

Loading-Aware Bit-Interleaver Design

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Abstract—The design of bit interleavers for convolutionally encoded orthogonal frequency-division multiplexing transmission using rate and power loading is analyzed. The study is based on an equivalent channel model consisting of several independent parallel binary input channel (aka. bit levels). Using insights on the non-iterative decoding of bit-interleaved coded modulation and some effects of rate and power loading on the characteristics of the bit levels, we propose a novel design guideline for bit interleavers which leads to significantly better bit-error ratios than the commonly employed random bit interleavers. The advantages of the new design are in particular observable for high spectral efficiencies, where conventional bit-interleaver designs cannot benefit from an adapted rate distribution. The proposed approach does not require additional side information beyond the adapted rate distribution.

I. INTRODUCTION

We consider high-data rate transmission over frequency-selective channels. Due to the dispersive characteristics of the channel the receive symbols suffer from intersymbol interference (ISI). Orthogonal frequency-division multiplexing (OFDM) is employed in order to compensate for this effect. The dispersive channel is decomposed into a set of frequency-flat carriers. Although ISI may be overcome by the use of OFDM, the received signal may still exhibit some unfavorable properties. In particular, the signal attenuations experienced within the individual carriers usually are rather different, resulting in high variations of the reliability of the respective receive symbols.

Rate and power loading is a well-known measure to resolve this particular problem and significantly improves the performance of (uncoded) multicarrier transmission. Numerous algorithms have been proposed in literature which either adapt rate or power distribution or both to the conditions experienced on the carriers, e.g., [2], [5], [6], [4], [8], [13]. All of these loading algorithms share the need for channel state information (CSI) for the derivation of optimized rate and power distributions. Although, channel knowledge is not necessarily required at both side of the transmission. In this paper, we assume a scenario where optimized rate and power distributions are sent back to the transmitter from the receiver, i.e., perfect CSI is only available at the receiver side.

In digital transmission systems forward error correction commonly is employed to preserve the transmitted information from being corrupted. Preferring a low-complexity, low-latency implementation of channel coding, the well-known concept of bit-interleaved coded modulation (BICM) [1] with non-iterative decoding is chosen in this work. Codewords span over the parallel

carriers and are limited to a single OFDM symbol (coding over carriers). In [10], [11] we have already demonstrated the crucial role of the bit interleaver for the performance of BICM schemes. An interleaver design tailored to the channel encountered by the codewords, here the individual carriers, significantly enhances the performance of BICM.

In the following, we study the interaction of rate and power loading and the applied coding scheme. Thereby, our focus is on the design of the bit interleavers. Previous publications, e.g., [11], have revealed rather poor results for the plain combination of rate and power loading with BICM at higher spectral efficiency if random bit interleavers are used. On the other hand, bit-interleaver designs taking into account the available channel state information have shown to be rewarding in terms of the bit-error ratio (BER), cf. [10], [11].

II. SYSTEM DESCRIPTION

We employ a generic system model for coded adaptive OFDM transmission over frequency-selective channels using D carriers, cf. Fig. 1. A source sequence \mathbf{q} of K_{bs} binary symbols shall be communicated to the receiver and thus is encoded into a binary sequence \mathbf{c} of length N_{bs} by a non-recursive, non-systematic convolutional rate- $(k'/n' = R_c)$ encoder (ENC). The bit interleaver Π permutes and partitions \mathbf{c} into D binary R_d -tuples \mathbf{x}_d . Here, $d = 1, \dots, D$ denotes the carrier index and implicitly indicates a carrier dependency of the respective variable. The sizes of the binary tuples, i.e., the individual rates $R_d \in \{0, 1, 2, \dots, R_{\text{max}}\}$ form the rate distribution $\mathbf{R} = [R_1, R_2, \dots, R_D]$; R_{max} is the maximum number of bits per carrier. Clearly, the sum over the individual rates has to match the number of encoded bits to be transmitted, i.e., $\sum_d R_d = N_{\text{bs}}$ must hold. The respective power allocation is denoted by $\mathbf{P} = [P_1, P_2, \dots, P_D]$ with $\sum_d P_d = P_{\text{tot}}$, with the total transmit power P_{tot} and the individual powers per carrier P_d .

In the d -th carrier, an R_d -tuple $\mathbf{x}_d = [x_d^{(1)}, \dots, x_d^{(R_d)}]$ of binary symbols $x_d^{(\mu)} \in [0, 1]$, $\mu = 1, \dots, R_d$, is mapped onto channel symbols $a_d \in \mathcal{A}_d \subseteq \mathbb{C}$ by the bijective mapping $\mathcal{M}_d : \mathbf{x}_d \mapsto a_d$. In this paper, for $R_d > 0$ the carrier-dependent sets of channel symbols are restricted to M_d -ary ($M_d = 2^{R_d}$) amplitude-shift keying (M_d -ASK) with $\mathcal{A}_d = \{\pm 1, \pm 3, \dots, \pm(M_d - 1)\}$; for $R_d = 0$ the signal set is given as $\mathcal{A}_d = \{0\}$.¹

¹An extension of our analysis to square QAM (M_d^2 -QAM) constellations— M_d -ASK per dimension—is straightforward.

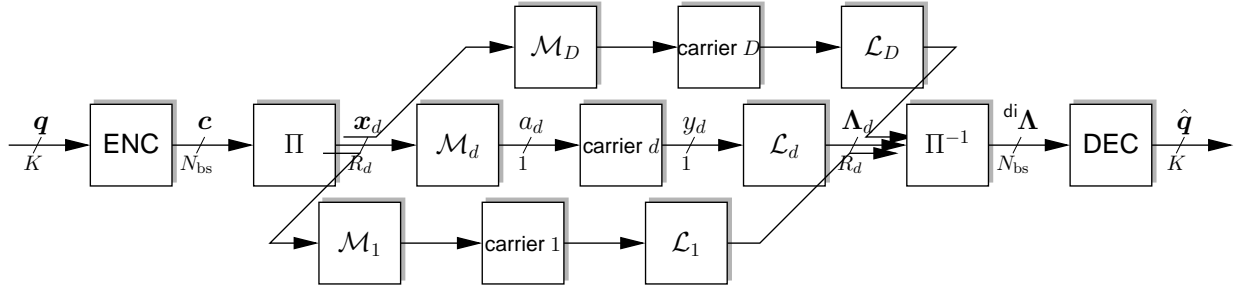


Fig. 1. Block diagram of coded multicarrier transmission with D parallel carriers.

After transmission over the channel with the complex channel coefficient h_d , the receive signal of the d -th carrier in frequency domain reads

$$y_d = h_d \cdot \rho_d \cdot a_d + n_d. \quad (1)$$

Here, n_d is a sample of the complex-valued additive white Gaussian noise (AWGN) with carrier-independent variance² $\sigma_N^2 = N_0/T_s$. N_0 denotes the one-sided noise power spectral density, T_s is the symbol duration. The real-valued scaling factor ρ_d ensures transmit power P_d and thus is given as $\rho_d = P_d/\sigma_{A_d}^2$ ($\sigma_{A_d}^2$: variance of transmit symbols $a_d \in \mathcal{A}_d$). Defining the carrier-dependent average energy per received channel symbol $(E_s)_d = |h_d|^2 \rho_d^2 \sigma_{A_d}^2 T_s$ we can write the carrier-dependent signal-to-noise-ratio (SNR)

$$(E_s)_d/N_0 = |h_d|^2 \cdot \rho_d^2 \cdot \frac{\sigma_{A_d}^2}{\sigma_N^2}, \quad (2)$$

and define the average energy per information bit for the entire OFDM symbol

$$\bar{E}_b/N_0 = \frac{\sum_d (|h_d|^2 \cdot \rho_d^2 \cdot \sigma_{A_d}^2)}{D \cdot \sigma_N^2 \cdot K_{\text{bs}}}. \quad (3)$$

At the receiver side the signals y_d are passed into metric generators (\mathcal{L}_d) to compute R_d -tuples of pairs of bit metrics $\Lambda_d = [\lambda_d^{(1)}, \dots, \lambda_d^{(R_d)}]$. After Π^{-1} the stream of deinterleaved bit metrics ${}^{\text{di}}\Lambda$ is fed into a non-iterative decoder (DEC) which finally returns an estimate \hat{q} on the initial binary source sequence q .

III. BIT-LEVEL-BASED ANALYSIS

In recent publications, e.g., [10], [11], [13], [12], the introduction of an alternative representation of bit mapping and channel has proven itself valuable for an analysis of the bit interleaver design resp. the rate and power loading problem. We replace the cascade of bit mapping \mathcal{M}_d and channel (any scaling plus additive noise) present in each carrier by an equivalent model which has been first studied in detail in [14]. The equivalent channel model consists of R_d parallel binary input channels, the so-called bit levels, which share a common scalar output y_d , cf. Fig. 2. As the mappings \mathcal{M}_d are bijective, the

²Upper case letters denote the respective random variables.

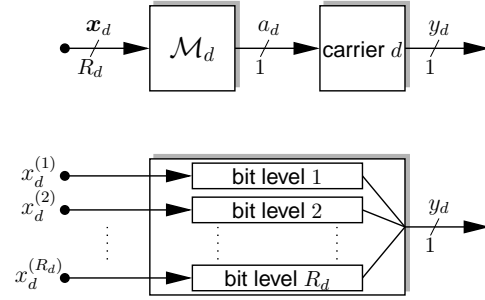


Fig. 2. Equivalent representation of mapping \mathcal{M}_d and d -th carrier by system model with R_d parallel bit levels.

mutual information between the binary inputs and the scalar outputs can be given as

$$I(A_d; Y_d) = I(X_d^{(1)} X_d^{(2)} \dots X_d^{(R_d)}; Y_d). \quad (4)$$

Exploiting the chain rule of information theory [3], the mutual information between a single binary input $X_d^{(\mu)}$ and Y_d reads

$$I(X_d^{(\mu)}; Y_d) = I(A_d; Y_d) - I(\tilde{\mathbf{X}}_d^{(\mu)}; Y_d | X_d^{(\mu)}), \quad (5)$$

where $\tilde{\mathbf{X}}_d^{(\mu)} = [X_d^{(1)} \dots X_d^{(\mu-1)} X_d^{(\mu+1)} \dots X_d^{(R_d)}]$ comprises all binary inputs except the μ -th one. Assuming equally distributed binary symbols, (5) constitutes the capacity of the respective bit level

$$\text{bl}C_d^{(\mu)} = I(X_d^{(\mu)}; Y_d). \quad (6)$$

Apart from an obvious dependency on $(E_s/N_0)_d$, the bit-level capacity $\text{bl}C_d^{(\mu)}$ also depends on the applied binary labeling rule \mathcal{M}_d .

The decoding of BICM realizes a so-called parallel decoding (PD) approach resulting in a (small) loss of information compared to the optimum solution serial decoding. The equivalent channel thus is characterized by the parallel-decoding capacity $\text{pd}C_d$ which for the d -th carrier is obtained by the sum of the R_d individual bit-level capacities [14]

$$\text{pd}C_d = \sum_{\mu=1}^{R_d} \text{bl}C_d^{(\mu)}. \quad (7)$$

In the SNR region of interest so-called binary-reflected Gray mappings maximize the parallel-decoding capacity for ASK signal constellations [9].

The parallel decoding capacity per channel symbol of an entire OFDM symbol reads

$$\text{pd}C = \frac{1}{D} \sum_{d=1}^D \sum_{\mu=1}^{R_d} \text{bl}C_d^{(\mu)}. \quad (8)$$

IV. PREREQUISITES

In order to provide a solid basis for the following considerations on the bit-interleaver design for rate loaded transmission, we first briefly recall some results of recent publications each covering only parts of the problem. Later these insights are combined into a novel bit-interleaver design.

A. Bit-Interleaver Design

Let us first focus on the design of bit interleavers for BICM in non-adapted transmission scenarios. In [10], [11], we have analyzed the decoding of BICM using a simple Viterbi decoder and identified the sliding-window characteristics of the latter as a crucial point to be taken into account in the design. We have furthermore established a relation between the bit-level capacity $\text{bl}C_d^{(\mu)}$ and the average reliability of the respective pair of bit metrics $\lambda_d^{(\mu)}$. Obviously, metrics originating from levels with lower capacity on average are less reliable than those related to levels with high capacities. The optimization goal for the bit interleaver design has then been formulated as the equalization of the average bit metrics reliability within each shifted version of the virtual window, i.e., local and global averages should coincide as far as possible. Formally, we aim at the minimization of the variance of the so-called *segmental path capacity*

$$\Pi_{\text{opt}} = \underset{\Pi}{\text{argmin}} \{ \text{Var} \{ {}^{\text{sp}}C_{\delta} \} \}, \quad (9)$$

which is defined as³

$${}^{\text{sp}}C_{\delta} = \sum_{\zeta=(n'-1)\delta}^{n'\delta} \text{bl}C_{\zeta} = \sum_{\zeta=(n'-1)\delta}^{n'\delta} \text{bl}C_d^{(\mu)}. \quad (10)$$

The segmental path capacity is related to the bit metrics used in the δ -th step of the decoding procedure, i.e., to the δ -th segment of the trellis diagram consisting of K_{bs}/k' segments in total. In a trellis segment n' bit metrics are combined into the segmental path metrics and k' estimates on the initial binary sequence are returned.

In [11] we have solved the optimization problem under the constraint of perfect CSI at both sides of the channel. The resulting concept called *adaptive bit interleaving* successively combines strongest and weakest bit metrics into segmental path metrics. If channel knowledge is not available at the transmitter, we can still combine

³Note, the index $\zeta = 1, \dots, N_{\text{bs}}$ relates the bit-level capacities $\text{bl}C$ to the respective pairs of bit metrics $\text{di}\lambda_{\zeta}$ contained in $\text{di}\mathbf{A}$ and has a one-to-one relation to the indices d and μ .

bit metrics resulting from higher bit levels with those of lower bit levels into segmental path metrics. This approach called *intralevel interleaving* has been discussed in detail in [10]. Both optimized bit interleaver design lead to significant gains over random bit interleaving in terms of the bit-error ratio.

B. Rate Loading

As already mentioned in the introduction, the problem of rate and power loading has been addressed in numerous publications. In the context of this paper, we prefer a bit-level-based approach recently introduced in [13] and exploit results presented in [12].

The optimization of the rate distribution \mathbf{R} can be interpreted as the selection of the best bit levels out of a set of potential candidates. Consider a system with at most R_{max} bits per carrier, D carriers, and N_{bs} bits to be transmitted. Then we have DN_{bs} potential binary input channels for the transmission of the N_{bs} bits and just select those with the highest bit-level capacities aiming at an overall maximized parallel-decoding capacity. Potential bit levels with weak capacities are eliminated. This new rate allocation strategy yields rate distributions very similar to those of other earlier approaches, e.g., [2], [5]. The performance in terms of the bit-error ratio of uncoded transmission virtually equals the results of these already well-known loading algorithms.

C. Adaptive BICM

In [12] we have shown that rate loading cannot increase the parallel-decoding capacity at higher spectral efficiency. Only for rather small average signal constellations an adapted rate distribution is rewarding. Power loading, in particular a power distribution according to the water-filling principle [3], can enhance the PDC. As the parallel-decoding capacity and the bit-error ratio of coded transmission are tightly related, the latter cannot necessarily gain from rate loading, especially if random bit interleavers are used. Power loading can slightly improve the BER. So far, the best approach to exploit channel knowledge for BICM presented in these previous works is adaptive bit interleaving.

V. LOADING-AWARE BIT-INTERLEAVER DESIGN

Based on these insights and the equivalent channel model, we now introduce a novel bit-interleaver design which does not need full CSI at the transmitter, but simply exploits an existing (adapted) rate distribution. In other words, the design can be employed in any existing environment using rate and power loading and does not increase the amount of side information to be communicated. This represent the main benefit of the proposed method compared to adaptive bit interleaving.

The main idea of intralevel interleaving [10] can be briefly summarized. Permute, i.e., interleave only bit metrics originating from identical bit levels, i.e., with similar reliabilities, and carefully rearrange them in order

to equalize the average reliability of the segmental path metrics. Taking now into account results presented in [13], [12], we can immediately see that in a scenario with an adapted rate distribution not necessarily bit levels with formally identical indices μ coincide wrt. ${}^{\text{bl}}C$. Due to the elimination of weak bit levels by the rate allocation algorithm, an inherent coarse equalization of the bit-level capacities of the highest level employed per carrier is achieved. Consider for example two neighboring carriers using a 4-ASK ($R_{d_1} = 2$) resp. an 8-ASK ($R_{d_2} = 3$) after the adaptation. Then, second resp. third bit level are most likely similar in terms of ${}^{\text{bl}}C$, i.e., ${}^{\text{bl}}C_{d_1}^{(2)} \approx {}^{\text{bl}}C_{d_2}^{(3)}$. Translated to the bit metrics $\lambda_{d_1}^{(2)}$ resp. $\lambda_{d_2}^{(3)}$, we can assume similar average reliabilities. These maybe rather coarse but reasonable assumptions can be made only regarding the rate distribution \mathbf{R} ; no further channel state information is required.

Using these observations, the bit interleaver resp. the deinterleaver is designed such, that we finally simply can employ the design rules already developed for intralevel interleaving. Therefore, first $E\{R_d\}$ D -ary sets of bit metrics which differ in their average bit metric reliabilities have to be created. We propose the following procedure starting with the bit metrics related to the highest, i.e., weakest employed bit level per carrier (see Fig. 3 for an illustration):

- 1) Collect the bit metrics of the respective highest, i.e., weakest employed bit level per carrier in a set. Repeat this step for the next highest levels of each carrier until $E\{R_d\}$ sets are generated.
- 2) Complete these sets with the remaining bit metrics until D pairs of bit metrics $\lambda_d^{(\mu)}$ are contained in each set. The assignment follows the average bit metric reliability (weakest remaining bit metrics to 'weakest' set and so on).
- 3) Randomly permute the elements of each set.
- 4) Rearrange the sets such that a succession of weak and strong bit metrics is ensured in the decoder, cf. Fig. 3.

Note, the first steps of this procedure describe just one of the many reasonable ways to generate $E\{R_d\}$ sets of bit metrics with similar reliabilities. The crucial part for the bit interleaver design and the performance in terms of the bit-error ratio is the rearrangement of the sets according to the findings of [10]. The last two steps actually realize intralevel interleaving with an optimized bit level arrangement. The term 'intralevel' here refers to the sets of roughly equally reliable bit metrics.

VI. NUMERICAL RESULTS

In the following, we present numerical results to corroborate the advantages of the proposed bit-interleaver design. We studied three scenarios with $E\{R_d\} = 2$, $E\{R_d\} = 4$, and $E\{R_d\} = 6$, which for non-adapted rate distributions means 4-ASK, 16-ASK, and 64-ASK transmission. For the simulations, we assume OFDM transmission with $D = 128$ carriers, where the channel

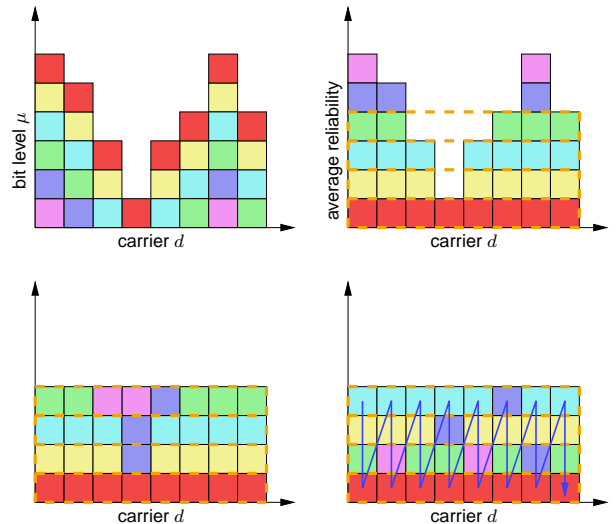


Fig. 3. Illustration of proposed deinterleaver Π^{-1} design for $D = 8$, $N_{\text{bs}} = 32$, and $E\{R_d\} = 4$. Colors indicate bit metrics with similar average reliability. Top left: pairs of bit metrics $\lambda_d^{(\mu)}$ over carriers acc. to rate distribution \mathbf{R} . Top right: generation of 4 set of pairs of bit metrics (surrounded by dashed orange lines); five unassigned $\lambda_d^{(\mu)}$. Bottom left: completion of set. Bottom right: permutation within sets and rearrangement of sets; blue arrow indicates succession of bit metrics in decoder.

coefficients were i.i.d. zero-mean unit-variance complex Gaussian distributed $h_d \sim \mathcal{CN}(0, 1)$, cf. [7]. The rate distribution was optimized according to [13] and water-filling [3] was employed to optimize the power distribution. Six different combinations of rate and power loading and bit-interleaver designs were assessed:

- (S1) no rate and power adaptation; random bit interleaving (black)
- (S2) no rate and power adaptation; adaptive bit interleaving acc. to [11] (blue)
- (S3) optimized rate and power distribution; random bit interleaving (cyan)
- (S4) optimized rate and power distribution; no bit interleaving/coincides with symbol interleaving if carrier coefficients are correlated (magenta)
- (S5) optimized rate and power distribution; adaptive bit interleaving (red)
- (S6) optimized rate and power distribution; novel bit-interleaver design (green)

In Fig. 4 the bit-error ratios obtained for the three simulated scenarios are depicted. Starting with the results of the scenario with lowest spectral efficiency given in the left plot of Fig. 4, we can clearly see the gap between the BER of entirely non-adapted transmission (S1) and the BERs of the five other approaches. Here, any kind of adaptation is advantageous. On the other hand, we can already observe slight variations in the gains of (S2)–(S6). Rate loading with random interleaving (S3) is worse than our novel approach (S6); both are outperformed by (S2), (S4), and (S5). Interestingly, pure symbol interleaving (S4)—obviously contradicting the main idea behind BICM—yields a BER identical to those

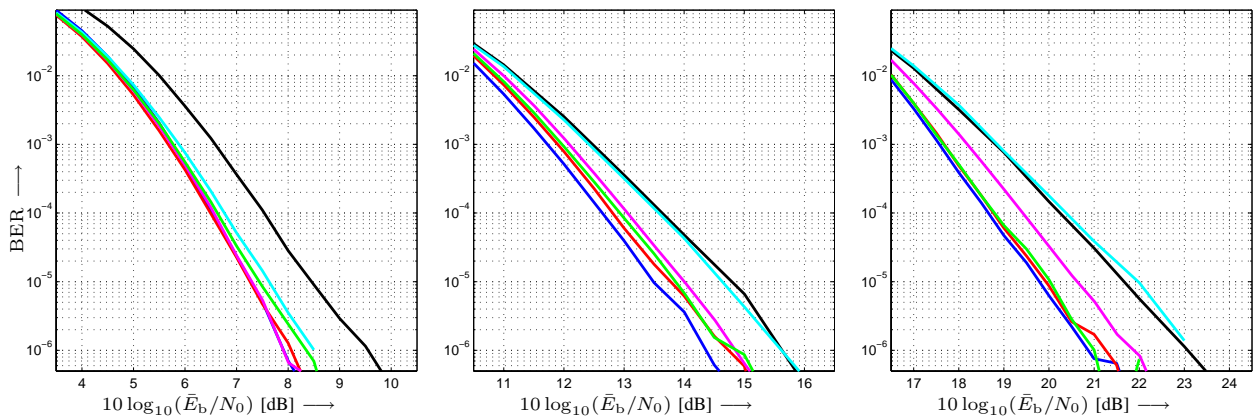


Fig. 4. Bit-error ratio of coded multicarrier transmission over $10 \log_{10}(\bar{E}_b/N_0)$ for several bit interleaver designs and loading strategies. Code rate $R = 1/2$, constraint length $\nu = 10$, number of carriers/block length $D = 128$. Black: (S1). Blue: (S2). Cyan: (S3). Magenta: (S4). Red: (S5). Green: (S6). Left: $E\{R_d\} = 2$. Center: $E\{R_d\} = 4$. Right: $E\{R_d\} = 6$.

of adaptive bit interleaving with/without rate adaptation.

The remaining plots of Fig. 4 clearly reveal the poor performance of (S3), which has been already documented in [11], [12]. Adapting the rate distribution and using just a simple random bit interleaver even leads to losses compared to non-adapted transmission in these settings. The results emphasize the importance of apt bit-interleaver design. Interestingly, pure symbol interleaving (S4) again performs quite well and leads to significant gains over (S1). In contrast to the scenario with $E\{R_d\} = 2$, the gap to (S2), (S5), and (S6) widens for higher spectral efficiencies, though. In both scenarios adaptive bit interleaving without rate adaptation yields the lowest bit-error ratios. Here again, a rate adaptation may be even disadvantageous as the results for (S5) are slightly worse. The novel bit-interleaver design (S6) introduced in this work comes close to the performance of (S5) but with big advantage of requiring no additional side information except for the adapted rate distribution. Adaptive bit interleaving in contrast significantly increases the amount of required side information, since either perfect channel state information has to be available or the entire ordering of the N_{bs} pairs of bit metrics $\lambda_d^{(\mu)}$ in $^{di}\lambda$ has to be transmitted.

VII. CONCLUSION

In this paper, we have proposed a novel bit-interleaver design for coded multicarrier transmission using rate (and power) loading. Applying a bit-level-based analysis and exploiting some recent insights on non-iterative decoding of BICM and effects of rate loading on the characteristics of the bit levels, the bit interleaver is designed such that an equalized average reliability of the bit metrics is ensured for the entire decoding procedure. The proposed approach does not require any additional side information if the adapted rate distribution is known. The presented numerical results have shown significant gains in terms of the bit error-ratio compared to conventional BICM or adaptive BICM with random bit interleavers. The significance of the results grows with increasing spectral efficiency.

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