Comparison of ConvOFDM and Wavelet–OFDM for Narrow–Band Power Line Communications

Martin Hoch
Lehrstuhl für Informationsübertragung
Friedrich–Alexander–Universität Erlangen–Nürnberg, Germany
hoch@LNT.de

***Abstract*— Systems for Power Line Communications (PLC) mostly apply multicarrier modulations in order to cope with the time dispersive nature of the power line transmission channel. In the emerging IEEE 1901.1 standard on broad–band communications via power line, two alternative techniques, conventional OFDM (convOFDM) and Wavelet–OFDM, are included. However, up to now, only convOFDM systems can be found for narrow–band PLC. Hence this paper compares the suitability of Wavelet–OFDM for this area to that of convOFDM. The main focus is on non–coherent detection, which turns out to fail for Wavelet–OFDM.**

I. Introduction

A plenty of Orthogonal Frequency Division Multiplexing (OFDM) methods has been developed during a long process of research [1], and some of them are deployed in Power Line Communications (PLC), as power line is a time dispersive medium for communications and OFDM is able to cope well with that effect—OFDM techniques can be said to divide the channel into narrow–band (NB) quasi–distortion–free subchannels ("narrow–band approximation", cf. [1]).

The most widely spread variant of OFDM is the one based on Discrete Fourier Transform (DFT) and the cyclic–prefix technique, and in this paper will be referred to as conventional OFDM (convOFDM). Coming up as Discrete Multitone (DMT) for DSL (digital subscriber line) context, this technique is nowadays used in a high variety of modern communication standards, and of course both broad–band and narrow–band PLC systems of different manufactures [2], [3], [4], [5] rely on convOFDM.

In *broad–band* power line communications, however, an alternative, "Wavelet–OFDM", has been established, too [6]. Before this OFDM technique was implemented in PLC devices, it had been proposed under the term "Discrete Wavelet Multitone" (DWMT) for DSL in the 90ies [7]. While at that time DWMT lost the race, Wavelet–OFDM will coexist with convOFDM in the PLC context—made possible by an "inter–PHY" layer as suggested in the PLC standard IEEE 1901 [8].

On the other hand, Wavelet–OFDM has not been used for *narrow–band* PLC systems up to now. These systems mainly apply differential modulation and non–coherent detection, thus avoiding elegantly channel estimation [9]. This topic has not been studied for Wavelet–OFDM in literature until now, but will be the main focus in the following. Thereby, we refer to a discrete time system model for (equivalent complex) base-band signal processing, i.e. the transmitted and received signal are denoted as $s[k]$ and $r[k]$, respectively, and the real analog world including the power line channel is modeled by an impulse response $h[k]$ plus a noise term $n[k]$. Hence the equation of data transmission reads

$$r[k] = s[k] * h[k] + n[k], \tag{1}$$

where $*$ denotes the convolution operator.

At first we review both convOFDM and Wavelet–OFDM in sections II and III, respectively, pointing out their advantages and drawbacks for narrow–band PLC. Then non–coherent detection is discussed for the two OFDM techniques in section IV, followed by the concluding section V.

II. ConvOFDM

In general, an OFDM transmit signal $s[k]$ can be expressed by

$$s[k] = \sum_n \sum_{\mu=0}^{M-1} A_\mu[n] g_\mu[k - nM], \tag{2}$$

i.e. the superposition of M mutually orthogonal subbands μ ($\mu = \{0, 1, \dots M-1\}$) with transmit pulses $g_\mu[k]$, each of them modulated by symbols $A_\mu[n]$, the parallelized stream of data.

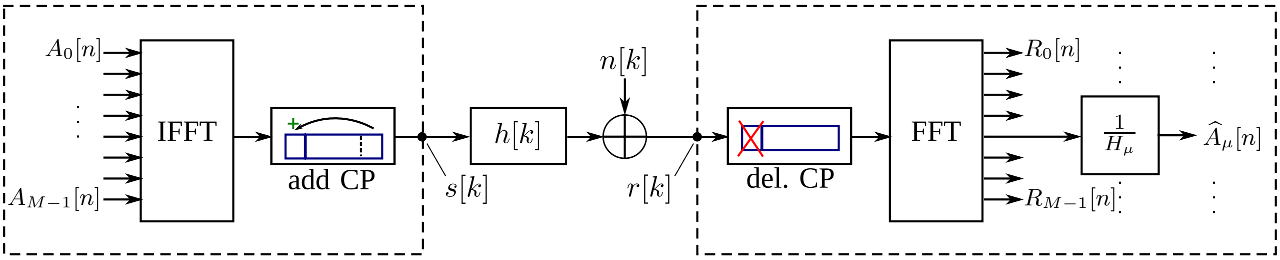

Fig. 1. Block diagram of convOFDM.

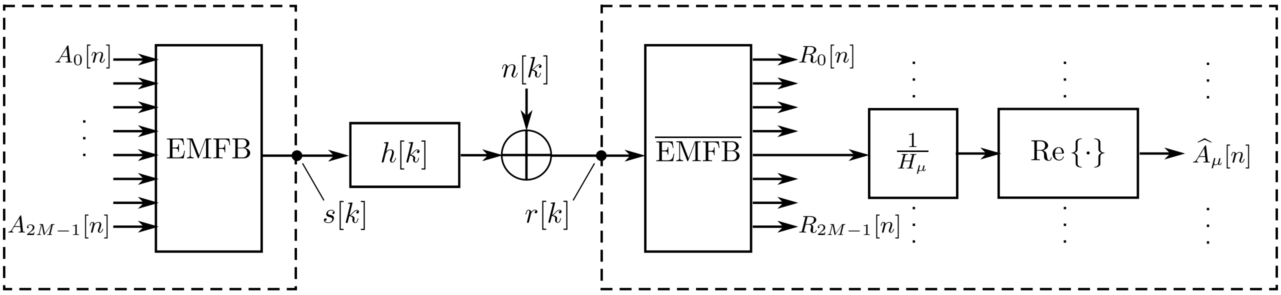

Fig. 2. Block diagram of Wavelet–OFDM.

Conventional OFDM, as depicted in Fig. 1, applies an M size Inverse Fast Fourier Transform (IFFT) to the data symbols and adds a cyclic prefix to the IFFT output.

This means, convOFDM uses the complex exponential pulses

$$g_\mu[k] = \frac{1}{\sqrt{M}} \exp\left(\mathrm{j}2\pi \frac{\mu k}{M}\right), \tag{3}$$

$k \in \{0, 1, \dots M-1\}$ as "subband filters", which provide orthogonality both in time and frequency:

$$\sum_k g_\mu[k + nM] \cdot g_\nu^*[k] = \delta[\mu - \nu]\delta[n], \tag{4}$$

where $\delta[k]$ denotes the Kronecker symbol and $()^*$ the complex conjugate.

As the exp–pulses are eigen–functions of cyclic convolution, adding L_{CP} cyclic repeated samples as a prefix results in the fact that—when transmitting over a time–dispersive channel, modeled by its impulse response $h[k]$—they each are multiplied with their corresponding eigen–value, which read

$$H_\mu = \sum_k h[k] \mathrm{e}^{-\mathrm{j}2\pi \frac{\mu}{M} k}, \tag{5}$$

i.e. the values of the channel transfer function $H(\mathrm{e}^{\mathrm{j}\Omega})$ of $h[k]$ at frequency $\Omega = 2\pi \frac{\mu}{M}$.

Consequently, one can make up a simple transmission model in frequency domain:

$$R_\mu[n] = H_\mu \cdot A_\mu[n] + N_\mu[n], \tag{6}$$

where $N_\mu[n]$ represents the additional noise.

Hence, the narrow–band approximation, mentioned in section I, is mathematically exactly true for convOFDM.

As convOFDM can deal with time–dispersive channels in such an elegant and low–cost way, it qualified for Power Line Communications.

However, one point to be considered carefully is the signaling overhead introduced by the cyclic prefix, which counts

$$\frac{L_{\text{CP}}}{M + L_{\text{CP}}} \tag{7}$$

and is independent of how many symbols are sent.

Secondly, the power line channel is a very hostile environment because of narrow–band disturbers, so that data signals should have a concise spectrum with steep slopes. The convOFDM transmit pulses $g_\mu[k]$ are sequences with a rectangular window in time domain, what corresponds to a sinc–type frequency response. In frequency domain these responses of adjacent subbands overlap (without causing interference to each other), however, the side lobes of the sinc functions are only $13\,\text{dB}$ below the main lobe, cf. Fig. 3. Consequently, a strong narrow band interferer will harm not only a single subchannel, but several ones. Furthermore, switching of a couple of subcarriers in order to prevent interference both from and to third–party narrow–band communication systems is not sufficient, as the notches produced by this technique are not deep enough.

As a remedy windowing has been introduced to convOFDM, i.e. the time domain data blocks

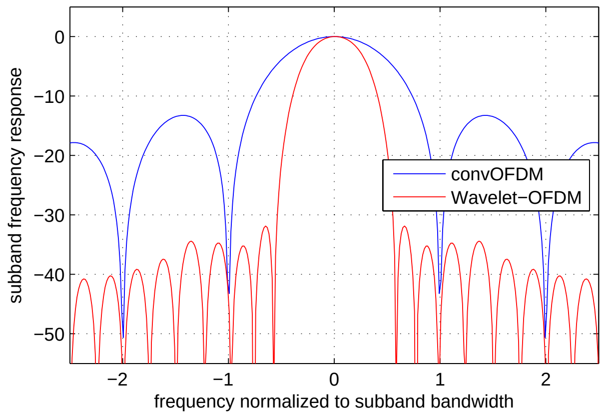

Fig. 3. Frequency responses of subbands for convOFDM and Wavelet–OFDM.

are pre- and suffixed with some additional cyclic repeated samples and in this pre- and suffix, smooth windowing slopes are realized [10], [11].

III. Wavelet–OFDM

As an alternative to convOFDM, Wavelet–OFDM has been introduced in broad–band PLC [6]—an OFDM variant that is based on cosine modulated filter banks (CMFB) [12][1].

A main feature of this technique is that the subband pulses $g_\mu[k]$ have a length $N > M$, i.e. the pulses of consecutive symbols $A_\mu[n]$ and $A_\mu[n+1]$ overlap, but can be separated by the receiver filter bank without any interference as long as no distortion is introduced by the transmission channel. Due to their length $N > M$ the pulses $g_\mu[k]$ provide a more advantageous spectral shape with larger side lobe attenuation than the pulses of convOFDM. In practice $N = 4M$ has been selected [12], resulting in a side lobe attenuation of $35\,\mathrm{dB}$, cf. Fig. 3.

While Wavelet–OFDM was originally developed from CMFB as a starting point, it can be described by an exponentially modulated filter bank (EMFB) in a more general way [13], [14], allowing for an equivalent complex base-band (ECB) signal model for modulated communications, too (see Fig. 2).

The impulse responses $g_\mu[k]$ of an EMFB are equal to

$$p[k]\exp\left(\mathrm{j}\tfrac{2\pi}{2M}\left(\mu+\tfrac{1}{2}\right)\left(k-\tfrac{N-1}{2}\right)-(-1)^\mu\tfrac{\pi}{4}\right), \qquad (8)$$

for $k \in \{0,\dots,N-1\}$ and $\mu \in \{0,\dots,2M-1\}$. Hereby, the prototype filter $p[k]$ is the crucial point to achieve orthogonality in time and frequency and the desired side lobe attenuation. Its design has been studied well in literature [15]. A typical prototype filter for a system with $M = 512$ and $N = 2048$ is depicted in Fig. 4.

[1]The name “Wavelet–OFDM” arises from the fact that these filter banks with perfect reconstruction property are realizations of wavelet transforms.

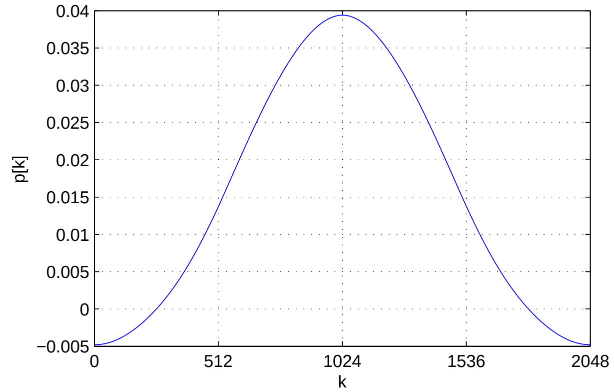

Fig. 4. Typical prototype filter for Wavelet–OFDM with $M = 512$ and $N = 2048$.

The pulses $g_\mu[k]$ can be easily proved to fulfill the orthogonality condition

$$\mathrm{Re}\left\{\sum_k g_\mu[k+nM]\cdot g_\nu^*[k]\right\} = \delta[\mu-\nu]\delta[n], \qquad (9)$$

but not eq. 4.

Consequently, the symbols $A_\mu[n]$ must not be chosen from a complex–valued signal constellation (QAM), but from a real–valued ASK constellation, only. However, Wavelet–OFDM is able to transmit the same data rate as convOFDM using QAM on M subcarriers, as the missing imaginary dimension is compensated by a doubled number of subbands $2M$.

While thus a real–valued $A_\mu[n]$ is transmitted, the receiver generates complex–valued $R_\mu[n]$ and, obeying eq. 9, estimates of the corresponding symbols $A_\mu[n]$ are obtained:

$$\widehat{A}_\mu[n] = \mathrm{Re}\left\{R_\mu[n]\right\}. \qquad (10)$$

As, in contrast to eq. 9,

$$\mathrm{Im}\left\{\sum_k g_\mu[k+nM]\cdot g_\nu^*[k]\right\} \neq \delta[\mu-\nu]\delta[n], \qquad (11)$$

applies, we have—in the case of no linear dispersive distortion and noise (no channel, $r[k] = s[k]$)—

$$R_\mu[n] = A_\mu[n] + \mathrm{j}I_\mu[n] \qquad (12)$$

with defining a real–valued interference term

$$I_\mu[n] = \sum_{\upsilon=0}^{2M-1}\sum_l A_\nu[l]\cdot \qquad (13)$$
$$\cdot\,\mathrm{Im}\left\{\sum_k g_\nu[k+lM]\cdot g_\mu^*[k+nM]\right\}.$$

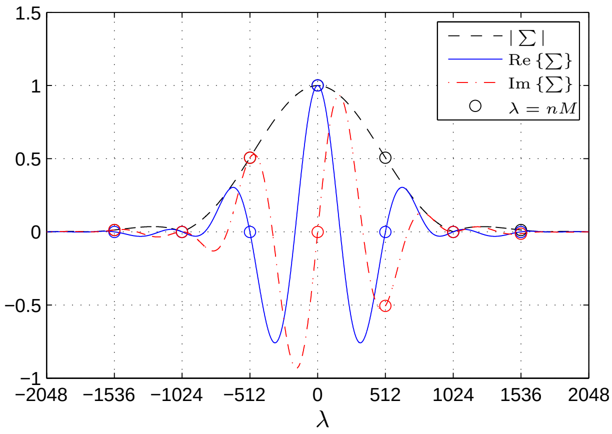

Fig. 5. Illustration of $\sum_k g_1[k+\lambda]\cdot g_1^*[k]$, $p[k]$ of Fig. 4.

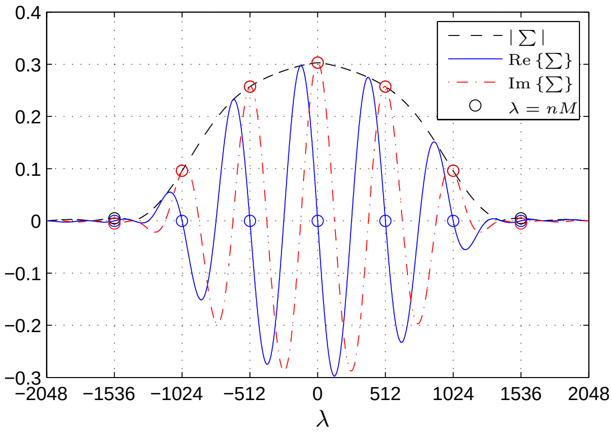

Fig. 6. Illustration of $\sum_k g_1[k+\lambda]\cdot g_2^*[k]$, $p[k]$ of Fig. 4.

For illustration see Figs. 5 and 6, where magnitude and real and imaginary part of $\sum_k g_1[k+\lambda]\cdot g_1^*[k]$ and $\sum_k g_1[k+\lambda]\cdot g_2^*[k]$ is evaluated for a Wavelet–OFDM system based on the prototype in Fig. 4, respectively.

If we consider a Wavelet–OFDM system operating at a time dispersive channel, we firstly have to state that the pulses of Wavelet–OFDM (eq. 8) do not have any eigen–function nature, like the ones of convOFDM have. However, it is justified to assume the narrow–band approximation to be fulfill sufficiently well, because the subbands' frequency responses are well–localized in frequency with a narrow pass–band, steep slopes and a high stop–band attenuation, cf. Fig. 3.

Thus the subchannels are approximated by complex–valued factors H_μ, whereby

$$H_\mu \overset{\text{def}}{=} H(\mathrm{e}^{\mathrm{j}\Omega_\mu}), \quad \Omega_\mu = 2\pi\tfrac{\mu+0.5}{2M}. \qquad (14)$$

and in the receiver, the reciprocal of H_μ will correct for amplitude and phase distortion of the transmitted symbol just like in convOFDM by good approximation. In case of strong channel distortion, a finite impulse response (FIR) filter can be utilized instead of the single compensating factor to equalize the resulting inter–symbol interference linearly [13].

Wavelet–OFDM systems are usually implemented by discrete cosine and discrete sine transform (DCT / DST) [12], as DCT and DST can be calculated at low complexity.

A signaling overhead arises for Wavelet–OFDM from the dying out of the final pulses of data transmission ($3M$ Sample), and hence equals to

$$\frac{3M}{(N_{\text{sym}}+3)M}, \qquad (15)$$

where N_{sym} is the number of OFDM symbols transmitted. Therefore, Wavelet–OFDM is very inefficient if only short packets with a few OFDM symbols have to be transmitted—a scenario, which will occur often in NB–PLC communications.

IV. Non–coherent Detection

As reported so far, the OFDM receiver has to know or at least estimate well the transmission coefficients H_μ. Yet, established and proposed NB-PLC systems based on convOFDM [3], [4] utilize a certain technique that even allows to avoid channel estimation at the receiver and additionally offers a remarkable robustness subject to rather severe symbol timing errors (jitter), and a moderate carrier frequency mismatch [9]. (A carrier phase synchronization and gain estimation, i.e. estimation of complex–valued factors H_μ, is not necessary at all.)

Hereby, the transmitter applies differential phase shift keying (DPSK)[2] across the subcarriers[3], i.e. data is represented by the phase rotation of a symbol $A_\mu[n]$ relative to its predecessor $A_{\mu-1}[n]$, which is expressed by $A_\mu[n]\cdot A_{\mu-1}^*[n]$, where $|A_\mu[n]|, |A_{\mu-1}[n]| = 1$ hold.

Then the receiver can detect the data in a non–coherent fashion from the phase of

$$\begin{aligned} Y_\mu[n] &= R_\mu[n]\cdot R_{\mu-1}^*[n] \qquad (16) \\ &= A_\mu[n]A_{\mu-1}^*[n]|H_\mu||H_{\mu-1}|\mathrm{e}^{\mathrm{j}(\phi_\mu-\phi_{\mu-1})} \end{aligned}$$

with $\phi_\mu = \arg\{H_\mu\}$ and $\phi_{\mu-1} = \arg\{H_{\mu-1}\}$, and while discarding the noise.

[2]It is also possible to use differential amplitude shift keying, additionally (DAPSK).

[3]DPSK *per* subcarrier will work in a similar way.

In (16) the amplitude distortion $|H_\mu||H_{\mu-1}|$ is irrelevant for PSK symbols, and the phase distortion $\phi_\mu - \phi_{\mu-1}$ is close to zero, for H_μ and $H_{\mu-1}$ normally are strongly correlated and hence $\arg\{H_\mu\} \approx \arg\{H_{\mu-1}\}$ applies.

Together with a forward error correcting code, this technique builds a robust communication system with very simple signal processing architecture.

But now, let's consider Wavelet–OFDM.
Digital communications theory requires

$$\Big|\sum_k g_\mu[k+nM]\cdot g_\nu^*[k]\Big| = \delta[\mu-\nu]\delta[n] \qquad (17)$$

for orthogonal non–coherent detection [16], what is not valid for Wavelet–OFDM, cf. eq. 9 and Figs. 5 and 6.

This principle impossibility of orthogonal non–coherent detection of Wavelet–OFDM can be figured out in detail as follows.

Assuming differential encoded symbols (only binary DPSK is possible), the information carried by the phase rotation would have to be extracted from

$$\begin{aligned} Y_\mu[n] \;=\; & (A_\mu[n]A_{\mu-1}[n] + I_\mu[n]I_{\mu-1}[n]) \qquad (18)\\ & + \mathrm{j}\,(-A_\mu[n]I_{\mu-1}[n] + A_{\mu-1}[n]I_\mu[n]). \end{aligned}$$

The real part of $Y_\mu[n]$ indeed contains the information represented by $A_\mu[n]A_{\mu-1}[n]$, but it is not a reliable estimate, as the unknown interference term $I_\mu[n]I_{\mu-1}[n]$ cannot be eliminated.

For example, ante- and postcursor in time, $A_\mu[n-1]$ and $A_\mu[n+1]$, contribute to $I_\mu[n]$ with coefficient ± 0.5, as can be seen in Fig. 5, and adjacent subbands with ± 0.3, cf. Fig. 6. The random data may effect the summands of $I_\mu[n]$ (eq. 13) to add up to a harmful value, so that $-I_\mu[n]I_{\mu-1}[n]$ might be greater than $A_\mu[n]A_{\mu-1}[n]$ and hence cause unpredictable[4] phase shifts of $180°$.

A dispersive channel will at least introduce an additional phase rotation, thus making the problem even worse.

V. Conclusion

As a conclusion, we recapitulate that convOFDM in conjunction with DPSK and non–coherent detection makes up a robust and simple communications system that is suited well for NB–PLC.

Wavelet–OFDM, which is in principle attractive because of its spectral sharpness, however, fails at non–coherent detection and is unhandy for transmission of short data packets.

Therefore convOFDM is the method of choice for narrow–band power line communications.

[4] As the interference term is produced by many data symbols (cf. eq. 13), a sequence detection procedure with exponential complexity would be necessary.

References

[1] S.B. Weinstein, "The history of orthogonal frequency-division multiplexing," *Communications Magazine, IEEE*, vol. 47, no. 11, pp. 26–35, November 2009.

[2] HomePlug Powerline Alliance, *HomePlug AV White Paper*.

[3] iAd GmbH Großhabersdorf, *DLC-2000 Node Flyer*.

[4] PRIME-Project, *Draft Standard for Powerline-Related Intelligent Metering Evolution: Physical Layer Specifications*, Iberdola et al., 8.0 edition, 2008.

[5] Électricité Réseau Distribution France, *PLC G3 Physical Layer Specification*.

[6] H. Koga, N. Kodama, and T. Konishi, "High-speed Power Line Communication System based on Wavelet OFDM," in *International Symposium on Power Line Communications and Its Applications, 2003. ISPLC 2003.*, March 2003, pp. 226–231.

[7] M.A. Tzannes, M.C. Tzannes, J. Proakis, and P.N. Heller, "DMT systems, DWMT systems and digital filter banks," in *IEEE International Conference on Communications, 1994. ICC '94*, May 1994, pp. 311–315 vol.1.

[8] S. Galli and O. Logvinov, "Recent Developments in the Standardization of Power Line Communications within the IEEE," *Communications Magazine, IEEE*, vol. 46, no. 7, pp. 64–71, July 2008.

[9] H. Hampel, J. Huber, K. Lehmann, and T. Wagner, "Digital information transmission system, corresponding process and devices," Patent WO 98 / 12880, September 1997.

[10] S. Müller-Weinfurtner, *OFDM for Wireless Communications: Nyquist Windowing, Peak-Power Reduction, and Synchronization*, Shaker, Aachen, 2000.

[11] G. Bumiller, *Single Frequency Network Technology for Fast ad hoc Communication Networks over Power Lines*, Ph.D. thesis, Technische Fakultät, Universität Erlangen–Nürnberg, 2009.

[12] S. Galli, H. Koga, and N. Kodama, "Advanced signal processing for PLCs: Wavelet-OFDM," in *IEEE International Symposium on Power Line Communications and Its Applications, 2008. ISPLC 2008.*, April 2008, pp. 187–192.

[13] T. Ihalainen, T.H. Stitz, M. Rinne, and M. Renfors, "Channel Equalization in Filter Bank Based Multicarrier Modulation for Wireless Communications," *EURASIP Journal on Advances in Signal Processing*, 2007.

[14] A. Viholainen, J. Alhava, and M. Renfors, "Efficient Implementation of $2\times$ Oversampled Exponentially Modulated Filter Banks," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 53, no. 10, pp. 1138–1142, Oct. 2006.

[15] R.D. Koilpillai and P.P. Vaidyanathan, "Cosine-modulated FIR filter banks satisfying perfect reconstruction ," *Signal Processing, IEEE Transactions on*, vol. 40, no. 4, pp. 770–783, Apr 1992.

[16] J.G. Proakis and M. Salehi, *Digital Communications*, McGraw–Hill, New York, 5th edition, 2009.