Improved Differential Demodulation of OFDM over Two-Path Channels

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Abstract—**In this paper, improved differential demodulation for OFDM over two-path channels is presented. The optimum post-processing subsequent to conventional differential detection across the carriers of an OFDM block is derived. The gains of the proposed method are covered by numerical simulations.**

Keywords—**Differential demodulation, two-path channels, OFDM.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is an attractive modulation scheme especially for transmission over multi-path channels. When adding a cyclic prefix to each OFDM block whose length covers the delay spread of the channel, equalization of the intersymbol interference is simply performed by scaling each carrier individually [1].

If, moreover, *differential modulation* is used, i.e., information is represented in symbol transitions rather than actual symbols, non-coherent demodulation can be applied without the need of performing any kind of equalization. Here, in particular, we focus on differential modulation *across* the carriers, i.e., in frequency direction, see e.g. [5], [3]. Using this approach, each OFDM block can be detected (or decoded) separately.

Non-coherent demodulation is based on the assumption that adjacent carriers are affected by (nearly) the same attenuation and phase distortion due to the channel. However, if the guard time is of the order of 20–25% of the frame duration and the delay spread of the channel spans the entire guard period, significant fluctuations of the channel spectrum can be observed within only 2 or 3 carrier spacings. Hence, strong degradation of the differential detection scheme is visible. Furthermore, improved demodulation using multiple-symbol differential detection [3]—which utilizes the slow frequency variance of the channel and allows performance approaching that of coherent transmission—is impossible.

In this paper, an improved demodulation scheme, especially for channels which have two dominant paths, is proposed. In Section II, the system model is set up. Improved differential demodulation is derived in Section III. Numerical results (Section IV) show the performance of the proposed algorithm.

II. System Model

In differential phase-shift keying (PSK), user data is first mapped onto phase increments a_μ^{inc} ($\mu = 1, 2, \dots, D-1$, is the carrier number), taken from an M-ary PSK signal constellation $\mathcal{A} = \{\mathrm{e}^{\mathrm{j}\frac{2\pi}{M}m},\ m = 0, 1, \dots, M-1\}$. Using $a_0 = 1$ as phase reference for each OFDM block, differential encoding (phase accumulation) $a_\mu = a_{\mu-1} \cdot a_\mu^{\mathrm{inc}}$ is performed across the carriers. Via IDFT (inverse discrete Fourier transform) a block of D symbols a_μ is transformed to time domain (For simplicity, we expect all D carriers to be active). Each block or frame of D transmit symbols is prefixed with the D_0 last samples, the so-called guard interval [1], of the same block at the transmitter.

After transmission over the channel with impulse response $h[k]$, $k = 0, 1, \dots, D_h - 1$ (equivalent low-pass domain), D out of $D + D_0$ received symbols are used for further processing. Assuming that the length D_0 of the guard interval is chosen such that $D_0 \geq D_h - 1$, the linear convolution of transmitted signal and channel impulse response is converted into a cyclic convolution. Subsequently, we always assume that $D_0 \geq D_h - 1$ holds. The IDFT/DFT pair of OFDM resolves this cyclic convolution such that the transmission between the IDFT input and the DFT output is performed in D parallel, independent subchannels. Defining the channel transfer function as $H_\mu \stackrel{\mathrm{def}}{=} \sum_{k=0}^{D_h-1} h[k] \cdot \mathrm{e}^{-\mathrm{j}\frac{2\pi}{D}\mu k}$, the receive symbols in frequency domain are given as

$$e_\mu = a_\mu H_\mu + n_\mu\ , \quad \mu = 0, 1, \dots, D-1\ , \qquad (1)$$

where n_μ denotes additive white Gaussian noise.

Differential Demodulation

Neglecting for the moment the channel noise and regarding only the signal part, differential demodulation

[4] produces the decision variables

$$v_\mu = e_\mu \cdot e^*_{\mu-1} = a^{\text{inc}}_\mu \cdot H_\mu H^*_{\mu-1} \; . \qquad (2)$$

If—as usually assumed—the channel transfer function H_μ changes only very slightly from carrier to carrier, the phase of the channel cancels out and only a gain term ($\approx |H_\mu|^2$) remains. However, in some situations this assumption does not hold.

Two-Path Channels

In particular, we study transmission over a channel with two (dominant) taps. The channel impulse response is given as ($\delta[k]$: discrete-time delta sequence)

$$h[k] = c_1 \cdot \delta[k] + c_2 \cdot \delta[k - k_0] \; , \quad c_1, c_2 \in \mathbb{C} \; , \qquad (3)$$

with $k_0 \le D_h - 1$. Obviously, the channel transfer function reads

$$H_\mu = c_1 + c_2 \cdot \mathrm{e}^{-\mathrm{j}\frac{2\pi}{D}\mu k_0} \; , \qquad (4)$$

and the complex gain factor, expired by symbol a^{inc}_μ, calculates to

$$\begin{aligned} H_\mu H^*_{\mu-1} &= |c_1|^2 + |c_2|^2 \mathrm{e}^{-\mathrm{j}\frac{2\pi}{D}k_0} \\ &\quad + 2\mathrm{Re}\{c_1 c_2^*\}\mathrm{e}^{-\mathrm{j}\frac{\pi}{D}k_0} \cos\left(\frac{\pi}{D}k_0(2\mu-1)\right) \\ &\stackrel{\text{def}}{=} c_a + c_b \cdot \cos\left(\frac{\pi}{D}k_0(2\mu-1)\right) \; . \qquad (5) \end{aligned}$$

Hence, the noise-less receive symbols after differential demodulation lie on a straight line passing (assuming differential signal point $1 + \mathrm{j}0$ has been transmitted) the point $c_a = |c_1|^2 + |c_2|^2 \mathrm{e}^{-\mathrm{j}\frac{2\pi}{D}k_0}$ and having direction $c_b = 2\mathrm{Re}\{c_1 c_2^*\}\mathrm{e}^{-\mathrm{j}\frac{\pi}{D}k_0}$.

The situation is visualized in Figure 1, where a scatter plot of the decision symbols v_μ in the complex plane after conventional differential detection is shown. DQPSK modulation in each carrier and a typical two-path channel are assumed. Noticeable, the signal points after differential detection exhibits some nonlinear distortion. Assuming (D)QPSK the undisturbed signal points no longer lie on the coordinate axes and are not only affected by a pure real-valued fading gain.

III. Improved Differential Demodulation

In order to improve performance of the differential detector, at the receiver an additional rotation of the symbols v_μ by an angle φ_μ is admitted to compensate for a systematic phase offset. The new decision variable hence reads

$$v'_\mu = v_\mu \cdot \mathrm{e}^{\mathrm{j}\varphi_\mu} = a^{\text{inc}}_\mu \cdot \mathrm{e}^{\mathrm{j}\varphi_\mu} H_\mu H^*_{\mu-1} \; . \qquad (6)$$

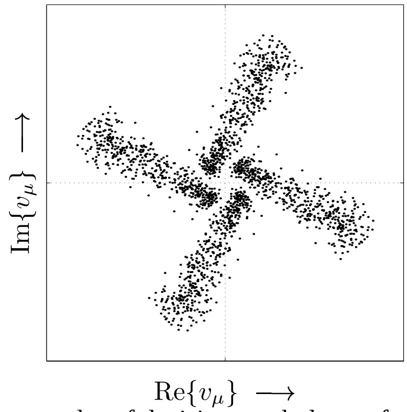

Fig. 1. Scatter plot of decision symbols v_μ after differential demodulation for two-path channel.

Obviously, the natural choice for φ_μ is the negative argument of $H_\mu H^*_{\mu-1}$. This rotation, depending on the channel transfer function, can be seen as some kind of a simple synchronization algorithm. From each OFDM block in frequency domain, suited parameters should be extracted and used for the de-rotation. However, since non-coherent reception is desired, we do not want to estimate the channel transfer function H_μ explicitly.

Derivation

Subsequently, it is more convenient to denote real and imaginary part of the complex variable v as $x = \mathrm{Re}\{v\}$ and $y = \mathrm{Im}\{v\}$, respectively, i.e., $v = x + \mathrm{j}y$. Moreover, we restrict ourselves to transmit symbols $a^{\text{inc}}_\mu = 1$. Rotating the signal space by multiples of $2\pi/M$ the results are valid for all points drawn from an M-PSK constellation.

The points v_μ lie on a curve $y = f(x)$ (i.e., $\mathrm{Im}\{v\} = f(\mathrm{Re}\{v\})$), which for the two-path channel is a linear function

$$f(x) = a + b \cdot x \; . \qquad (7)$$

From (5), the coefficients readily calculate to

$$\begin{aligned} a &= \mathrm{Im}\{c_a\} - \mathrm{Re}\{c_a\}\mathrm{Im}\{c_b\}/\mathrm{Re}\{c_b\} \; , \qquad (8) \\ b &= -a/(\mathrm{Re}\{c_a\} - \mathrm{Im}\{c_a\}\mathrm{Re}\{c_b\}/\mathrm{Im}\{c_b\}) \; . \qquad (9) \end{aligned}$$

Due to channel noise, the actual receive symbols scatter around this line.

Unfortunately, for channel with more than two (dominant) taps, no bijective function $f(x)$ can be found. Here, the signal points are located at trajectories of higher-order.

Since the magnitude of the complex variable v is uniquely related to its experienced phase deviation, we propose to choose the rotation angle φ (Eq. (6)) according to

$$\varphi = \Phi(|v|^2) \; . \qquad (10)$$

Since rotation is dependent on the distance from the origin, a torsion of the signal space is performed.

To determine $\Phi(\cdot)$, we demand that the (noise-less) signal points are transformed back onto the positive part of the real axis. Mathematically, for each $v = x + \mathrm{j}y$ we require

$$\mathrm{Im}\left\{(x + \mathrm{j}f(x)) \cdot \mathrm{e}^{\mathrm{j}\Phi(|v|^2)}\right\} \stackrel{!}{=} 0 , \quad (11)$$

which results in

$$\Phi(|v|^2) = \Phi(x^2 + f^2(x)) = -\mathrm{atan}\left(\frac{f(x)}{x}\right) . \quad (12)$$

In order to obtain a function, depending only on $|v|^2$, we regard (7) and express $|v|^2 = x^2 + f^2(x)$ as

$$|v|^2 = x^2 + (a+bx)^2 = (1+b^2)x^2 + 2abx + a^2 . \quad (13)$$

Solving this quadratic equation for x gives (the other solution may be negative and hence an undesired phase offset by 180° may be caused)

$$x = \frac{-ab + \sqrt{|v|^2(1+b^2) - a^2}}{1+b^2} , \quad (14)$$

which inserted into (12) yields $\Phi(\cdot)$ as

$$\Phi(|v|^2) = \mathrm{atan}\left(\frac{a + b\sqrt{|v|^2(1+b^2) - a^2}}{ab - \sqrt{|v|^2(1+b^2) - a^2}}\right) . \quad (15)$$

Noteworthy, the above function is only defined for $|v|^2(1+b^2) - a^2 \geq 0$; otherwise x are complex-valued. In this case, the rotation may be adjusted such that the angle φ is continuous (or simply nothing is done). Since

$$\lim_{|v|^2 \to a^2/(1+b^2)} \Phi(|v|^2) = \mathrm{atan}\left(1/b\right) , \quad (16)$$

this value may be chosen in case of $|v|^2 < a^2/(1+b^2)$.

Post Processing

In summary, improved differential demodulation for two-path channels is obtained by applying a rotation subsequent to conventional differential demodulation. Based on this post-processed signal, metric for channel decoding is calculated or threshold decision is performed.

The new decision variable v'_μ is obtained from that (v_μ) of conventional differential demodulation as

$$v'_\mu = v_\mu \cdot \mathrm{e}^{\mathrm{j}\varphi_\mu} , \quad \mu = 1, 2, \ldots, D-1 , \quad (17)$$

with

$$\varphi_\mu = \begin{cases} \mathrm{atan}\left(\frac{a+b\sqrt{|v_\mu|^2(1+b^2)-a^2}}{ab-\sqrt{|v_\mu|^2(1+b^2)-a^2}}\right) , & |v_\mu|^2 \geq \frac{a^2}{1+b^2} \\ \mathrm{atan}\left(\frac{1}{b}\right) , & |v_\mu|^2 < \frac{a^2}{1+b^2} \end{cases} . \quad (18)$$

The parameters a and b are thereby extracted (see next section) from the $D-1$ decision variables v_μ comprised in one OFDM block.

Example

The effect of rotating the signals points depending their magnitude is visualized in Figure 2. Since a (D)QPSK signal constellations is assumed, only one quadrant, i.e., $|\arg(v)| \leq \pi/4$ is shown. A two-path channel which results in the parameters $a = 1.0$ and $b = -0.5$ is assumed . The undisturbed signal points ($1 + \mathrm{j}0$ transmitted) hence lie on the straight line $y = f(x) = 1.0 - 0.5x$, see the solid line. On the left hand side, the complex plane for the symbols v_μ after conventional differential demodulation is shown. The dashed circle with radius $\sqrt{a^2/(1+b^2)}$ marks the boundary between the two cases for calculating the optimum angle, see (18).

On the right hand side, the situation after a torsion of the complex plane is depicted. The dotted grid exactly correspond to the dotted grid on the left hand side. As can be seen, the points lying on the straight line $y = f(x)$ are transformed back on the real axis. Noteworthy, all points with $|v|^2 = a^2/b^2 = 4$ (e.g. $z = 2 + \mathrm{j}0$) are fixed points of the mapping of the complex plane onto itself.

Determination of the Parametes

Having received one OFDM block conventional differential demodulation is performed resulting in the symbols v_μ. Form these, the parameters of the straight line have to be extracted.

In the first step, via a modulo operation into the sector $|\arg(v)| \leq \pi/M$ the M-PSK modulation is removed. This yields the situation if only (differential) signal points $1+\mathrm{j}0$ would have been transmitted. Then, a least-squares fit of a straight line with parameters a and b to the data is calculated (regression line). Denoting the real and imaginary part of v_μ again by $x_\mu = \mathrm{Re}\{v_\mu\}$ and $y_\mu = \mathrm{Im}\{v_\mu\}$, $\mu = 1, 2, \ldots, D-1$, we demand

$$(a, b) = \underset{(\tilde{a},\tilde{b})}{\mathrm{argmin}} \sum_{\mu=1}^{D-1} (y_\mu - (\tilde{a} + \tilde{b} \cdot x_\mu))^2 . \quad (19)$$

The solution to this problem reads [2]

$$b = \frac{\sum_{\mu=1}^{D-1}(x_\mu - \bar{x}) \cdot y_\mu}{\sum_{\mu=1}^{D-1}(x_\mu - \bar{x})^2} , \qquad a = \bar{y} - \bar{x} \cdot b , \quad (20)$$

with $\bar{x} \stackrel{\mathrm{def}}{=} \frac{1}{D-1}\sum_{\mu=1}^{D-1} x_\mu$ and $\bar{y} \stackrel{\mathrm{def}}{=} \frac{1}{D-1}\sum_{\mu=1}^{D-1} y_\mu$.

Note, for calculating the parameters a and b it is important that points v_μ belonging to the transmit point

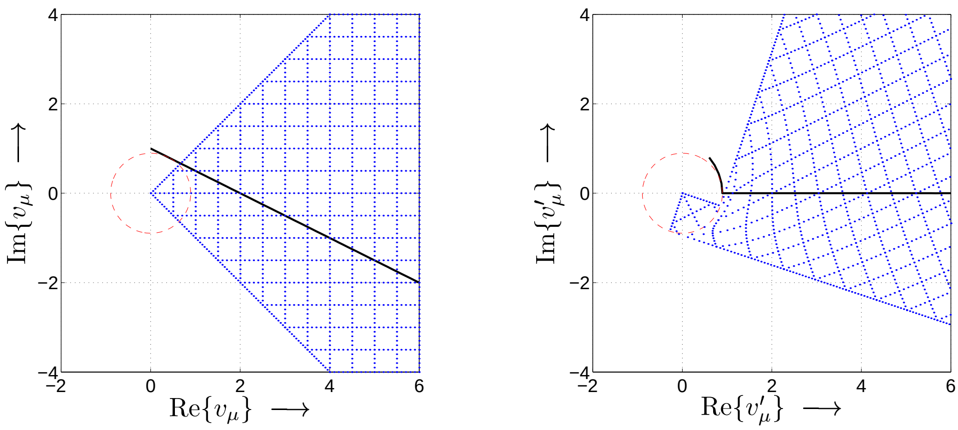

Fig. 2. Visualization of the torsion of the signal space.

$a_\mu^{\mathrm{inc}} = 1 + \mathrm{j}0$ are not affected by the modulo operation. This can e.g. be achieved by first performing a rotation which compensates for a systematic phase offset. When the signal points concentrate around the coordinate axes, the torsion of the signal diagram is applied.

IV. Simulation Results and Conclusion

The performance of the proposed differential demodulation scheme is shown by numerical simulations. OFDM uses a DFT of size 512; all carriers are active. Within one OFDM block channel coding (64-state rate-$1/2$ convolutional code) is performed—after random interleaving the coded bits are mapped onto QPSK signal points and are differential encoded. The power-delay profile of the channel has 6 taps. The relative power of taps number (cf. (3)) $k \in \{0, k_0\}$ is 1, that of taps $k \in \{1, k_0+1\}$ is 0.1, and that of taps $k \in \{2, k_0+2\}$ is 0.01. For each OFDM block, the channel taps are generated randomly (complex Gaussian distributed). To eliminate the effect of flat fading, the channel impulse response is normalized to unit energy.

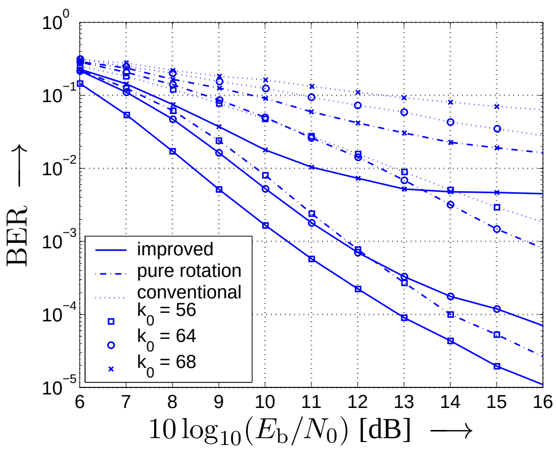

Fig. 3. Bit error rate over E_b/N_0 in dB. Solid: improved demodulation; Dotted: conventional differential demodulation; Solid: improved demodulation; Dash-dotted: pure rotation. $k_0 = 56, 64, 68$.

Figure 3 shows the bit error rate over the signal-to-noise ratio (E_b: received energy per information bit, ignoring guard period; N_0: one-sided noise power spectral density). Using the proposed post processing, significant gains can be achieved compared to conventional differential demodulation. For reference, results are plotted where a pure rotation (same rotation angle over the whole block) is applied. By taking the 4^{th} power of the received symbols v_μ the QPSK modulation is removed. The phase angles of these points are averaged and each symbol v_μ is rotated back a quarter of the average angle, i.e., $\bar{\varphi} = -\frac{1}{4}\sum_{\mu=1}^{D-1}\arg(v_\mu^4)$. Noteworthy, in order to avoid ambiguities, this rotation is also done in the improved differential demodulation, prior to the torsion of the signal space.

Significant gains can already be achieved by applying a pure rotation (simple synchronization algorithm). However, a torsion of the signal space according to (17) and (18) provides additional gains. Especially for impulse responses which exhibit large delays between the two dominant taps, the proposed improved demodulation is rewarding. Only if the delay is too large and systematic phase offsets exceeding $45°$ occur, differential modulation across the carriers degrades significantly.

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