Low Complexity Multilevel Coding for Multiple–Symbol Differential Detection

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Abstract — **Power and bandwidth–efficient multilevel coding matched to noncoherent transmission with multiple–symbol differential detection is discussed. We propose a simplified metric calculation, which has a computational complexity comparable to conventional differential detection, but offers almost the entire performance gain provided by multiple–symbol differential detection.**

Index Terms: multilevel coding, multiple–symbol differential detection

1 Introduction and System Model

In this letter, we consider classical differentially encoded M–ary phase–shift keying (MDPSK) over a stationary, time–varying, *flat* Rayleigh fading channel. The fading conditions are expected to be (almost) constant over N consecutive symbols. In such communication scenarios, noncoherent transmission requiring no *explicit* channel state information (CSI) at the receiver and thus, avoiding the drawbacks of channel estimation circuits used to approximately implement coherent detection, is attractive.

Multiple–symbol differential detection (MSDD) [1] is a promising approach for noncoherent reception. With it, channel coherence time can be exploited and thus, the power efficiency gap between noncoherent and coherent transmission with perfect CSI can be bridged to some extent. Blocks $\boldsymbol{y}_N \triangleq [y_1, \ldots, y_N]$ of N consecutively received symbols y_ν, overlapping each other by one symbol, are jointly processed to deliver decoding metrics for the corresponding blocks $\boldsymbol{a}_{N-1} \triangleq [a_1, \ldots, a_{N-1}]$ of $N-1$ PSK data symbols a_ν prior to differential encoding. After interleaving (deinterleaving) based on blocks $\boldsymbol{a}_{N-1}$ ($\boldsymbol{y}_N$), a practically memoryless *block fading channel* between $\boldsymbol{a}_{N-1}$ and $\boldsymbol{y}_N$ is obtained. This *vec-*

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tor channel is completely characterized by the single complex N–dimensional probability density function (pdf) $p_{\boldsymbol{Y}_N}(\boldsymbol{y}_N|\boldsymbol{a}_{N-1})$ of $\boldsymbol{y}_N$ for given $\boldsymbol{a}_{N-1}$. Details are given in [2].

In this letter, the application of multilevel coding (MLC) with multistage decoding (MSD) [3, 4] to noncoherent transmission with MSDD is treated. In particular, a simple metric generation for MSD and MSDD with $N > 2$ is presented, which requires a similar computational complexity as conventional differential detection ($N = 2$). But since longer blocks ($N > 2$) are processed, power efficiency can considerably be improved over conventional differential detection.

The performance is assessed by the associated random coding exponent, and the information theoretic results are confirmed by means of simulations.

For brevity, in this letter we only consider Rayleigh fading channels. However, the proposed low complexity noncoherent MLC scheme works for general flat Ricean fading channels and MSDD with arbitrary observation sizes $N > 2$.

2 Multilevel Coding and Low Complexity Decoding

As natural choice when performing MSDD, coding is done with respect to the vectors $\boldsymbol{a}_{N-1}$. For the resulting coded modulation scheme with $\ell \triangleq (N-1)\cdot\log_2(M)$ binary address symbols b^i per modulation symbol $\boldsymbol{a}_{N-1}$, we apply well–known multilevel coding (MLC) [3, 4]. In MLC, the ℓ binary digits b^i stem from individual encoding.

For MSDD and MLC with MSD, at level i metrics are based on the two pdfs $p_{\boldsymbol{Y}_N}(\boldsymbol{y}_N|b^i,[b^0,\dots,b^{i-1}])$ of $\boldsymbol{y}_N$ given the decisions $b^0,\dots,b^{i-1}$ of lower levels and a trial symbol $b^i \in \{0,1\}$. Optimally, these pdfs are obtained by averaging the vector channel pdf $p_{\boldsymbol{Y}_N}(\boldsymbol{y}_N|\boldsymbol{a}_{N-1})$ over all $\boldsymbol{a}_{N-1}$ which represent the given binary symbols $b^0,\dots,b^i$. Accordingly, for optimal metrics all M^{N-1} N–dimensional pdfs $p_{\boldsymbol{Y}_N}(\boldsymbol{y}_N|\boldsymbol{a}_{N-1})$ have to be evaluated. Hence, computational effort increases *exponentially* in N, which is the main drawback of MSDD.

We assume that the coding levels addressing the same symbol a_ν are ordered consecutively in the MLC scheme. To reduce complexity, instead of a fixed N, a linearly increasing observation window of $N'_\nu = \nu + 1$, $1 \le \nu \le N-1$, symbols is applied for metrics corresponding to the $\log_2(M)$ address bits of a_ν. As usual in MLC, hard decisions of lower levels are incorporated. But in contrast to conventional MLC, where the higher levels are assumed to be equiprobable, here they are simply ignored. In turn, only

2–dimensional pdfs have to be evaluated, which is the same as for $N = 2$, i.e., conventional differential detection. Consequently, independent of N, this simplified MSDD for MLC with MSD has a *comparable computational complexity* as conventional differential detection. This in turn allows the use of larger N, and the exploitation of the respective MSDD gain. For details of the metric calculation we refer to [5].

3 Performance Assessment

For assessing the performance of noncoherent transmission using MLC/MSD and (simplified) MSDD, the associated random coding exponent $E_r(R)$ [6] is regarded. For MLC/MSD with (simplified) MSDD this exponent is derived from that of the *equivalent channels* [3, 4] corresponding to the ℓ coding levels. Denoting the code rate at level i as R^i, the random coding exponent $E_r^i(R^i, N)$ of equivalent channel i is obtained as

$$E_r^i(R^i, N'_\nu) = \max_{0 \le \rho \le 1} \{E_0^i(\rho, N'_\nu) - \rho R^i\} \ , \tag{1}$$

where Gallager's function $E_0^i(\rho, N'_\nu)$ is given by ($\mathcal{E}$: expectation)

$$E_0^i(\rho, N'_\nu) = \mathcal{E}_{b^0,\ldots,b^{i-1}} \left\{ -\log_2 \left(\mathcal{E}_{\boldsymbol{y}_{N'_\nu}} \left\{ \mathcal{E}_{b^i} \left\{ \left(\frac{p_{\boldsymbol{Y}_{N'_\nu}}(\boldsymbol{y}_{N'_\nu} | b^i, [b^0, \ldots, b^{i-1}])}{p_{\boldsymbol{Y}_{N'_\nu}}(\boldsymbol{y}_{N'_\nu})} \right)^{\frac{1}{1+\rho}} \right\}^{1+\rho} \right\} \right) \right\} . \tag{2}$$

Clearly, for optimal MSDD $N'_\nu = N$, $1 \le \nu \le N-1$, holds.

From (1), for given code length n, the word error rate of level i is bounded by

$$p_w^i \le 2^{n \cdot E_r^i(R^i, N'_\nu)} \ . \tag{3}$$

For usually desired (low) error rates and well–designed MLC, the effect of error propagation through levels can be neglected, and word error rates of levels are almost equal. Hence, the overall word error rate p_w reads approximately $p_w \approx p_w^i$, $0 \le i \le \ell - 1$.

Using the formulae given above, the minimum required signal–to–noise ratio (SNR) resulting a random coding exponent, which in turn leads to a word error rate p_w lower than a given threshold, has been evaluated. As an example, 8DPSK with target rate of 2 bit/(channel use) is considered. For partitioning of vector symbols, simple separate Ungerboeck labeling of the constituent PSK constellation is chosen, cf. [2].

In Fig. 1, the curves of $\bar{E}_b/N_0$ ($\bar{E}_b$: average received energy per information bit, N_0: one–sided noise power spectral density) required for $p_w = 10^{-3}$ and MSDD with

$N = 2, 3, 4$ are plotted over the code length n. As performance limit, the curves for overall maximum–likelihood decoding (MLD) are also plotted.

First, we notice that for increasing code length n, the curves of MLC/MSD with optimal metrics converge to the curves of MLD. This means, MLC/MSD is asymptotically optimum for noncoherent transmission with MSDD (cf. [4] for coherent transmission). Second, the presented curves indicate that the proposed metric simplification causes almost no loss in power efficiency. Of course, for $N = 2$ optimal and simple scheme are identical. Since for larger N more terms are neglected during simplified metric calculation, for $N = 4$ a slightly more distinct difference between the curves of optimal and simplified decoding can be observed than for $N = 3$. By enlarging the observation interval of MSDD from $N = 2$ to $N = 4$ a gain of more than 1 dB in required $\bar{E}_b/N_0$ is achieved. Remarkably, using the proposed low complex MSDD approach, this gain is for free.

The proposed system has been simulated using powerful Turbo codes [7] as MLC component codes. The measured bit–error rates (BER) over $\bar{E}_b/N_0$ for noncoherent transmission over the Rayleigh fading channel using MSDD and MLC/MSD with optimal and simplified metric computation are presented in Fig. 2. Turbo codes [7] of length 2000 with random interleaver, and 16–state constituent codes are employed. For rate design, the target rate of 2 bit/(channel use) is divided into individual rates R^i according to the random coding exponents of the equivalent channels corresponding to $p_w^i = 10^{-3}$, cf. [4].

Apparently, the simulated curves of BER in Fig. 2 are in good accordance with the information theoretic results given above. Enlarging N from 2 to 4 increases power efficiency of MSDD by more than 1 dB. Almost the whole gain is achievable if the simplified metric is applied. Again, for $N = 4$ a somewhat larger gap due to simplification occurs than for $N = 3$. In summary, for comparable computational complexity considerable performance improvements over conventional differential detection are obtained.

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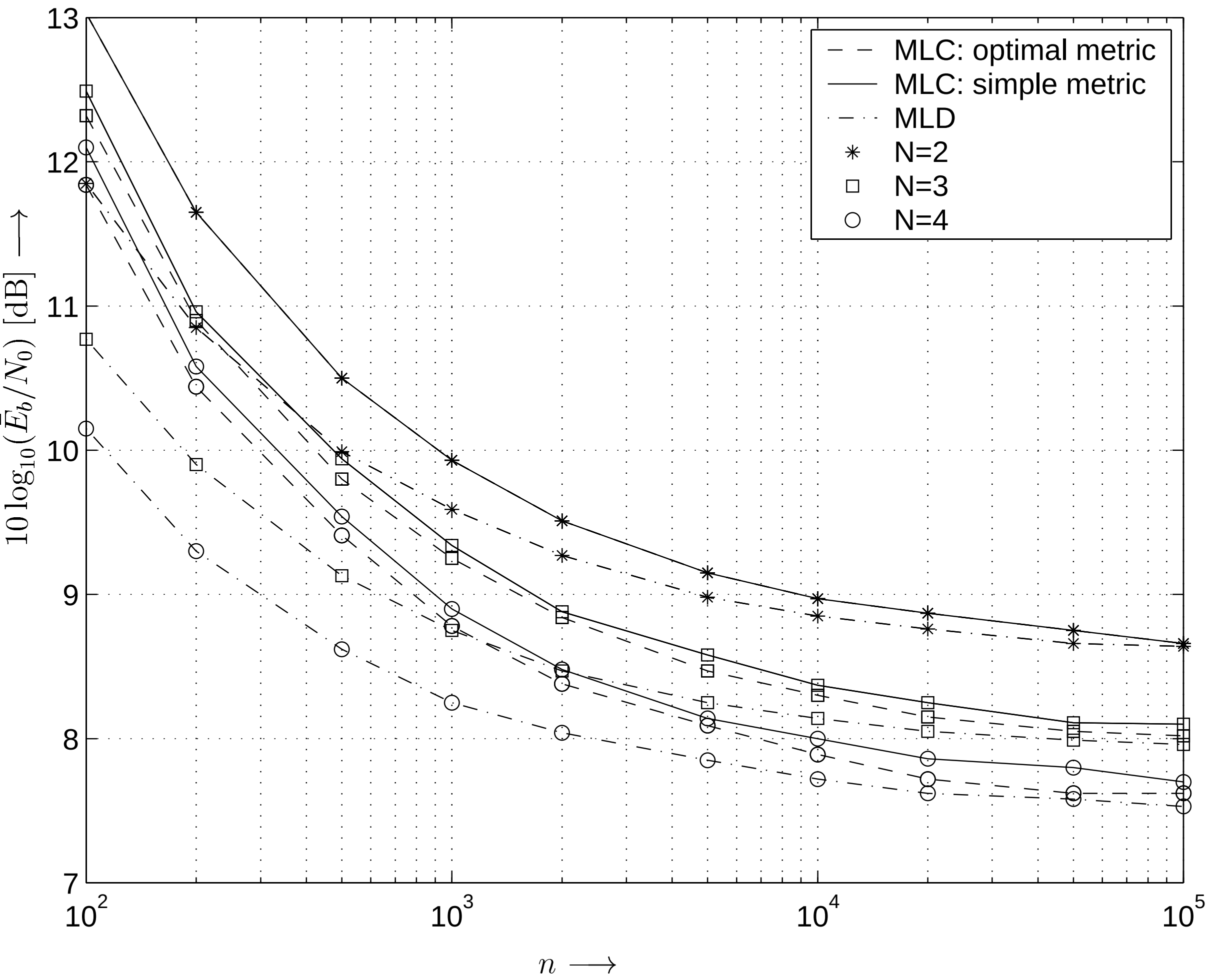

Figure 1: *Required $\bar{E}_b/N_0$ over code length n for $p_w = 10^{-3}$. 8DPSK with 2 bits/(channel use) over Rayleigh fading channel. MLC/MSD and optimal (dashed lines) vs. simplified metric computation (solid lines) for MSDD with $N = 2, 3, 4$. Dash–dotted lines: MLD.*

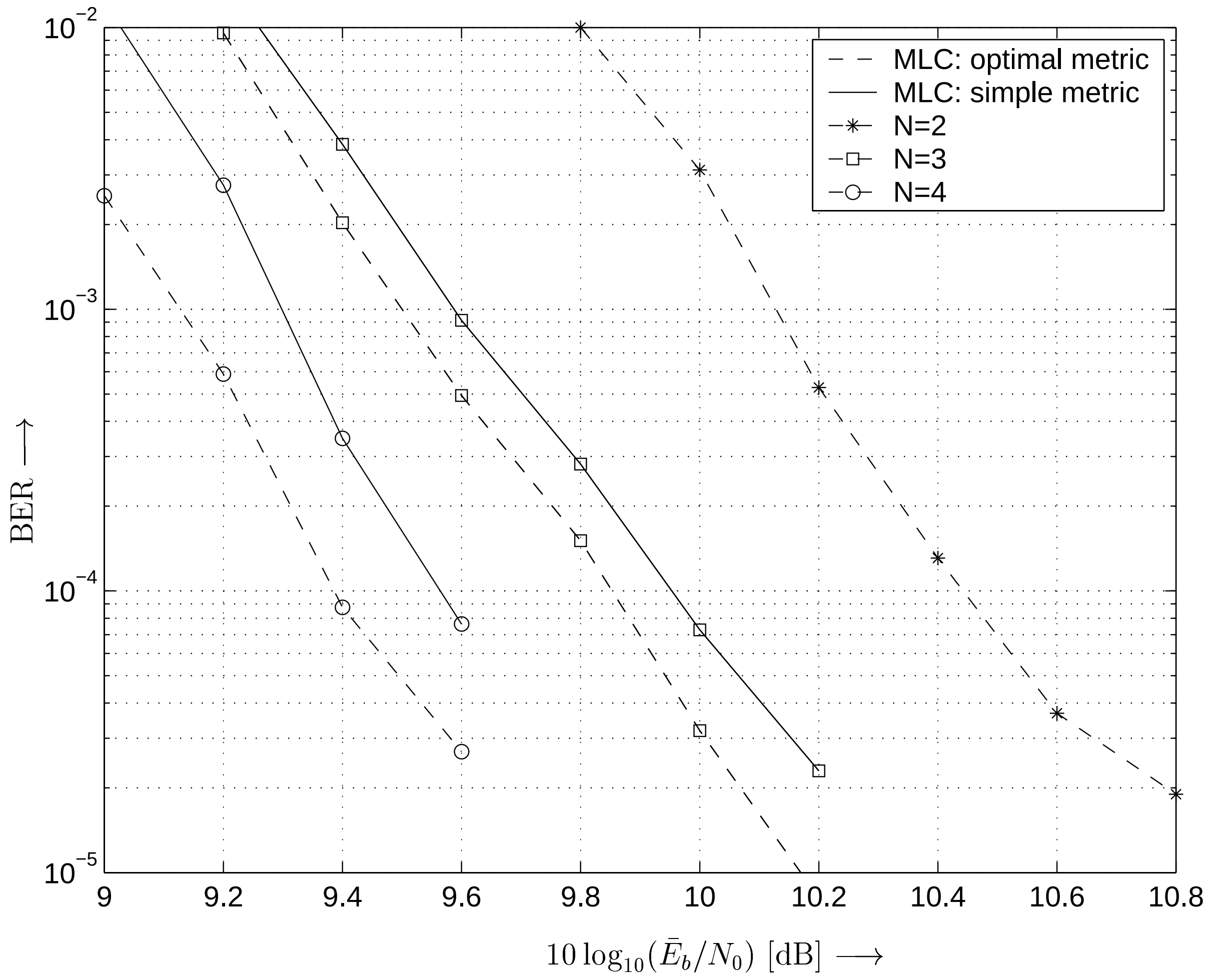

Figure 2: *BER over $\bar{E}_b/N_0$ for 8DPSK with 2 bits/(channel use) over Rayleigh fading channel. MLC/MSD and optimal (dashed lines) vs. simplified metric computation (solid lines) for MSDD with $N = 2, 3, 4$. Turbo codes with 16 states and code length $n = 2000$.*