# Soft-FEC Implementation for High-Speed Coherent Optical OFDM Systems

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## Abstract

Nowadays one of the discussed solutions for optical transmission systems at 100 Gbit/s (N x 100 Gbit/s) and beyond is coherent optical orthogonal frequency division multiplexing (CO-OFDM). Commonly hard-decision forward error correction (FEC) with the redundancy of 7% is assumed. Recently the improvement of the receiver sensitivity was analyzed by using a low density parity check (LDPC) code that is one of the promising candidates for 100 Gbit/s soft-FEC. In this work we assess the efficiency and investigate the implementation of soft-FEC with LDPC codes with overheads between 7% and 50% in coherent optical OFDM systems.

## 1 Introduction

High spectral efficiency and robustness against fiber dispersion make OFDM attractive for high speed transmission. Spectral efficiency could be further boosted by combining Optical OFDM (O-OFDM) with multi-level modulation. On the other hand the larger the number of levels of a modulation format the worse the signal sensitivity compared to that of the single-carrier transmission. One of the possible solutions of the arising problem is to apply FEC.

In this paper we discuss the advantage of the application of soft FEC in coherent O-OFDM systems. As soft FEC we chose LDPC codes, whose performance could be very close to the Shannon limit. But LDPC codes tend to have an error-floor. Moreover in simulations and in experiments burst-errors were observed [12]. Reed-Solomon (RS) codes are able to effectively eliminate error-floors and correct burst-errors following the LDPC decoding. Consequently, the concatenation of LDPC with RS codes serving as inner and outer code, respectively, could be efficiently used to reach the minimal required net coding gain (NCG) of 9.6 dB as it was formulated in [1] for 100 Gbit/s application.

The organization of the paper is as follows. In the section 2 two system models are presented and are used to assess the performance of the code itself and the transmission system operating with FEC, respectively. In section 3 the basic FEC requirements are described. In parallel we discuss the main features of standard LDPC codes looking at irregular LDPC codes with 7%, 10%, 16% and 50% overheads, show the corresponding simulation results and assess implementation complexity of LDPC and RS codes. Some details about repeat-accumulate (RA) codes as well as simulation results and complexity analysis are given in section 4. In section 5 the simulated performance is compared with OFDM offline lab experiments by looking at one 20Gb/s OFDM slice of an up to 250-Gbit/s super channel. Finally in section 6 the conclusions are drawn.

## 2 System Model

In order to assess the basic performance of the isolated code a simple system is used consisting of a signal modulated using binary phase shift keying (BPSK) and of additive white Gaussian noise (AWGN) channel. Bit error ratio (BER) is calculated, by using Monte-Carlo (MC) technique. We refer to this system model by AWGN simulation setup.

### 2.1 AWGN Simulation Setup

The performance of LDPC codes strongly depends on number of iterations. The more iterations are done the better the performance but the higher the implementation complexity. Therefore in this work we got results only for 10, 16 and 100 iterations.

Due to the fact that sequences of length 1e+13 cannot be simulated BER values after the LDPC decoder were analytically extrapolated by using the formula to calculate BER after the RS decoder given the BER before it. This formula serves only as rough extrapolation means for our simulations and measurements, since it assumes that errors are independent from each other and incorrect decoding is not probable [5].

$$\begin{cases} \mathrm{SER}_{\mathrm{output}} = \sum_{i=t+1}^{N} \frac{i}{N} \cdot \binom{N}{i} \cdot \mathrm{SER}_{\mathrm{input}}^{\ i} \cdot \left(1-\mathrm{SER}_{\mathrm{input}}\right)^{N-i} \\ \mathrm{BER}_{\mathrm{input}} = 1-\left(1-\mathrm{SER}_{\mathrm{input}}\right)^{1/m} \\ \mathrm{BER}_{\mathrm{output}} = 1-\left(1-\mathrm{SER}_{\mathrm{output}}\right)^{1/m} \end{cases}$$

Here $\mathrm{SER}_{\mathrm{input}}$ and $\mathrm{SER}_{\mathrm{output}}$ are symbol error rate before and after the RS decoder, respectively.

## 2.2 OFDM

### 2.2.1 Simulation Setup

Transmission of polarization multiplexed OFDM signals is considered. Quadrature phase shift keying (QPSK) is chosen as a basic subcarrier modulation format. AWGN channel is assumed (Fig. 1). We transmit 392 OFDM symbols each of which is carried by 272 subcarriers. One OFDM symbol consists of 544 bits and includes 540 time samples. With a sampling rate of 10 GHz the resulting bit rate on a single polarization is about 10 Gb/s. Typically such an OFDM signal has a spectral width of about 7 GHz [8, 9, 10].

For numerical study of a coherent optical OFDM system with the soft-FEC LDPC the OFDM simulation model was extended by a LDPC encoder at the transmitter and a soft-decoder for LDPC followed by a RS decoder and a decision unit at the receiver (Fig.1).

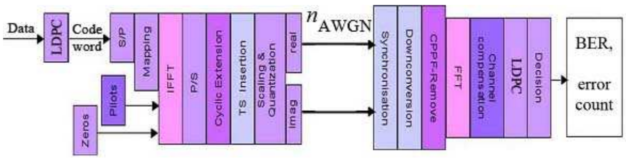


**Fig. 1.** OFDM simulation setup. Offline processing.

### 2.2.2 Experimental Setup

The OFDM system is organized in such a way that OFDM signals' generation and the signal processing at the receiver side are done off line. I- and Q-components of an OFDM signal are calculated in Matlab and then generated by an arbitrary waveform generator (AWG) with sampling speed of 10 Gsample/s. They drive an optical I-/Q-modulator with a nested Mach-Zehnder modulator (MZM) structure and followed by polarization multiplexer emulator (Fig. 2).

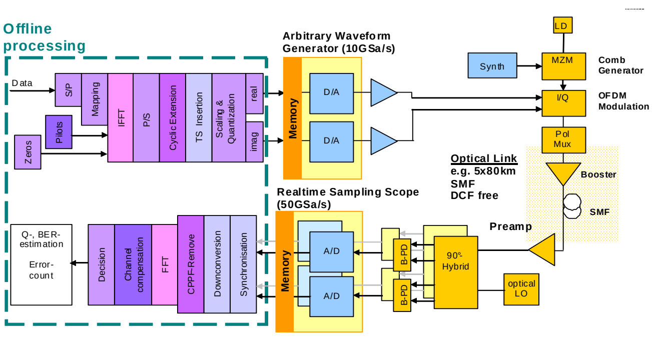


**Fig. 2.** OFDM experimental setup.

One continuous waveform (CW) laser is used. The successive MZM-based comb generator produces 15 lines with 5.96 GHz spacing. The lines, which form 15 sub bands after modulation, are separated into 'odd' and 'even' paths each of which is OFDM modulated in an independent I/Q-modulator. Afterwards 15 OFDM sub bands are combined again and put into a polarization division multiplex emulator that forms one super channel of about 150 Gb/s in each polarization.

We use a polarization diversity coherent receiver detecting two orthogonal polarizations. One OFDM sub band is selected by a local oscillator (LO). Two 90°-hybrides aligned to orthogonal polarization directions are followed by four balanced receiver frontends that convert the incoming four optical signals into electrical domain. Sampling of the four output signals is done by real time sampling oscilloscope at a rate of 50 Gsample/s. The resulting data is then processed on a PC [8, 9, 10].

# 3 Soft FEC

As aforementioned, one of the requirements for FEC in 100 Gbit/s systems is to achieve a NCG larger than 9.6 dB at post-FEC BER of 1e-13 [1]. This value might also serve as benchmark if we are looking at future applications beyond 100 Gbit/s.

A class of linear block codes LDPC codes is defined by a parity-check matrix $H$ with low density of ones. A LDPC code is called regular, if $H$ contains a constant number of ones in each column and a constant number of ones in each row. Otherwise the LDPC code is irregular. In this paper we concentrate on soft decision irregular LDPC codes for the reason that they perform closer to the channel capacity. Although powerful, LDPC codes tend to have an error-floor that results from suboptimality of LDPC decoders and turns FEC to unusable. Hence the second requirement is to remove the possible error-floor for BER of 1e-13.

The length of the shortest cycle in parity-check matrix (PCM) is called girth. Larger girths lower an error-floor and therefore improve the performance of LDPC codes for high signal-to-noise ratios (SNR). It is possible to increase girth by expansion of either the block size or redundancy of a code. However, in the first case the implementation complexity increases, whereas in the second case the signal bandwidth should be extended.

The girth 8 that was assumed in this work does not guarantee no error-floor in the region of interest [1, 7] and a concatenation of an inner LDPC code together with an outer low-overhead hard-decision code such as RS that is able to eliminate unwanted error-floors is often favored [1].

In high speed systems a severe problem that draws attention is circuit complexity. There is a trade-off between efficiency and implementation complexity of a code. Hence an error-correcting code optimal in terms of performance and realization complexity has to be found. In order to do this LDPC codes with different percent overheads, namely 7%, 10%, 16%, 50% were investigated and compared (Fig. 3). It was demonstrated that the LDPC code with 50% overhead has best performance. For BER of 1e-3 its NCG is 0.2 dB better than that of 16.1% overhead LDPC(9252,7967).

In [1] LDPC(9252,7967) code with the redundancy of 16.1% was already proposed. Its error-floor appeared at the post-FEC BER of 1e-9 is eliminated by the outer RS(992,956) code, whose redundancy is 3.8% and error correcting capability is 18. Analytically estimated that BER after RS(992,956) decoder is already lower than 1e-13, when input BER is 2.7e-4. Note that for the input BER larger than 1e-3 the correction power of the RS(992,956) decoder becomes negligible.

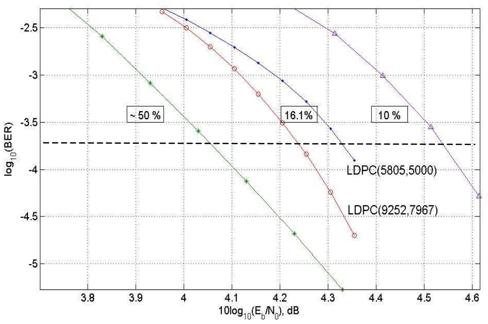


**Fig. 3.** AWGN system model. LDPC codes with different overheads, namely 10%, 16.1% and 50%, and different codeword lengths. At the LDPC decoder maximum number of iterations is set to 16. Horizontal line shows RS input BER for the output BER of 1e-13.

The concatenated LDPC(9252,7967)+RS(992,956) code has the total redundancy of 20.5%. Simulation results based on AWGN system model showed that LDPC(9252,7967)+RS(992,956) has no error-floor until the post-FEC BER of at least 1e-13 [1]. We made simulations with the similar LDPC(9252,7967)+RS(992,956) code and analytically estimated that if the maximum number of iterations at the LDPC decoder is set to 16, then NCG is about 10 dB (Fig. 4).

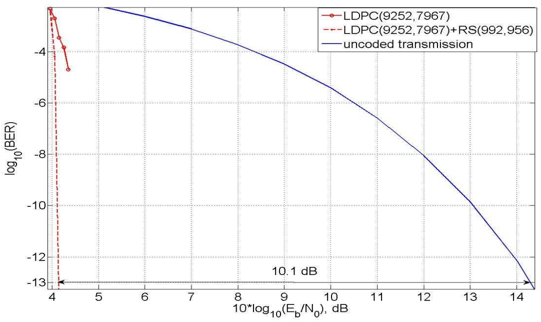


**Fig. 4.** AWGN channel model. LDPC(9252,7967)- solid dotted line, LDPC(9252,7967)+RS(992,956) analytical extrapolation- dashed line, uncoded transmission- solid line (right). The maximum number of iterations at the LDPC decoder is set to 16. The two-side arrow indicates NCG of about 10.1 dB at BER of 1e-13.

As an alternative a shorter LDPC(5805,5000) code also with 16.1% overhead was observed in this work exhibiting a negligible NCG reduction of only 0.1 dB with 16 iterations, although its realization complexity is essentially reduced as the block length of the code is more than 1.5 times less than that of LDPC(9252,7967) (Fig. 5).

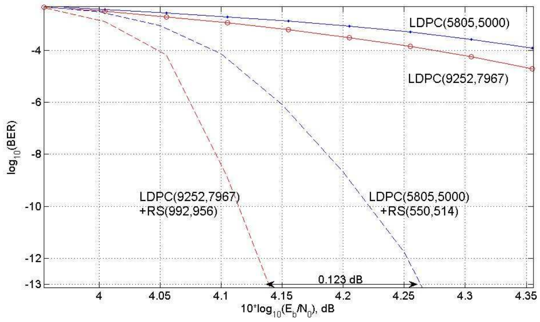


**Fig. 5.** AWGN channel model. LDPC(9252,7967) +RS(992,956) and LDPC(5805,5000)+RS(550,514). Maximum number of iterations at the LDPC decoder is set to 16.

LDPC(5805,5000) was combined with RS(550,514), whose overhead is the same as that of standard RS(255,239), namely 7%, and error-correcting capability is the same as that of RS(992,956) proposed in [1], namely 18. The efficiency of RS(550,514) is better than that of RS(992,956), although its block size twice less and hence its realization complexity is much lower.

## 3.1 Implementation Complexity

### 3.1.1 LDPC codes

In general the number of operations required for LDPC encoding is $O(n^2)$, where $n$ is block length. The total decoding complexity of LDPC codes is linear in block length and defined as a product of block length, number of iterations and operations per bit per iteration [6].

In [12] it was demonstrated that by using a certain approximation of the LDPC decoding algorithm it is possible to implement a decoder of LDPC+RS with 20.5% overhead for 100 Gb/s transport systems and to reach NCG of 9.0 dB at the output BER of 1e-13.

### 3.1.2 RS codes

In [3] it was shown that for high-rate RS codes the syndrome-based decoder architecture is preferred because less hardware is required and higher throughput and shorter latency are achievable. In Tab. 1 it is seen that RS decoder complexity is proportional to a codeword length
$N$. $t = (N\text{-}m)/2$ is the symbol error correcting capability of a code, where $m$ is the number of bits pro symbol [5].

| / | Multipliers | Adders | Registers | Muxes | Latency | Throughput$^{-1}$ |
|---|---|---|---|---|---|---|
| $\sum$ | $7t$+4 | $7t$+2 | $N$+53$t$+15 | 19$t$+8 | $N$+21$t$ | 12$t$ |

**Tab. 1.** Syndrome-based decoding. RS decoder complexity.

# 4 IRA-LDPC

Repeat-accumulate (RA) codes gain linear encoding complexity of turbo codes and powerful decoding of LDPC codes [4]. If the first constituent repetition code is irregular, then RA codes are called irregular (IRA). Compared to RA IRA-LDPC codes operate very close to a theoretical limit and allow high code rates. Long irregular LDPC codes operate within a fraction of a decibel from the Shannon limit [4]. The code performance depends on the column weight. The lower the column weight the lower the error-floor, but the worse the code performance for low OSNR values. The higher the column weight the higher the error-floor, but the better the code performance for low OSNR values. The optimal column weight distribution could be found by using the so-called differential evolution optimization algorithm.

In general IRA codes are nonsystematic. Nevertheless, we concentrate on systematic codes and starting from the standard communication overhead propose systematic IRA-LDPC(30000,27870) with 7% redundancy. The Shannon limit of this code is 3.7071 dB assuming AWGN channel model. If the codeword length goes to infinity the theoretical limit of $E_b/N_0$ = 4.06 dB is reached. It means that the distance of IRA-LDPC(30000,27870) from the Shannon limit is 0.3529 dB.

In the Fig. 6 it is seen that the performance of the designed IRA-LDPC is indeed comparable to that of the standard irregular LDPC(30000,27870), while its encoding complexity is much reduced. At the LDPC decoder maximum number of iterations is set to 100. NCG estimated by means of RS analytical extrapolation is 8 dB at BER of 1e-13.

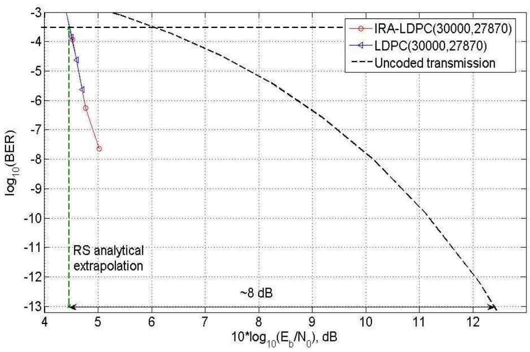


**Fig. 6.** AWGN channel model. IRA-LDPC(30000,27870) and standard irregular LDPC(30000,27870) with 100 iterations. Horizontal line shows RS input BER for the output BER of 1e-13.

## 4.1 Implementation Complexity

RA codes could be viewed as a serial concatenation of repetition and convolutional codes with an interleaver in between, where convolutional code plays a role of accumulator. According to the check equations, specific numbers of bits are summed up modulo 2, forming the input sequence of a binary accumulator that produces the parity bit sequence (Fig. 7) [4].

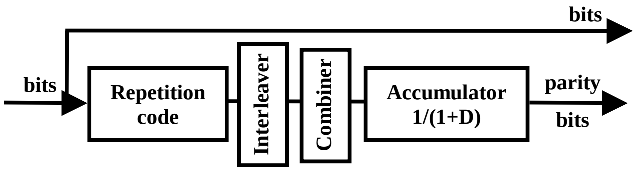


**Fig. 7.** Systematic encoding of IRA codes

The accumulator has a role of columns with weight two in the parity-check matrix. Repetition code and the interleaver determine weights and structure of the remaining columns, respectively. The complexity of repetition encoder together with an interleaver is linear, as they could be viewed as multiplication with a sparse matrix. The complexity of accumulator is negligible. Consequently, RA codes allow for a linear encoding complexity.

On the other hand RA codes are a class of LDPC codes, whose decoding is powerful from one side and has a low complexity in comparison to turbo-codes from another side [4]. Due to their Tanner graph presentation (Fig. 8), sum-product algorithm based on exchanging extrinsic reliability information in form of log-likelihood ratios (LLR) between variable nodes and check nodes is well suited to decode IRA codes. Note that the variable nodes of parity bits are of degree two plus one node of degree one, and that the degree of information bits variable nodes should not be smaller than 3 (extended IRA codes) [11].

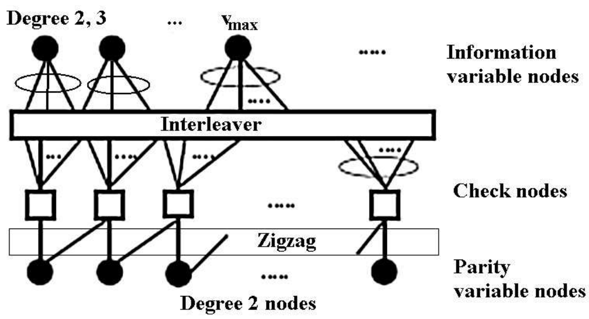


**Fig. 8.** Tanner graph of an IRA code with check nodes and variable nodes for information bits and parity bits.

Using canonical scheduling, in every first half-iteration all variable nodes, i.e. the information and the parity bits, and in the second half-iteration all check nodes have to be updated. Since individual nodes operate independently within one half iteration, processing can be parallelized extensively. In general, the decoder complexity characterized by

- variable nodes with a low complexity additive LLR update,
- check nodes update with a more complex, favorably table-based LLR update,
- quasi-random permutation network of the information bits,
- simple zigzag-like permutation network of the parity bits.

In addition, the number of iterations determines the throughput and/or the decoder latency.

# 5 Experimental Results

In the OFDM offline lab experiment the BER results were received only for certain OSNR values. They are marked by boxes in the Fig. 10, 11. The intermediate points were found with the help of analytical interpolation. First of all we analyzed the uncoded transmission over OFDM channel model. The comparison of the simulation and experimental results shows the OSNR-offset of roughly speaking 2 dB (Fig. 9). Some origins for degradation might be:

- phase noise of the two lasers, which was not included in the simulation model;
- I/Q imbalance caused by differences in the signal path for I and Q, i.e. cables, modulators, drivers, photodiodes;
- Small amplitude of the signal at the receiver, which limits the effective resolution of the analog to digital converters in the receiver.

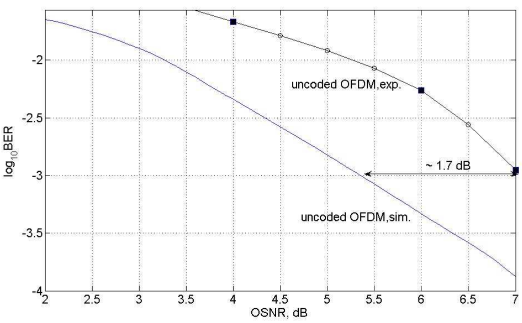


**Fig. 9.** Uncoded transmission over OFDM channel model. OSNR-offset between the simulation (left) and experimental (right) results is constant and could be rounded till 2 dB.

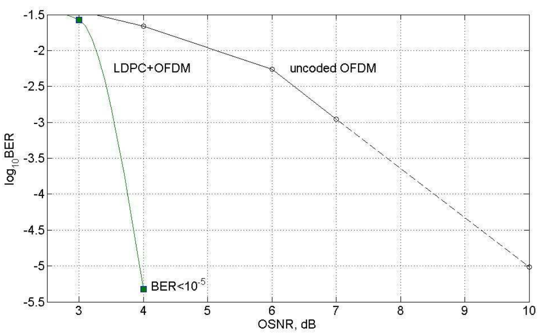


**Fig. 10.** Experimental results for uncoded transmission and transmission with soft FEC LDPC(9252,7967). Maximum number of iterations at the LDPC decoder is set to 10. Dotted line corresponds to the analytical extrapolation.

In the Fig. 10 the left curve shows the signal performance when soft FEC, namely 16.1% overhead LDPC(9252,7967), is used. Due to the limited number of the transmitted code words only BER above 1e-5 could be measured. Therefore only at BER of 1e-3 NCG of 3.5 dB can be directly extracted from the measured BER/OSNR points. For the OSNR of 4 dB BER after the LDPC decoder is already less than 1e-5. Assuming the fact that the OSNR difference between simulation and experimental results is about 2 dB it was estimated that in the case of the uncoded transmission the BER of 1e-5 could be reached at the OSNR of 10 dB. Hence for the BER of 1e-5 NCG of the soft LDPC with 16.1% redundancy is approximately 6 dB.

In the Fig. 11 simulation as well as experimental results of the transmission over OFDM channel model with and without FEC are presented. In the case of the transmission with soft LDPC encoding the OSNR penalty of the experimental setup is 2 dB as it was already shown in the Fig. 9 for the uncoded transmission. Consequently we can conclude that NCG in simulations and experiments stays unchanged. Analytical extrapolation of the outer RS decoder output helps to assess NCG for BER of 1e-13 that is at least 10 dB (Fig. 11).

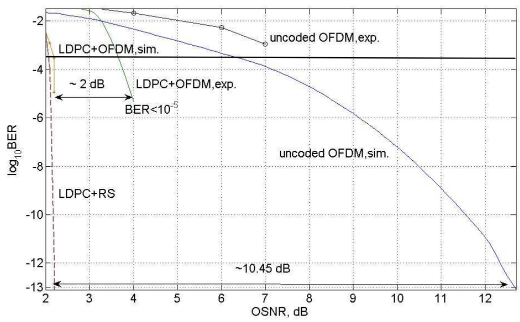


**Fig. 11.** Comparison of simulation (leftmost dashed and blue solid curve) and experimental results (green solid and rightmost curve) for transmission over OFDM channel model with (left curves) and without (right curves) soft FEC. For BER of 1e-13 NCG of the concatenated 20.5% overhead LDPC+RS code is about 10 dB. Horizontal line shows RS input BER for the output BER of 1e-13.

We also performed the OFDM offline lab experiment for IRA-LDPC with 7% redundancy and code word length of 30000 bits. As in the simulations the maximum number of iterations at the IRS-LDPC decoder is set to 100. Opposite to the standard LDPC, for the 7% overhead IRA-LDPC only few words were transmitted. The implication of this limited number of codes is shown in the Fig. 12, revealing a visible variation of the BER curves. The OSNR-offset between simulation and experimental results lies in the range from 2 to 2.5 dB. Depending on the transmitted block of bits the results could change within 1 dB for the BER of 2.7e-4. By applying outer 3.8% overhead RS decoder this BER value could be reduced till 1e-13. Therefore NCG varies from 8 to 9 dB.

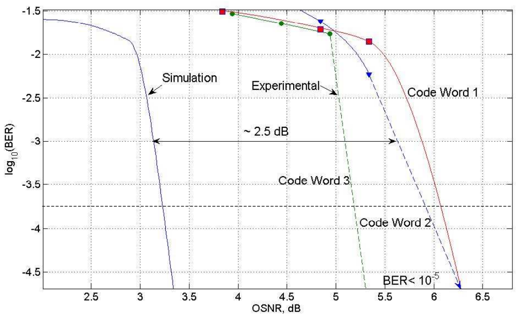


**Fig. 12.** 7% overhead IRA-LDPC code is transmitted over the OFDM channel model. Simulation and experimental results for three different IRA-LDPC code words reveal the OSNR difference changing from 2 to 2.5 dB. Horizontal line shows RS input BER for the output BER of 1e-13.

# 6 Conclusion

Implementation of soft-FEC for future transport Nx100 Gbit/s systems based on CO-OFDM was investigated. In this work different candidates for soft-FEC, namely LDPC and concatenated LDPC+RS with overheads from 7% to 50%, were analyzed. We looked at the code performance for a limited number of iterations of 10, 16 and 100 in order to take realization constraints into account. Simulation of LDPC(9252,7967)+RS(992,956) with the total redundancy of 20.5% over AWGN channel exhibited the NCG of at least 9.6 dB. The same code was used in an offline OFDM laboratory system with 10.07 Gbit/s bitrate. Experimental results confirmed the net coding gain of 10 dB for 16.1% overhead LDPC codes. However, they also revealed penalty of at least 2 dB, which we relate to optical and electrical signal path degradation due to realization, such as crosstalk of clock frequencies at the arbitrary waveform generator (AWG).

Based on the simulation results it was demonstrated that the LDPC code with 50% overhead has best performance. For BER of 1e-3 its NCG is 0.2 dB better than that of 16.1% overhead LDPC(9252,7967). But the increase of a FEC-overhead means the increase of signal bandwidth.

Besides the traditional LDPC code, whose encoding complexity is quadratic in the block length, we also investigated an IRA-LDPC code that can be encoded in linear time. According to simulation the proposed IRA-LDPC with 7% overhead possesses the similar performance as the irregular LDPC (30000,27870). Analytically estimated that these codes could reach NCG of 8 dB at BER of 1e-13.

Trying to keep the complexity low and the efficiency of an error-correcting code comparable we also looked at LDPC codes with shorter codeword lengths. A good alternative for the LDPC(9252,7967)+RS(992,956) could be LDPC(5805,5000)+RS(550,514) with the same overhead. Based on the analytical estimation the use of this code enables NCG of 0.1 dB, namely 10 dB, at BER of 1e-13, whereas the number of operations required for encoding and decoding is reduced in 2.5 and 1.5, respectively.

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