Out-of-Band Power Reduction using Selected Mapping with Power-Amplifier-Oriented Metrics

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***Abstract*— OFDM systems suffer from a high dynamic of the transmit signal, which leads to out-of-band radiation due to nonlinearities in the transmitter front-end. In order to overcome this issue there exist a variety of schemes reducing the out-of-band power. Most of these schemes consider the peak-to-average power ratio (PAR) as optimization criterion. In this paper, the PAR reduction scheme selected mapping (SLM) is extended to be used with other selection metrics such as the cubic metric (CM) or an amplifier oriented metric (AOM). Their influence on the out-of-band radiation is analyzed applying various amplifier models. It can be shown that out-of-band power is mainly caused by the clipping characteristic of an amplifier, which is sufficiently reflected by the PAR.**

I. Introduction

Digital transmission over channels with large bandwidths causes intersymbol interference, which has to be equalized at the receiver. Due to its simple implementation *orthogonal frequency-division multiplexing (OFDM)* has become very popular for transmission over such channels.

One of the most serious drawbacks of OFDM is the dynamic of the time-domain transmit signal. Nonlinearities at the transmitter front-end, mainly at the power amplifier, cause undesirable out-of-band radiation, which disturbs adjacent transmission channels. In order to avoid violation of spectral masks by this out-of-band radiation, operation at large input power back-off is required to operate at the linear range of the amplifier. This, in turn, reduces the power efficiency of the transmitter significantly.

Over the last years, several schemes to overcome this issue have been developed. Mostly, these schemes focus on the *peak-to-average power ratio (PAR)* of the discrete transmit signal as performance measure. One of their most popular representatives is *selected mapping (SLM)* [1].

Recently, in 3GPP [2] the *cubic metric (CM)* of the transmit signal has been proposed as suited performance measure replacing PAR. This metric is chosen, as it should give a better representation of the influence of the amplifier, which is modeled in 3GPP as a 3^{rd}-order polynomial [3]. Considering such an amplifier model, it is obvious that the third moment of the transmit signal, given by the CM, is the best representation of the nonlinearities.

Subsequently, we extend SLM to be used with other selection metrics. In addition to PAR and CM an *amplifier oriented metric (AOM)* is defined, which reflects the influences of the power amplifier on the transmit signal.

This paper is organized as follows: in Section II the system model is defined; Section III explains the SLM scheme and the applied selection metrics. Numerical results in terms of out-of-band power are analyzed in Section IV. Section V draws some conclusions.

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II. System Model

A. OFDM System Model

In this paper, we consider an OFDM system with D subcarriers. The information carrying symbols are drawn from an M-ary QAM constellation and are put into a vector[1] $\boldsymbol{A} = [A_d]$, $d = 1, \ldots, D$, the frequency-domain OFDM frame. This vector is then transformed into time domain (time-domain OFDM frame $\boldsymbol{a}$) via *inverse-discrete Fourier transform (IDFT)* [5]. The continuous-time transmit signal $s(t)$ is obtained by insertion of the cyclic prefix, pulse shaping through filter $g(t)$, and modulation to *radio frequency (RF)*. This signal is further processed by a (nonlinear) *power amplifier (PA)*. The resulting signal is denoted as $\tilde{s}(t)$.

The energy of the pulse filter $g(t)$ is given by $E_{\text{g}} = \int_{-\infty}^{\infty} |g(t)|^2 \, \mathrm{d}t$. Moreover, the duration of the time-domain samples is denoted by T_{c} (chip duration).

For the subsequent analysis, we omit insertion of the cyclic prefix for convenience. Moreover, instead of considering the modulation to radio frequency, we assess all signals in the *equivalent complex baseband (ECB)* [4]. A block diagram of this system model is shown in Fig. 1.

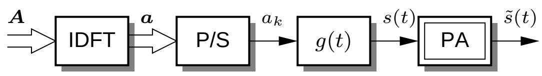

Fig. 1. System model of the OFDM transmitter.

In order to obtain a reasonable estimate for the continuous time signal $s(t)$ pulse shaping with L-times oversampling should be applied to all signal candidates according to [10], [11]. The resulting estimate is denoted by the vector $\boldsymbol{s} = [s_k]$, $k = 1, \ldots, LD$, with variance $\mathrm{E}\{|s_k|^2\} = \sigma_s^2$.

B. Power Amplifier Models

In order to analyze the out-of-band power, suited nonlinear models of power amplifiers are required. For that, we define the *power back-off*

$$\mathrm{PBO} = \frac{\hat{s}^2}{\mathrm{E}\{|s(t)|^2\}}, \quad (1)$$

[1]Notation: Vectors in frequency domain are denoted as bold capital letters, vectors in time domain as bold lower case letters. $\lfloor z \rceil_a$ clips the amplitude of a complex number z to the real value a whereby the argument of z remains. $\mathcal{H}\{\cdot\}$ denotes the Hilbert transform [4].

where $\hat{s}$ is the maximum possible power at the amplifier output (saturation level).

As the influence of the nonlinear behaviour of the model is of interest, we normalize the gain of all power amplifiers to one. Moreover, it is assumed that the amplifier influences only the amplitude of the ECB signal (pure AM/AM conversion).

The most common nonlinear model is the soft limiter

$$\tilde{s}_{\text{lim}}(t) = \lfloor s(t) \rceil_{\hat{s}} \tag{2}$$

which has (ideal) linear behaviour as long as the amplitude of the input signal $s(t)$ falls below the saturation level $\hat{s}$, otherwise, the signal is clipped to $\hat{s}$.

In 3GPP, the power amplification of the signal at radio frequency (RF) modeled by a 3^{rd}-order polynomial [3]

$$\tilde{s}_{\text{RF}}(t) = g_{\text{lin}} \cdot s_{\text{RF}}(t) + g_{\text{nonlin}} \cdot (s_{\text{RF}}(t))^3, \tag{3}$$

where g_{lin} describes the gain of the linear part of the PA and g_{nonlin} of the nonlinear (3^{rd}-order) part.

In this paper, we consider all signals in the equivalent complex baseband [4], thus the amplifier model (3) has to be transformed into ECB domain (with transformation frequency f_{c}). Using the PA model (3) and the relation

$$s_{\text{RF}}(t) = \sqrt{2} \cdot \text{Re}\{s(t) \cdot \text{e}^{\text{j}2\pi f_{\text{c}} t}\}, \tag{4}$$

between the RF signal $s_{\text{RF}}(t)$ and the ECB signal $s(t)$, the amplified signal $\tilde{s}(t)$ in the ECB reads

$$\begin{aligned} \tilde{s}(t) &= \frac{1}{\sqrt{2}}(\tilde{s}_{\text{RF}}(t) + \text{j}\mathcal{H}\{\tilde{s}_{\text{RF}}(t)\}) \cdot \text{e}^{-\text{j}2\pi f_{\text{c}} t} && (5) \\ &= g_{\text{lin}} \cdot s(t) + \frac{3 g_{\text{nonlin}}}{2} \cdot s(t) \cdot |s(t)|^2 \\ &\quad + \frac{g_{\text{nonlin}}}{2} \cdot s^3(t) \cdot \text{e}^{\text{j}4\pi f_{\text{c}} t} && (6) \\ &\approx s(t) \cdot \left(g_{\text{lin}} + \frac{3 g_{\text{nonlin}}}{2} \cdot |s(t)|^2 \right). && (7) \end{aligned}$$

The approximation (7) follows from the assumption that f_{c} is much larger than the signal bandwidth, and all signal components around $2f_{\text{c}}$ can be neglected.

In order to ensure unit gain, we choose $g_{\text{lin}} = 1$. Moreover, to limit the maximum amplitude to the saturation level $\hat{s}$, the nonlinear gain has to be chosen to $g_{\text{nonlin}} = -8/(81\hat{s}^2)$. Hence, the resulting ECB characteristic reads

$$\tilde{s}_{\text{poly}}(t) = s(t) \cdot \left(1 - \frac{4}{27} \cdot \left(\frac{|s(t)|}{\hat{s}}\right)^2\right). \tag{8}$$

This amplifier model does not consider any clipping effects. For amplitudes $|s(t)| > 3\hat{s}/2$ the amplitude of the output signal decreases. In order to include clipping effects this model can be extended to[2]

$$\tilde{s}_{\text{poly+lim}}(t) = \lfloor s(t) \rceil_{\hat{s}} \cdot \left(1 - \frac{4}{27} \cdot \left(\frac{|\lfloor s(t) \rceil_{\hat{s}}|}{\hat{s}}\right)^2\right). \tag{9}$$

[2]Noteworthy, this model is implemented in MATLAB (version 7.7.0.471) as a SIMULINK model for 3^{rd}-order polynomial nonlinearities.

Another very popular model in literature is the Rapp model [6], which is defined as

$$\tilde{s}_{\text{Rapp}}(t) = s(t) \cdot \left(1 + \left(\frac{|s(t)|}{\hat{s}}\right)^{2p}\right)^{-\frac{1}{2p}}. \tag{10}$$

This model is additionally parameterized with the knee-factor p, which specifies the transition from the linear region to the clipping region. For $p \to \infty$ the Rapp model corresponds to the soft limiter (2).

III. Selected Mapping and Amplifier-Oriented Selection Metric

A. Review of Selected Mapping

Selected mapping (SLM) has originally been presented as a peak-to-average power ratio reduction scheme for OFDM systems [1]. However, it can straightforwardly be extended to reduce any other metric associated with an OFDM frame. All metrics used in this paper are defined in Section III-B.

With SLM, the information carrying data, namely the frequency-domain OFDM frame $\boldsymbol{A}$, is mapped to U different, statistically independent signal candidates $\boldsymbol{A}^{(u)}$, $u = 1, \dots, U$, via U different bijective mappings. How these mappings are implemented is irrelevant as long as the resulting signal candidates are statistically independent. However, in this paper, we use the mapping from the original approach of SLM [1]. All U signal candidates are then transformed into time domain and out of these the "best" one, according to a certain metric, is chosen for transmission.

In order that the receiver is able to revert the mapping, it has to be aware of the applied mapping on the actual received OFDM frame. Hence, the transmission of side-information, a label representing the number of the chosen signal candidate, is essential. Several approaches for an efficient transmission of this side-information are known (see, e.g., [7], [8], [9]). However, this issue is not considered for convenience.

B. Selection Metrics for Selected Mapping

Aim of the application of SLM is the reduction of out-of-band radiation at the transmitter. Hence, the actual applied metric within the SLM algorithm should represent the resulting out-of-band power. The following metrics are considered and their information about the out-of-band radiation is analyzed.

1) Peak-to-Average Power Ratio (PAR): The most popular metric, to be reduced in this context, is the *peak-to-average power ratio*

$$\text{PAR} = \max_{k=1,\dots,LD} |s_k|^2 / \sigma_s^2. \tag{11}$$

2) Cubic Metric (CM): Another metric, which has become popular is the *cubic metric* [2]

$$\text{CM}^2 = \frac{1}{LD} \sum_{k=1}^{LD} \left(|s_k|^2 / \sigma_s^2\right)^3. \tag{12}$$

This metric reflects more closely the behaviour of the power amplifier (3) defined in [3].

3) Amplifier-Oriented Metric: In order to define a metric, which describes exactly the behaviour of the power amplifier, we define the *amplifier oriented metric*

$$\mathrm{AOM}^2 = \frac{1}{LD} \sum_{k=1}^{LD} |s_k - \tilde{s}_k|^2 / \sigma_s^2, \qquad (13)$$

where $\tilde{s}_k$ represents the sample, where the respective power amplifier model from Section II-B has been applied. This measure considers both, in- and out-of-band distortion Noteworthy, in case of the polynomial amplifier model (8) the AOM corresponds to the CM (12) except for a constant scaling factor.

IV. Out-of-Band Power Reduction using Selected Mapping

Subsequently, we consider an OFDM system with $D = 512$ subcarriers. All signals are drawn from an $M = 4$-QAM constellation. The pulse filter is a root-raised cosine filter [4] with bandwidth-excess-factor $\alpha = 0.3$. In order to approximate continuous-time signals $(L = 8)$-times oversampling has been applied.

As performance measure, we consider the normalized power spectral density $\Phi_{\tilde{s}\tilde{s}}(f)/(E_\mathrm{g}T_\mathrm{c})$ of the amplified transmit signal $\tilde{s}(t)$. The maximum normalized frequency fT_c of the undistorted signal $s(t)$ is given by $f_\mathrm{m}T_\mathrm{c} = (1+\alpha)/2 = 0.65$. In order to quantify the gains in terms of reduced out-of-band radiation introduced by SLM, we consider the gain

$$G = \frac{\int_{f_\mathrm{m}}^{\infty} \Phi_{\tilde{s}\tilde{s}}(f)\big|_{\text{original signal}} \,\mathrm{d}f}{\int_{f_\mathrm{m}}^{\infty} \Phi_{\tilde{s}\tilde{s}}(f)\big|_{\text{SLM}} \,\mathrm{d}f} \qquad (14)$$

between the (one-sided) out-of-band power of the original distorted signal $\tilde{s}(t)$ and the one with SLM assessing U signal candidates.

A. Soft Limiter

Fig. 2 shows numerical results for the soft limiter (2) as PA model and SLM with PAR (left column), CM (middle column), or AOM (right column) as selection metric.

Evidently, using the matched metric for this amplifier model, the AOM, offers the best performance in terms of out-of-band power reduction. However, using PAR as optimization metric leads to the same performance. Both metrics exhibit equal performance, because the maximum absolute squared value of one OFDM frame dominates the averaging over all samples (13). Hence, the PAR metric is a well suited and sufficient measure for the out-of-band radiation introduced by pure clipping. Using CM as selection metric results in poor performance, because this metric is not matched to this amplifier model at all.

B. Polynomial Model

Numerical results for the power spectral density with the pure polynomial model (8) are shown in Fig. 3. As this amplifier model exhibits another operating point than the soft limiter, the back-offs have been chosen to $10\log_{10}(\mathrm{PBO}) = 3\,\mathrm{dB}$, or $4\,\mathrm{dB}$, respectively.

Applying this polynomial model, consisting only of a linear and 3^{rd}-order part, leads solely to a triplication of the used frequency band, i.e., the out-of-band radiation ends at the frequency $3f_\mathrm{m}T_\mathrm{c}$. In the Appendix, an analytical expression for power spectral density of the amplified signal $\tilde{s}(t)$ has been derived. Even if this derivation is done for a rectangular pulse filter (i.e., $\alpha = 0$) it shows exactly this behaviour.

Unfortunately, it is not possible to reduce the out-of-band power significantly here; only gains of less than $1\,\mathrm{dB}$ are achievable. As all symbols within the frequency-domain OFDM frame $\boldsymbol{A}$ are statistically independent its spectrum (and consequently the ones of all its alternative representations) is almost white. Thus, the amplified version of all signal candidates exhibit almost the same power spectral density, wherewith it is hardly possible to choose one with low out-of-band power. However, using SLM with the AOM and CM (both metrics are equal and matched for this model) perform best.

Fig. 4 shows the results for the cubic amplifier model with additional soft limiting properties (9). As this model introduces signal clipping out-of-band radiation for frequencies above $3f_\mathrm{m}T_\mathrm{c}$ occurs. Hence, the frequency band can be divided into two regions: for frequencies less than $3f_\mathrm{m}T_\mathrm{c}$ the polynomial characteristic of the amplifier dominates; for frequencies greater than $3f_\mathrm{m}T_\mathrm{c}$ the clipping characteristic dominates. As it is hardly possible to overcome the out-of-band power introduced by the polynomial characteristic, the actual aim of SLM should be to reduce the out-of-band power introduced by clipping.

The largest gains are achieved if the PAR metric is used. AOM and CM perform almost equal but not as good as the PAR metric. Slight differences between AOM and CM are originated in the fact that AOM considers in-band and out-of-band distortion whereas the gain G only considers out-of-band distortion. Even if the AOM reflects all characteristics (3^{rd}-order polynomial and clipping behaviour) of the power amplifier, the cubic characteristic dominates this measure, whereas it is not possible to achieve a significant reduction of the out-of-band power caused by clipping. Finally, it can be stated that the PAR is a well suited performance measure for reducing the out-of-band radiation introduced by the clipping characteristic.

C. Rapp Model

Fig. 5 shows numerical results for the Rapp model (10) with knee factor $p = 3$, which reflects a heavily nonlinear model. Again, AOM offers the best capabilities for reduction of the out-of-band power. For $p = 3$ the PAR metric offers not as good performance as the AOM. The PAR metric only reflects the out-of-band power caused by clipping and not the one caused by nonlinearities for $s(t) < \hat{s}$. The CM is never a suited metric for this kind of model.

V. Conclusions

In this paper, we have assessed the influence of various selection metrics (PAR, CM, and AOM) applied in the

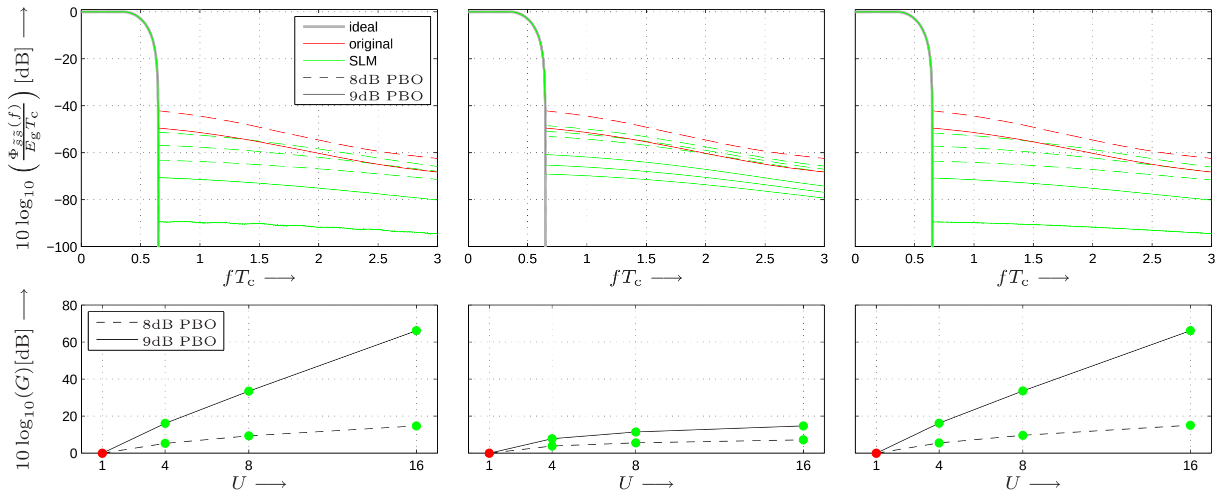

Fig. 2. Top: normalized average power spectral density of the original signal and for SLM ($U = 2$, 4, 8, and 16 assessed candidates) with soft limiter as amplifier model ($10\log_{10}(\mathrm{PBO}) = 8\mathrm{dB}$ (dashed) and $9\mathrm{dB}$ (solid)); bottom: gains in terms of out-of-band power reduction achieved by SLM; selection metrics of SLM are (columns from left to right) PAR, CM, and AOM; $M = 4$, $D = 512$, $L = 8$, $\alpha = 0.3$.

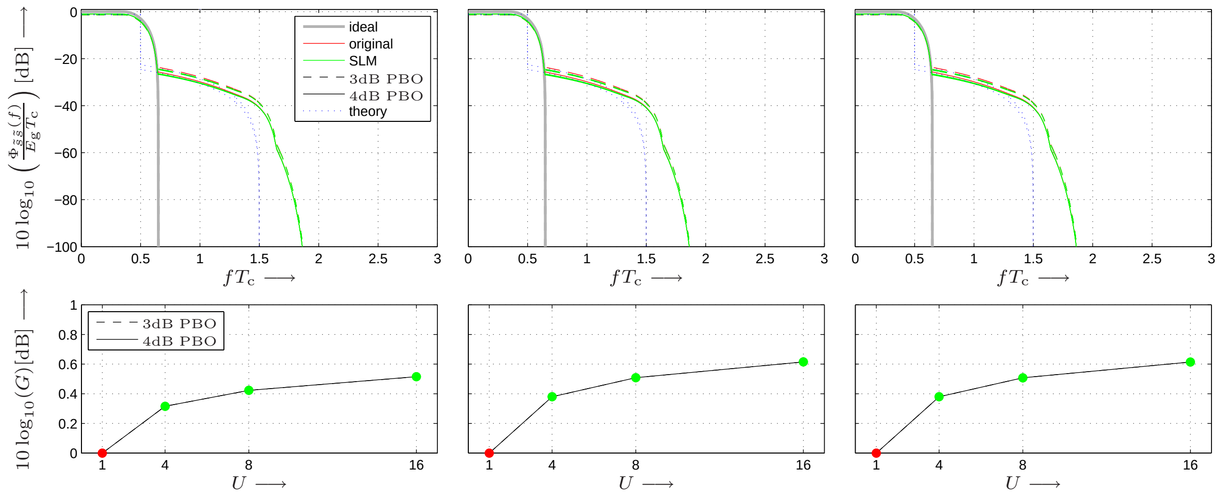

Fig. 3. Top: normalized average power spectral density of the original signal and for SLM ($U = 2$, 4, 8, and 16 assessed candidates) with 3$^{\mathrm{rd}}$-order polynomial as amplifier model ($10\log_{10}(\mathrm{PBO}) = 3\mathrm{dB}$ (dashed) and $4\mathrm{dB}$ (solid)); for reference, the theoretical result of the power spectral density (17) is depicted blue dotted; bottom: gains in terms of out-of-band power reduction achieved by SLM; selection metrics of SLM are (columns from left to right) PAR, CM, and AOM; $M = 4$, $D = 512$, $L = 8$, $\alpha = 0.3$.

SLM algorithm on the out-of-band power reduction. It has been shown that out-of-band radiation is mainly caused by the clipping characteristic of the respective power amplifier. Simple amplifier models, as the 3$^{\mathrm{rd}}$-order polynomial defined in 3GPP, do not consider any clipping. Hence, such amplifier models are useless in order to represent the behaviour of real power amplifiers, in turn the matched metric for these models, the CM, is useless as well.

In order to reduce out-of-band radiation caused by clipping, the PAR and the AOM show similar performance results. In summary it can be stated that it is more desirable to focus on these measures than on the CM.

Appendix

Subsequently, we assume a rectangular pulse filter $g(t)$ (i.e., $\alpha = 0$) and $s(t)$ being a stationary complex Gaussian random process with power spectral density

$$\Phi_{ss}(f) = E_\mathrm{g} T_\mathrm{c} \mathrm{rect}(f T_\mathrm{c}) \;\bullet\!\!-\!\!\circ\; \phi_{ss}(\tau) \tag{15}$$

and autocorrelation function $\phi_{ss}(\tau)$. The autocorrelation function $\phi_{\tilde s\tilde s}(\tau)$ of the signal processed by the pure 3$^{\mathrm{rd}}$-order amplifier model (8) (the index poly is dropped for brevity) can be calculated by applying the *Gaussian joint variable theorem* from [12] and reads

$$\phi_{\tilde s\tilde s}(\tau) = \mathrm{E}\{\tilde s(t)\tilde s^*(t+\tau)\} = \phi_{ss}(\tau)\left(\frac{32}{729\hat s^4}(\phi_{ss}^2(\tau) + 2E_\mathrm{g}^2T_\mathrm{c}^2) - \frac{16E_\mathrm{g}T_\mathrm{c}}{27\hat s^2} + 1\right). \tag{16}$$

Using the convolution theorem of the Fourier transform, the power spectral density can be calculated to

$$\Phi_{\tilde s\tilde s}(f) = E_\mathrm{g}T_\mathrm{c}\begin{cases} \frac{32E_\mathrm{g}^2T_\mathrm{c}^2}{729\hat s^4}\left(\frac{11}{4} - |fT_\mathrm{c}|^2\right) - \frac{16E_\mathrm{g}T_\mathrm{c}}{27\hat s^2} + 1 & ;\ |f| < \frac{1}{2T_\mathrm{c}} \\ \frac{16E_\mathrm{g}^2T_\mathrm{c}^2}{729\hat s^4}\left(|fT_\mathrm{c}| - \frac{3}{2}\right)^2 & ;\ \left||f| - \frac{1}{T_\mathrm{c}}\right| < \frac{1}{2T_\mathrm{c}} \\ 0 & ;\ \text{else} \end{cases}. \tag{17}$$

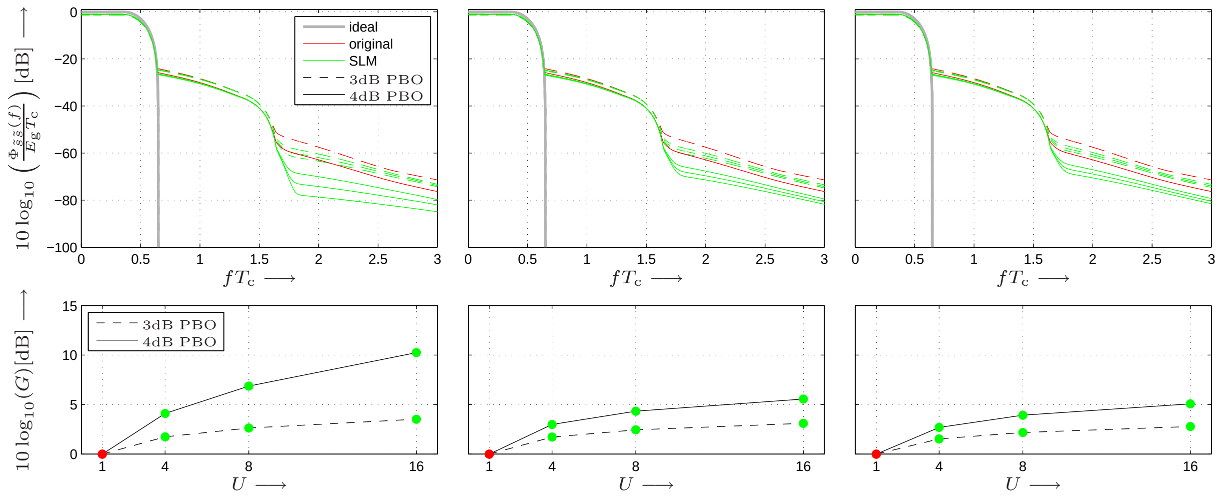

Fig. 4. Top: normalized average power spectral density of the original signal and for SLM ($U = 2, 4, 8$, and 16 assessed candidates) with 3[rd]-order polynomial with limiter as amplifier model ($10\log_{10}(\mathrm{PBO}) = 3\mathrm{dB}$ (dashed) and $4\mathrm{dB}$ (solid)); bottom: gains in terms of out-of-band power reduction achieved by SLM; selection metrics of SLM are (columns from left to right) PAR, CM, and AOM; $M = 4$, $D = 512$, $L = 8$, $\alpha = 0.3$.

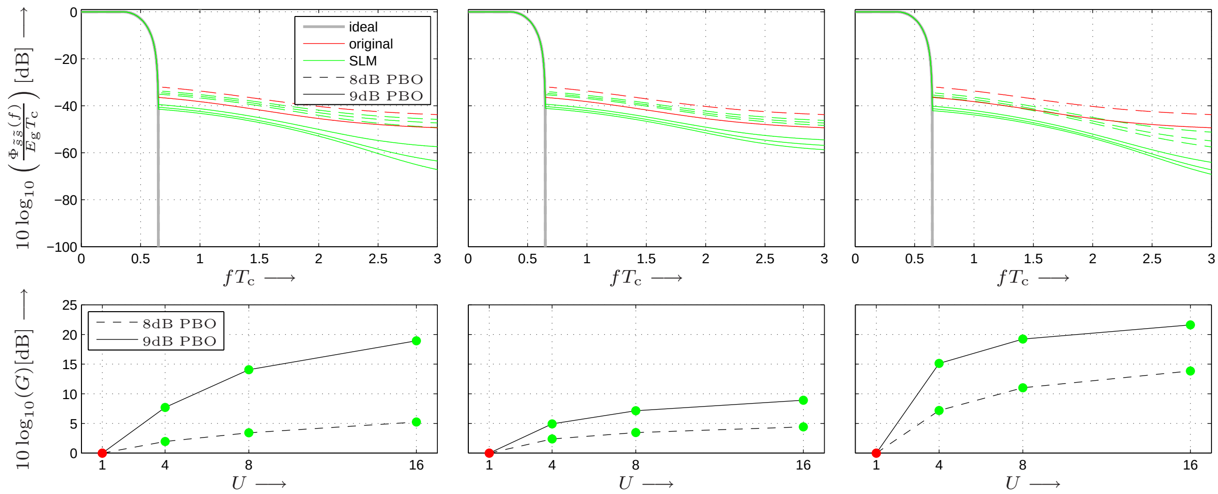

Fig. 5. Top: normalized average power spectral density of the original signal and for SLM ($U = 2, 4, 8$, and 16 assessed candidates) with Rapp model ($p = 3$) as amplifier model ($10\log_{10}(\mathrm{PBO}) = 3\mathrm{dB}$ (dashed) and $4\mathrm{dB}$ (solid)); bottom: gains in terms of out-of-band power reduction achieved by SLM; selection metrics of SLM are (columns from left to right) PAR, CM, and AOM; $M = 4$, $D = 512$, $L = 8$, $\alpha = 0.3$.

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