encoded signal modulated by QPSK or 8-QAM formats and transmitted over AWGN or OFDM channel models confirmed the expected least NCG of 9.6 dB at BER of 10^{-13}. We showed that the performance of standard irregular LDPC and IRA-LDPC codes is comparable, although the encoding complexity of IRA-LDPC codes is significantly lower. In order to futher reduce the realization complexity it is proposed to replace the BP algorithm, heretofore used at the IRA-LDPC decoder, by the cyclic approximated delta-minimum algorithm.

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VII. References

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