Spectral Efficiency Limitation by Fiber Non-linearity in Optical OFDM Transmission Systems

Markus Mayrock(1), Herbert Haunstein(2)

(1) Lehrstuhl für Informationsübertragung, Friedrich-Alexander-Universität Erlangen-Nürnberg, Cauerstraße 7/LIT, 91058 Erlangen, Germany, mayrock@LNT.de

(2) Alcatel-Lucent Deutschland AG, Thurn-und-Taxis-Str. 10, 90411 Nürnberg, Germany

Abstract—**In this paper performance limits of optical OFDM transmission systems are investigated. The maximum achievable spectral efficiency is limited by additive noise of optical amplifiers and non-linear fiber effects. In order to assess this measure, a "weakly non-linear" system model is used. For a given transmission scenario, there is a minimum power of signal distortion at a specific optical launch power; from this measure an estimate for the maximum achievable spectral efficiency can be deduced.**

I. Introduction

The demand for higher bitrates in optical long-haul transmission necessitates techniques which operate at high spectral efficiency. In this context coherent detection is an important technique, as it offers high sensitivity as well as the fact that linear distortion in the optical domain stays linear after opto-electronic conversion. Along with coherent detection orthogonal frequency division multi-plexing (OFDM) is interesting for such systems, as it allows for dense wavelength multiplexing. Furthermore the modulation alphabet for individual OFDM sub-carriers can be scaled easily and thus spectral efficiency can be adapted to given channel conditions.

Recently, results of an OFDM transmission experiment have been published, where the authors report 5.6 bit/s/Hz spectral efficiency by trans-mitting eight times 66.8 Gbit/s over 640 km of uncompensated standard single-mode fiber [1]. Some modification in the set of system parameters allows for enhancement to 7.0 bit/s/Hz [2].

However, the signal-to-noise power ratio cannot be increased to arbitrarily high values by means of increasing the optical transmit power. This limitation is caused by the Kerr effect, which describes the variation of the refractive index of an optical wave-guide under variation of the optical signal power. As a consequence, distorting phase modulation is caused, which is a function of the signal power and finally leads to non-linear signal distortion (self phase modulation). Furthermore the effect causes non-linear crosstalk between signals of different wavelengths and between orthogonally polarized signals.

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In this contribution we investigate an optical OFDM transmission system which deploys dual polarization transmission as well as wavelength division multiplexing. The system parameters are chosen according to the experimental setup of [1].

Our analysis is based on a system identification approach which treats the whole setup as a "weakly non-linear" system. This term reflects that the system characteristic is dominated by a linear transfer function. All kinds of distortion are treated as additive noise, i.e. besides actual noise, distortion due to non-linear effects is modeled as an additive noise-like contribution.

As a result, estimates for the noise variance versus optical launch power and transmission distance are obtained. For a given distance, there is a maximum achievable signal-to-noise power ratio (SNR) at a specific launch power. The simulation results indicate that – at these optimum operating points – the probability density of the observed distortion is Gaussian; thus computation of the maximum achievable spectral efficiency according to Shannon is applicable. We compare this bound with results from a straightforward AWGN-channel approach. Finally, the influence of quantization noise is studied.

II. Investigated System

Fig. 1 depicts the investigated OFDM transmission system. Two independent baseband signals modulate the orthogonally polarized parts of the transmit laser signal. To achieve this, the signal of the TX laser source is split by a polarization beam splitter. Next, two external optical I/Q-modulators are applied before both signal contributions are recombined and launched into the optical waveguide. At the receiver, polarization diverse coherent detection is deployed. Once again

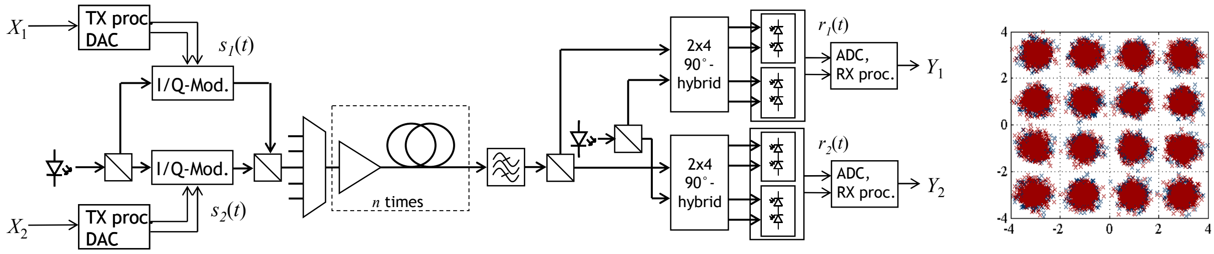

Fig. 1: Investigated system: Dual polarization OFDM transmission, coherent detection – RX constellation.

polarization beam splitters are required to provide orthogonally polarized contributions of the received signal as well as the local laser to optical hybrids. Balanced photo-detectors then convert their outputs to electrical representations of the inphase and quadrature components of both orthogonal RX signals.

For our simulations the gross bit-rate per polarization is set to 30 Gbit/s. $Q = 108$ sub-carriers are modulated with symbols from a 16-QAM-alphabet. The cyclic prefix length equals 1/8 of the original OFDM symbol duration. This choice of system parameters reproduces a setup published in [1]. Eight wavelength division multiplexing (WDM) channels (8.4 GHz bandwidth each) on a 9 GHz grid are simulated; at the receiver there is a 12.5 GHz optical band-pass filter. Laser phase noise is accounted for by multiplication of the complex-valued receive signal with $\exp(j\phi(t))$. The random process $\phi(t)$ is obtained as an integral [4]

$$\phi(t) = \int_0^t \phi'(\tau)\,d\tau\,. \qquad (1)$$

Here $\phi'(\tau)$ denotes zero-mean white Gaussian noise with power spectral density $2\pi\Delta\nu$. Finally, the parameter $\Delta\nu$ describes the laser line-width, which is set to 100 kHz in the sequel.

The transmission link itself consisted of identical spans of standard single mode fiber (length: 80 km, chromatic dispersion coefficient: 17 ps/nm/km, attenuation: 0.2 dB/km). The simulation model for the optical channel considers the Kerr effect with the non-linear coefficient $\gamma = 1.33$/W/km [5]; polarization dependent loss is neglected. Optical amplifiers compensate for attenuation; their noise-figure is assumed to be 4 dB. It should be mentioned that there are no fibers or devices for optical dispersion compensation. Fig. 1 shows the RX constellation after transmission over 8 spans at -8 dBm channel power. The resulting BER is below 10^{-4}, which is an order of magnitude lower than in published experimental results [1] and is attributed to implementation impairments which are not covered by the simulations up to this point. On the other hand, for this point of operation we expect a sensible estimate for the maximum achievable spectral efficiency.

III. Noise Variance Estimation

In order to determine an estimate for the maximum achievable spectral efficiency, at first the signal distortion shall be quantified. As mentioned above, the analysis is based on a system identification approach which treats the whole setup as a “weakly non-linear” system, i.e. the system’s characteristic is dominated by a linear transfer function. All kinds of distortion are treated as additive noise. Besides noise which is added to the signal by optical amplification, distortion due to non-linear fiber effects is modeled as an additive noise-like contribution. This assumption is not valid for arbitrary points of operation, but reasonable for values of optical powers where we expect best transmission performance.

In a first step, the linear transfer characteristic has to be estimated. This is usually done with the help of pilot symbols. In order to estimate the power of

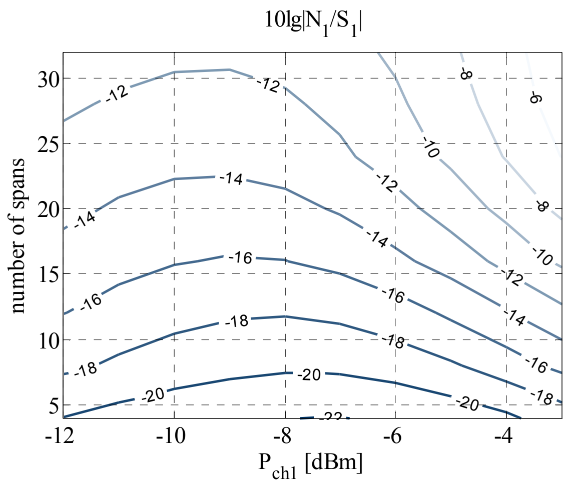

Fig. 2: Estimated relative noise power.

the total additive noise we subtract known data symbols (either further pilot symbols or data after decision) which have been affected by linear distortion through the channel from the received symbols; then we determine the inverse SNR for both receive branches [6]:

$$\begin{bmatrix} n_1(d) \\ n_2(d) \end{bmatrix} = \begin{bmatrix} Y_1(d) \\ Y_2(d) \end{bmatrix} - \begin{bmatrix} H_{11}(d) & H_{12}(d) \\ H_{21}(d) & H_{22}(d) \end{bmatrix} \cdot \begin{bmatrix} X_1(d) \\ X_2(d) \end{bmatrix}, d = 1...Q$$

$$\frac{N_i}{S_i} = \frac{\sum_{d=1}^{Q} |n_i(d)|^2}{\sum_{d=1}^{Q} |H_{i1}(d)X_1(d) + H_{i2}(d)X_2(d)|^2}, i \in \{1,2\} \quad (2)$$

The summation in the nominator and denominator represents integration over the discrete frequency spectrum.

In our simulations, the per-channel optical input power was varied from -12 to -3 dBm. The number of fiber spans ranges from 4 to 32. Based on the transmission of 100 OFDM symbols per polarization the inverse SNR of the orthogonal polarizations at the receiver is estimated according to (2). Fig. 2 shows a contour plot which depicts N_1/S_1 in logarithmic scale. The relative noise power increases with longer transmission distances: from less than -20 dB for short links to more than -14 dB beyond 23 spans. Furthermore the plot shows the interrelation between estimated noise variance and optical input power: For low input powers, a power increment reduces the variance of additive noise at the receiver. At a certain power level distortion due to fiber non-linearity comes into play and noise power increases.

IV. ACHIEVABLE SPECTRAL EFFICIENCY

According to Shannon the maximum information-rate which can be transmitted over a band-limited additive white Gaussian noise channel is

$$C = B \cdot \log_2\left(1 + \frac{S}{N}\right), \quad (3)$$

where S/N and B denote the signal-to-noise power ratio and the used bandwidth. Division by B results in the achievable spectral efficiency Γ.

In order to obtain an estimate for the maximum achievable spectral efficiency of the simulated OFDM system we determine Γ for both receive branches on sub-carrier basis

$$\Gamma_i(d) = \log_2\left(1 + \frac{E\{|H_{i1}(d)X_1(d) + H_{i2}(d)X_2(d)|^2\}}{E\{|n_i(d)|\}^2}\right). \quad i \in \{1,2\} \quad (4)$$

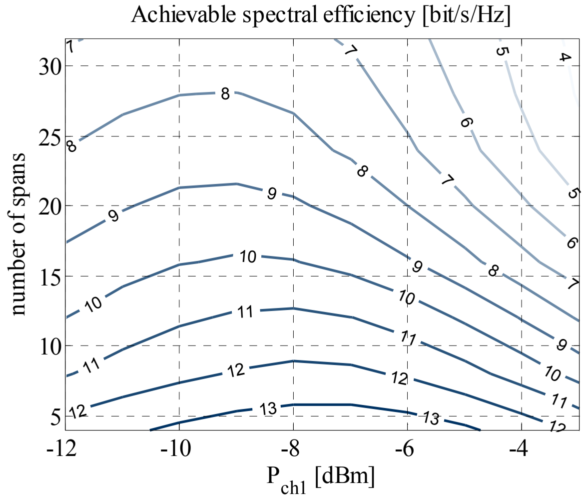

Fig. 3: Achievable spectral efficiency versus distance and launch power.

The optical channel is assumed to be free of polarization dependent loss. Thus the contributions of the orthogonal polarizations are added up. Furthermore we average over the OFDM sub-carriers

$$\Gamma = \eta \cdot \left(\frac{1}{Q}\sum_{d=1}^{Q}\Gamma_1(d) + \frac{1}{Q}\sum_{d=1}^{Q}\Gamma_2(d)\right). \quad (5)$$

The coefficient η accounts for guard bands between WDM channels; in this scenario there is an occupation ratio η = 8.4/9. The simulation data obtained for distances of 4 to 32 spans and optical powers of -12 to -3 dBm was analyzed according to these equations. The results are summarized in the contour plot in Fig. 3. The simulated transmission scenario allows for spectral efficiency of slightly more than 13 bit/s/Hz for a distance of 4 fiber spans. For each transmission distance there is a distinct optical power level which maximizes the SNR in the sense of a weakly non-linear system and at the same time yields the corresponding capacity. The extraction of the associated maximum achievable spectral efficiency versus distance is shown in Fig. 4. We observe a capacity decay to about 7.5 bit/s/Hz for a distance of 2500 km. Due to the non-linear characteristic of the channel these results are a valid estimate for the given optical setup, powers, and WDM-parameters. The channel capacity is limited by actual additive noise along with non-linear signal distortion and non-linear cross-talk.

For a comparison the diagram shows two curves which were obtained from approaches which consider linear AWGN channels. The investigated transmission system is assumed to consist of equally spaced, identical optical amplifiers. The

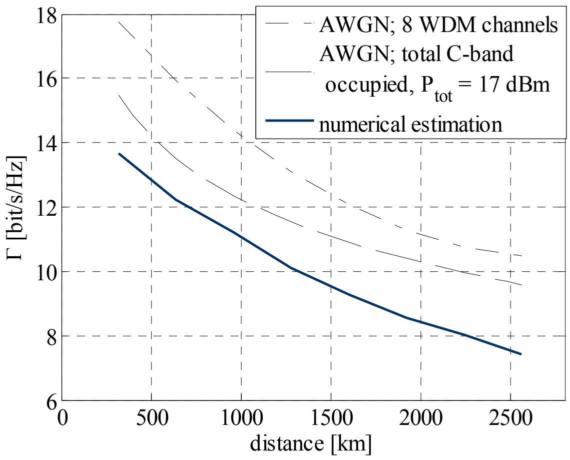

Fig. 4: Achievable spectral efficiency vs. distance.

optical SNR (OSNR) after N_{spans} amplifiers is given by [7]

$$\mathrm{OSNR} = \frac{P_{opt} \cdot \lambda_0}{G F_N \cdot h \cdot c \cdot N_{spans} \cdot B_{ref}}. \qquad (6)$$

Here h and c denote Planck's constant and the speed of light, respectively. G is the amplifiers' gain which shall equal the loss of one fiber span. The noise figure is given by F_N. Finally λ_0 and B_{ref} denote the reference wavelength (1550 nm) and bandwidth for noise power measurement. In optical communications usually B_{ref} equals 12.5 GHz. However, insertion of the actual OFDM signal bandwidth leads to an accurate SNR estimate which can be applied to Shannon's equation. The upper curve in Fig. 4 was obtained by this calculation, whereby for each transmission distance the optimum optical launch power found in Fig. 3 was inserted. It can be observed that the numerical estimates, which consider the fiber non-linearity as an additive noise contribution, deviate from the pure AWGN channel by approximately 3 bit/s/Hz. Hence, at the optimum point of operation with respect to input power, the system still suffers from a distinct amount of distortion due to non-linear effects.

An alternative way of finding a sensible value for the per-channel launch power is based on WDM transmission parameters. Commercially availably optical amplifiers operate in the so-called C-band which approximately ranges from 1525 to 1565 nm. Equivalently a total band-width of 5.0 THz can be used for parallel transmission. The maximum optical power typically reaches 17 dBm, e.g. used in [8]. In the case where the total bandwidth shall be occupied by OFDM bands on a 9 GHz grid, the per-channel launch power amounts to P_{opt} = -10.4 dBm. These considerations are the basis for the dashed line in Fig. 4, obviously leading to reduced capacity compared to the 8 channel AWGN scenario. Full occupation of the C-band results in penalty due to increased non-linear signal distortion.

The numerical estimates for the achievable spectral efficiency in Fig. 4 exhibit a gap of approximately 6 bit/s/Hz compared to the reported values of transmission experiments [1][2]. These results suggest that an increase of spectral efficiency is possible for the used fiber links. However, first the influence of quantization noise has to be incorporated into the simulation model.

V. ADC/DAC RESOLUTION

High speed devices for analog-to-digital and digital-to-analog conversion (ADC and DAC) with sampling rates beyond 20 GS/s have been demonstrated [9] as well as the integration of four such devices, which are need for sampling of inphase and quadrature components of the received signals of both polarizations. At such high sampling rates these devices have a more or less limited number of quantization levels. The authors of [9] report 6 bits resolution while state-of-the-art measurement devices provide higher accuracy. Thus, quantization noise as well as clipping has to be taken into consideration as performance limiting factors for high speed optical OFDM transmission. For this purpose the simulation models of the transmitter and receiver were extended by DAC and ADC device characteristics.
In further simulations the number of quantization levels was varied. The clipping ratio (maximum amplitude over the signal's root-mean-square) was set to 10 dB.

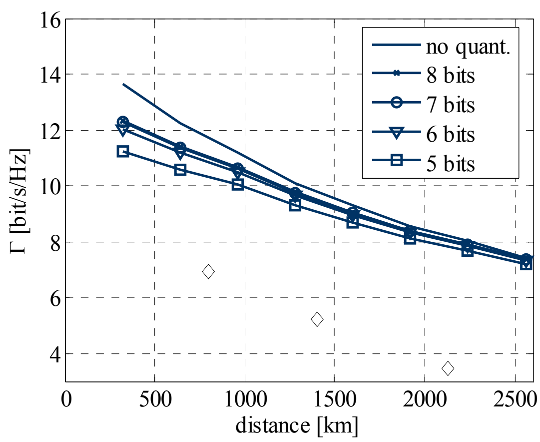

Fig. 5: Achievable spectral efficiency versus distance considering quantization noise. Diamonds denote actual spectral efficiencies; pre-FEC BER: 10^{-3}; code rate 0.93.

Fig. 5 depicts the resulting estimated achievable spectral efficiency. One can observe that there is marginal difference between the curves corresponding to 8, 7, and 6 bit resolution. In these cases there is a loss of approximately 1 bit/s/Hz for a distance of 500 km and just 0.5 bit/s/Hz for 1000 km. This loss is mainly attributed to distortion due to clipping. For longer transmission distances optical amplifier noise and fiber non-linear effects dominate. Usage of 5 bit DACs/ADCs introduces a distinct amount of quantization noise. Therefore, for actual implementations it will be desirable to use at least 6 bit quantization.

In a further analysis of the simulation data of Fig. 2, transmission distances were determined, which lead to a bit error ratio of 10^{-3} when different QAM alphabets are used. This value is a typical maximum pre-FEC bit error ratio, which can be handled by forward error correction codes, which have been standardized for optical communications. These codes exhibit a rate of 0.93. Diamonds in Fig. 5 show respective distances along with the achieved spectral efficiency for 16QAM, 8QAM, and 4QAM. The calculation of the spectral efficiency considers the code rate as well as occupation ratio of the WDM spectrum. Hence such systems are assumed to transmit over 800 km at a spectral efficiency of 7 bit/s/Hz. Error-free transmission at 3.5 bit/s/Hz is supposed to work over 2100 km. These results imply that transmission performance of future systems may be enhanced by introducing more powerful FEC coding schemes.

VI. Conclusions

In this contribution we investigated an optical OFDM transmission scenario with respect to maximum achievable spectral efficiency. This measure is limited by amplifier noise and non-linear fiber effects causing signal distortion and non-linear crosstalk between WDM channels. A weakly non-linear approach was applied which treats all these kinds of distortion as additive noise. Based on noise variance determination, estimates for the maximum achievable spectral efficiency are obtained, which amount to a range of 13 bit/s/Hz to 7.5 bit/s/Hz for distances from 320 km up to 2500 km. Realistic ADC/DAC characteristics turned out to slightly reduce the achievable spectral efficiency for short to medium distances. For long-haul transmission, quantization noise is not dominating.

Forward error correction codes standardized for optical transmission leave a gap of approximately 4 bit/s/Hz to the estimated upper bound. To reduce this gap, advanced FEC schemes are required, which are under investigation together with a partner project in the TakeOFDM framework.

VII. References

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