Improved Iterative Decoding of LDPC Codes from the IEEE WiMAX Standard

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***Abstract*— Multiple-bases belief-propagation is a parallel decoding setup which allows for improved decoding performance when compared to standard belief-propagation. Originally designed for decoding of high-density parity-check codes in an iterative manner, this method also shows good decoding results for well-designed low-density parity-check codes when signaling over the AWGN channel. We show the applicability of this scheme to channel codes defined in the IEEE WiMAX standard. It is challenging to find sets of well-performing parity-check matrices for these codes, all of them differing from each other. We propose an algorithm which makes use of the special structure of an underlying base matrix to accomplish this task. The results are compared to codes constructed by the progressive edge-growth algorithm and to bounds from information theory.**

Keywords: Channel Coding, Belief Propagation, LDPC codes, WiMAX IEEE 802.16e.

I. Introduction

Over the last years, the use of belief-propagation (BP) decoding [1] with redundant parity-check matrix representations has drawn a lot of attention. Using redundancy allows for significant performance improvements compared to BP decoding with parity-check matrices of standard size, i.e. $(n-k) \times n$, when signaling over the binary erasure channel (BEC). Several authors [2], [3], [4], [5] presented pioneering work on this subject and provided results on the number of redundant parity-check equations required to prevent certain decoder failures.

It is desirable to transfer the concepts used on the BEC to the additive white Gaussian noise (AWGN) channel. However, this cannot be done in a straightforward manner. The main reason is the fact that additional feedback loops are created by the redundant parity-check equations. These loops are often short and impair the decoding performance. Consequently, special algorithms to use redundant code descriptions for that type of BP decoding were designed. A proof of concept using the extended Golay code of length 24 was already given in [6]. In [7] and [8], adaptive BP algorithms were proposed. These algorithms adjust the parity-check matrix after each iteration, taking into account the reliability of the tentative decision for each variable node. This method requires additional operations which have to be completed in between two consecutive iterations. As a consequence, they increase the delay caused by the decoder. The random redundant decoding (RRD) algorithm [9] uses a slightly different approach. It deploys multiple parity-check matrix representations in a serial fashion for decoding. After a given number of iterations it stores the current decoding state, changes the parity-check matrix to a different representation and resumes decoding. The RRD algorithm has to conduct many iterations and thus imposes a high decoding delay. A recent paper by the authors of the RRD algorithm [10] indicates that the field of application of this algorithm is limited to algebraic codes.

Contrary, we proposed the multiple-bases belief-propagation (MBBP) algorithm [11], [12] which uses redundant parity information in a completely parallel setting. Originally designed to allow for high-performance iterative decoding of algebraic codes [11], this algorithm was shown to be applicable to LDPC codes optimized by the progressive edge-growth (PEG) algorithm [13]. Finding a set of parity-check matrices which lead to good decoding performance is the most challenging task when designing an MBBP system. This holds in particular for constructed LDPC codes. In order to improve the decoding situation if only a low number of matrices is available, we introduced a further algorithm which is supportive to the multiple-bases approach, the leaking algorithm [13]. This algorithm can be understood as a BP algorithm with a scheduling that is optimized to prevent feedback of unreliable decoding information. It was shown that the combination of MBBP and the leaking algorithm is a valuable tool to improve the decoding performance if a low number of redundant parity checks is available.

In this paper, we extend the field of application of MBBP to iteratively decoded channel codes from the IEEE 802.16e Worldwide Interoperability for Microwave Access (WiMAX) standard [14] and demonstrate the effectiveness of the algorithm for this class of codes when signaling over the AWGN channel. It is shown that the presented results can be generalized to block fading channels. We use both the MBBP approach and the leaking algorithm for decoding. The special structure of this class of codes can be used to find a set of well-performing parity-check matrices. Further, we compare the performance of the codes from the WiMAX standard to the performance of optimized PEG codes of comparable length, both for BP and MBBP decoding.

The paper is structured as follows. In Section II we describe the transmission setup and review MBBP decoding. Section III states how a set of different parity-check matrix representations is generated, and Section IV presents a selection of results.

II. Transmission setup and channel coding

We describe the transmission setup and introduce a consistent notation. The MBBP decoding approach is briefly recapitulated. Further, we describe the channel codes used in this paper.

A. Transmission setup

A source emits non-redundant binary information symbols u. We deploy $[n, k, d]$ block codes and use systematic encoding on blocks of source symbols[1]. The encoded symbols are denoted as vectors $\boldsymbol{x}$ of length n. Each element of this vector is mapped to a binary antipodal symbol (binary phase-shift keying, BPSK) and transmitted over the AWGN channel. The corresponding noisy received vector is denoted by $\boldsymbol{y}$.

B. Decoding

At the receiver, we use an iterative decoding scheme to estimate $\boldsymbol{x}$ and the corresponding source symbols. This scheme is either a standard BP decoder or the MBBP decoding setup. The MBBP decoding scheme was first introduced in [11]. It runs multiple instances of standard BP decoding. Each BP decoding unit is provided with the received signal $\boldsymbol{y}$. In total, l decoding units are run in parallel. We denote the parity-check matrix representations by $\boldsymbol{H}_1$ to $\boldsymbol{H}_l$. The corresponding codeword estimates are $\hat{\boldsymbol{x}}_1$ to $\hat{\boldsymbol{x}}_l$. In order to find the candidate which is forwarded to the information sink, we consider all decoders which converge to a valid codeword. In [16] we introduce several methods for making this choice. The best-performing method uses a full search among all candidates, i.e. evaluates

$$\hat{\boldsymbol{x}} = \underset{s \in \mathcal{S}}{\operatorname{argmin}} \sum_{\nu=1}^{n} |y_\nu - \hat{x}_{s,\nu}|^2 , \tag{1}$$

where we assume signaling over the AWGN channel. In this context, we denote the vector forwarded to the information sink by $\hat{\boldsymbol{x}}$. The set of successful decoders is denoted by $\mathcal{S}$ and we use the index s to refer to the elements of this set.

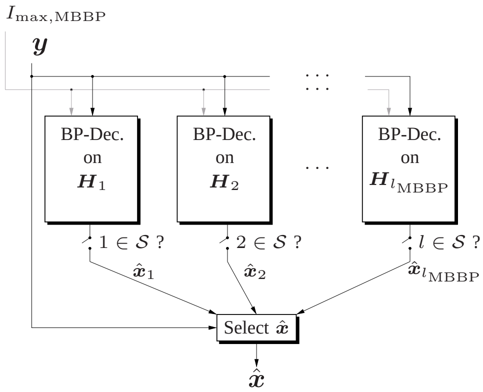

Fig. 1. MBBP decoding setup

The MBBP approach is motivated by the fact that different parity-check matrices allow for decoding of different error patterns, what is again a consequence of the fact that the BP algorithm is suboptimal. Consequently, the probability that all decoding units fail is lower and a performance improvement compared to standard BP is possible. This can be understood as "decoder diversity". Figure 1 visualizes this approach. Here, $I_{\max,\text{MBBP}}$ denotes the maximum number of iterations each BP decoding unit may perform at most. The switches indicate that only a subset of decoders is considered for finding $\hat{\boldsymbol{x}}$.

We have observed in [13], [11] that it is in general a complicated task to find a set of well-performing parity-check matrices. The situation is similar for LDPC codes from the WiMAX standard. In order to gain an additional performance improvement we add further decoders to the MBBP setup by making use of the leaking algorithm. In a nutshell, this algorithm keeps channel information from the decoder and allows it only to "leak" into the decoding progress with rising iteration number. It was shown that this method mitigates the problems of BP decoding with short cycles. The leaking approach allows more reliable information to leak into the decoding process before unreliable information is admitted. To realize this, the decoder sets a probability-based threshold for each iteration and includes only values exceeding this threshold in terms of reliability. A variable $p_\mathcal{L}$ denotes the probability of a variable node being informed on the channel output in the first iteration. With this value and the channel characteristics, the threshold for the first iteration can be calculated. This threshold is lowered linearly in order to find the threshold for all following iterations. A second parameter $I'_{\max}$ denotes the iteration number for which all channel information is included in the decoding process. This number is usually a hypothetical iteration number, i.e. it is larger than $I_{\max,\text{MBBP}}$. Detailed information on this algorithm can be found in [13]. We refer to an MBBP setup using the leaking algorithm by L-MBBP.

C. Channel codes and properties

The WiMAX standard considers a multitude of channel codes. A performance comparison for signaling over the AWGN channel, confirming that the class of LDPC codes is among the most powerful codes in this setup, can be found in [17]. Inspired by its short length and a discussion on the practical relevance of these codes [18], the rate-$1/2$ LDPC codes proposed in the IEEE WiMAX standard are investigated in this work. The larger focus of this work [12] considers codes of length up to $n = 1000$. As a consequence, we restrict our attention to codes of length $576 \le n \le 960$. Nevertheless, the proposed approach and the results can easily be transferred to codes of longer length. For comparison, we also consider PEG-optimized codes of rate $1/2$ and length $500 \le n \le 1000$.

III. Parity-check matrix representations

In order to design a well-performing MBBP system, one needs to find a possibly large number of parity-check matrices, all of them substantially differing from each other. It would be desirable to have a large set of minimum-weight codewords from the dual code available[2]. In that case, different parity-

[1] In [15] we have discussed the advantages that come with systematic encoding.

[2] The low weight is postulated to approximate the property "low-density" for the additional checks.

check matrices of full rank could be created by choosing appropriate disjoint subsets of these parity checks. Algebraic codes allow to generate these parity checks by means of permutations from the automorphism group of the code [19]. For constructed LDPC codes, these sets are not readily available. Often minimum-weight codewords besides the ones used in the constructed parity-check matrix do not exist in the code dual to the considered one. The methods discussed in [20] allow for an efficient search of low-weight codewords. Using these methods on the dual of the IEEE WiMAX rate-$1/2$-code of length 576 did not return any novel codewords of weight below 10 while the minimum possible weight is 6.

Choosing codewords from the dual code with higher weight leads to a degraded performance of the decoding unit and consequently, the decoding performance of the MBBP approach does not differ significantly from standard BP. Hence, sophisticated algorithms to construct these matrices are required.

We present a construction algorithm which is tailored for LDPC codes from the WiMAX standard and makes use of the special structure of the matrices. Prior to this, we describe the structure of the parity-check matrix of a WiMAX LDPC code as well as a generic construction algorithm presented in [5]. The latter algorithm is used for comparison reasons.

A. Parity-check matrices for codes specified in the IEEE 802.16e WiMAX standard

We describe the standard parity-check matrices of the rate $1/2$ LDPC codes from the IEEE WiMAX standard [14]. All codes are deducted from one base matrix. The parity-check matrices for codes of different lengths are created from this matrix by *lifting* [21], i.e. replacing all entries by submatrices of a given size. Prior to this step, a renormalization is done. The normalized matrix reads

$$H_{\mathrm{b}}(i,j) = \begin{cases} \left\lfloor \frac{H'_{\mathrm{b}}(i,j)\cdot z}{96} \right\rfloor & \text{if} \quad H'_{\mathrm{b}}(i,j) > 0 \\ H'_{\mathrm{b}}(i,j) & \text{if} \quad H'_{\mathrm{b}}(i,j) \leq 0 \end{cases}, \tag{2}$$

where $\boldsymbol{H}'_{\mathrm{b}}$ is the underlying matrix defined in the standard [14, p. 628]. The matrix $\boldsymbol{H}'_{\mathrm{b}}$ is shown in Equation (4). In this context, z is the *expansion factor*. It depends on the code realization and determines the length of the resulting code.

The lifting procedure is described as follows. Each negative entry in the base matrix $\boldsymbol{H}_{\mathrm{b}}$ is replaced by a $z \times z$ zero matrix and each non-negative element $H_{\mathrm{b}}(i,j)$ is substituted by an identity matrix which is cyclically shifted to the right by $H_{\mathrm{b}}(i,j)$ positions. Note that performing the lifting approach leads to the binary matrix $\boldsymbol{H}$ used in the decoder.

Recall that a permutation matrix has weight 1 in all rows and columns. Consequently, all resulting lifted matrices have the same weight distribution over the rows, regardless of the length n of the code. This also holds for linear combinations of parity-check equations. A linear combination leading to a redundant parity check of low weight for a given code length will lead to a redundant check of the same weight for a different length. This motivates us to find redundant parity-check equations for the base matrix and use them to identify redundant checks for any of the resulting lifted codes. Prior to this, we review a general method which allows us to find redundant checks for any code.

B. General method for finding redundant parity checks

In [13] a general method to construct a set of redundant parity-check matrices for a given code was presented. This method was originally intended to provide good redundant parity-check matrices for PEG-constructed codes of short length. As these codes have no special structural properties, the method is applicable to any code. It relies on the fact that any parity-check matrix contains cycles of a given length c. Let $\mathcal{G}_c$ be one set of indices of parity checks closing a cycle of that length. A linear combination of the parity checks indexed by the set $\mathcal{G}_c$ leads to a novel parity-check equation with a Hamming weight of at most

$$w_{\mathrm{r}} = \sum_{i \in \mathcal{G}_c} w_i - c, \tag{3}$$

where w_i denotes the weight of parity check i.

C. Redundant parity checks for WiMAX LDPC codes

The novel approach for creating redundant parity-check equations uses the base matrix $\boldsymbol{H}_{\mathrm{b}}$ instead of the binary matrix $\boldsymbol{H}$ to find valid linear combinations. Let us elaborate on the generation of these checks. In a binary matrix, a redundant check can be found as a linear combination of two or more existing checks. This proceeding is in general not possible when the base matrix $\boldsymbol{H}_{\mathrm{b}}$ is considered, as the addition of two entries is not defined. However, the addition of a negative and a non-negative element, as well as the addition of two zero elements is a straightforward task. The result of the addition is the non-negative element and the element -1, respectively. Using this approach, redundant checks can be created by the linear combination of two existing checks which do not share a positive element in any column. The base matrix $\boldsymbol{H}_{\mathrm{b}}$ contains pairs of rows with this special property. Once a redundant check for the base matrix is created, the process of lifting can be applied to it. This leads to a set of z checks for the binary matrix $\boldsymbol{H}$ which are subsequently used to create sets of non-equal, binary parity-check matrices. As an example, we state that the linear combination of rows 11 and 12 in $\boldsymbol{H}_{\mathrm{b}}$ leads to z binary redundant checks of weight 10, as there are six non-negative entries in rows 11 and 12. Their column positions are disjoint, except for the last column which contains zero entries. See rows 11 and 12 of matrix $\boldsymbol{H}'_{\mathrm{b}}$ to verify this.

Depending on the length of the code, we replace 10 to 16 parity checks in the “original” binary parity-check matrix to generate a new representation. We ensure that each constructed parity-check matrix has full rank. The low number of replaced rows is due to the higher weight compared to the parity checks in the original parity-check matrix. This choice is a tradeoff between significantly different parity-check matrices and good performance of the single BP decoding units.

A technical detail in the proposed approach allows for efficient storing of matrices in an MBBP setup. If the additional matrix representations are created in such a way that all z

$$
\boldsymbol{H}_{\mathrm{b}}' = \begin{bmatrix}
-1 & 94 & 73 & -1 & -1 & -1 & -1 & -1 & 55 & 83 & -1 & -1 & 7 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 27 & -1 & -1 & -1 & 22 & 79 & 9 & -1 & -1 & -1 & 12 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & 24 & 22 & 81 & -1 & 33 & -1 & -1 & -1 & 0 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
61 & -1 & 47 & -1 & -1 & -1 & -1 & -1 & 65 & 25 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & 39 & -1 & -1 & -1 & 84 & -1 & -1 & 41 & 72 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & 46 & 40 & -1 & 82 & -1 & -1 & -1 & 79 & 0 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & 95 & 53 & -1 & -1 & -1 & -1 & -1 & 14 & 18 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 \\
-1 & 11 & 73 & -1 & -1 & -1 & 2 & -1 & -1 & 47 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 \\
12 & -1 & -1 & -1 & 83 & 24 & -1 & 43 & -1 & -1 & -1 & 51 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & 94 & -1 & 59 & -1 & -1 & 70 & 72 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 \\
\mathbf{-1} & \mathbf{-1} & \mathbf{7} & \mathbf{65} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{39} & \mathbf{49} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{0} & \mathbf{0} \\
\mathbf{43} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{66} & \mathbf{-1} & \mathbf{41} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{26} & \mathbf{7} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} & \mathbf{0}
\end{bmatrix}
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{matrix}
\tag{4}
$$

binary checks emanating from one redundant check of $\boldsymbol{H}_{\mathrm{b}}$ are deployed, the resulting binary matrix preserves the structural properties of the original parity-check matrix, i.e. it consists only of $z \times z$ zero matrices and cyclically shifted identity matrices of the same size.

Finally, let us compare this result to the approach from Section III-B, exemplary using the WiMAX code of length $n = 576$. The local girth of its parity-check matrix varies between $c = 6$ and $c = 8$. Using Equation (3) and $c = 6$, it can be deducted that additional parity-check representations have a weight of at most 12 to 15, depending on the weight of the parity checks used to create the linear combinations. There, weight 6 and weight 7 is possible. Computer simulations indicate that these bounds are met with equality with a high probability and hence it is difficult to find good representations using this algorithm.

IV. Results and Comparison

Simulation results for codes from the WiMAX standard of rate $1/2$ are presented. We consider all lengths available in the standard [14] with $n \leq 1000$. We show the superiority of (L)-MBBP decoding over standard BP decoding. As we are interested in a good decoding performance, we allow all BP decoding units to perform at most 200 iterations. This choice is motivated as follows. A further increase does not improve the standard BP decoding performance significantly while lowering the number of iterations leads to a performance degradation. In MBBP setups, we limit the number of BP decoding units with non-equal parity-check matrices to 15. In further decoding units, the leaking approach is applied with an initial setting of $p_{\mathcal{L}} = 0.9$. We set the parameter $I'_{\max} = 300$, as this choice leads to desirable results in our computer simulations. Consequently, we run a maximum number of 30 decoders in parallel. Investigations on good decoding performance and a low number of total iterations (summed over all decoders) have shown that it is advisable to use MBBP with $l > 1$. This was shown in [11] where the power efficiency[3] was considered as a function of the number of decoders while the total number of iterations was fixed. A local minimum at $l > 1$ was observed. In this work, we focus on the performance obtainable in principle and exceed this minimum. Further, the current development of multiprocessor techniques allows us to state that this setting can easily be parallelized with upcoming microcontroller techniques.

In Figure 2 performance results for two selected WiMAX codes with $n = 576$ and $n = 960$ are shown. In order to emphasize that the bigger part of the decoding gain is already obtained with a low number of decoder representations, we show different (L)-MBBP settings. To be precise, we allow $l = 7$ (MBBP), $l = 15$ (MBBP), and $l = 30$ (L-MBBP) representations to run in parallel. We observe that the most prominent part of the decoding gain is already achieved with $l = 7$ decoders in parallel and another small gain is achieved for $l = 15$. The usage of L-MBBP, together with $l = 30$ decoders in total, compares favorably but the difference is small in relation to the number of decoders additionally required. It is clear that the local minimum on power efficiency over the number of decoders is exceeded at this point, but nevertheless performance improvements are still obtainable. Using $l = 15$ decoding units, the proposed multi-decoding approach improves the performance of WiMAX codes for about $0.15\,\mathrm{dB}$ when compared to standard BP. Using this approach, we observe a performance improvement over a wide range of signal-to-noise ratios. We conclude that MBBP also allows for better decoding performance when signaling over a block Rayleigh fading channel, i.e. a Rayleigh fading channel with constant attenuation during the transmission of a codeword. It is reasonable to assume block fading as the codes of interest are short [22].

For comparison reasons, we show the random coding bound (Gallager bound) [23] which marks desirable frame error rates for codes of given length and rate. In order to provide performance results on the BER, we estimate the minimum distance d of a specified code by means of the Gilbert-Varshamov-bound [19]. Here, we assume that d errors happen in an erroneously decoded frame. Details on this approach can be found in [15].

The PEG codes are a welcome opportunity to assess our results to codes which are known to provide very good performance results when the code length is limited. We use the PEG algorithm to generate codes of rate $1/2$ and of comparable length, i.e. $500 \leq n \leq 1000$. For the PEG codes, we use the optimized degree distribution

[3] The required signal-to-noise ratio for obtaining a given quality-of-service

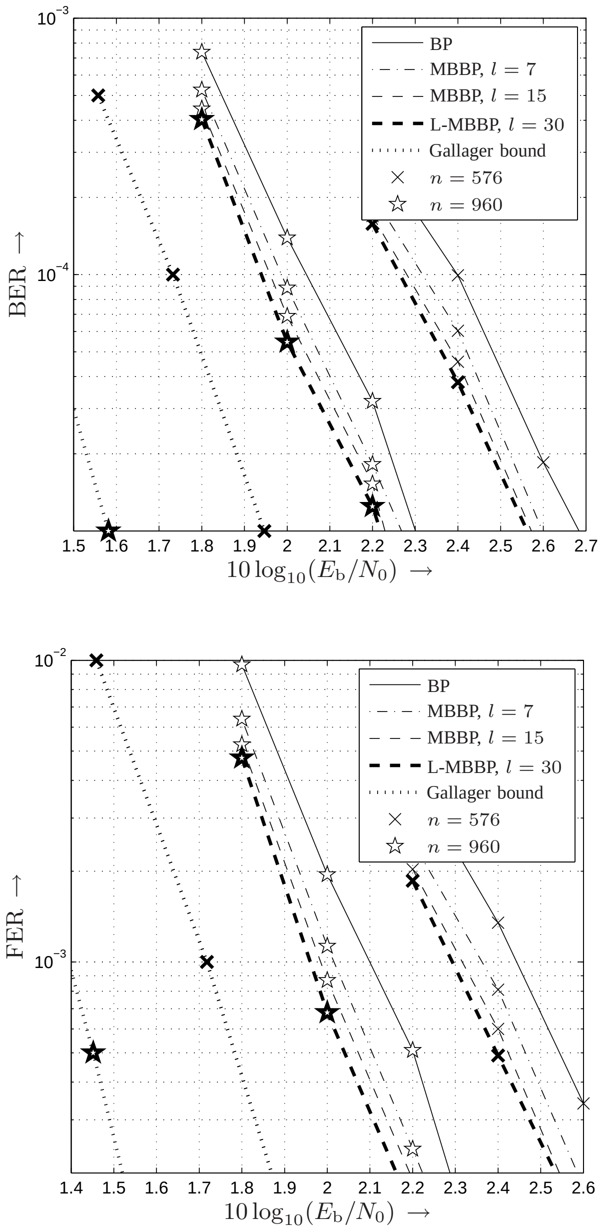

Fig. 2. BER and FER performance comparison of IEEE WiMAX codes with BP and the L-MBBP approach. The Gallager bound is shown for comparison reasons.

$$L(x) = 0.5043865558 \cdot x^2 + 0.2955760529 \cdot x^3 + 0.0572634080 \cdot x^5 + 0.0362602194 \cdot x^6 + 0.0049622081 \cdot x^7 + 0.0292344776 \cdot x^9 + 0.0650312477 \cdot x^{11} + 0.0072858305 \cdot x^{12} \quad (5)$$

from [24], which has a gap to capacity of about $0.2\,\mathrm{dB}$ and leads to desirable results for the code lengths of interest [15]. The given distribution is used to create channel codes in a random manner. Within the error region considered in our investigations, the created ensembles show strictly concentrated behavior, what allows us to study the subsequent results independent of the random seed used for the construction algorithm. Due to the random construction process, only general approaches for efficient encoding [25] can be used. However, due to short code-lengths, efficient encoding is not mandatory for producing simulation results.

Let us now compare these codes to the codes from the WiMAX standard, using both standard BP decoding and the (L)-MBBP setup. In the following, we discuss the signal-to-noise ratio $10 \cdot \log_{10}(E_\mathrm{b}/N_0)$ which is required to obtain the reliability criterion $\mathrm{BER} = 10^{-5}$ and $\mathrm{FER} = 10^{-3}$, respectively. Figure 3 shows this power efficiency over the code length. Plotted are results for WiMAX codes and PEG-optimized codes for both BP and L-MBBP decoding as well as the Gallager bound. Further, we include the sphere packing bound (SPB) which is a tight lower bound on the power efficiency when rate and length of the code are given. For details on calculating these values, the reader is referred to [26]. The capacity limit reads $10 \cdot \log_{10}(E_\mathrm{b}/N_0) \approx 0.19\,\mathrm{dB}$ for the considered code rates and error rates.

Let us first consider the results for the WiMAX codes. It can be observed that a gain of about $0.15\,\mathrm{dB}$ is achieved for all code lengths considered. From the plot for $\mathrm{FER} = 10^{-3}$ and the code of length $n = 960$ we observe that the gap to the Gallager bound reads about $0.7\,\mathrm{dB}$ for standard BP decoding. This gap can be lowered by $0.14\,\mathrm{dB}$ (or $20\ \%$) with the L-MBBP approach. This improvement can for example be used as a post-processing step in a WiMAX receiver and decode frames not decodable by the standard BP algorithm.

Similar results are presented for the PEG-optimized LDPC codes, where we also restrict the maximum number of decoders in parallel to 30. The actual number is however often lower as there exist no tailored methods for finding additional well-performing presentations [5]. Compared to WiMAX codes, the PEG codes show the desired performance results at about $0.15\,\mathrm{dB}$ lower signal-to-noise ratios. Again, the L-MBBP approach mitigates the gap to the random coding bound by about $20\,\%$.

It is worth mentioning that the codes defined in the WiMAX standard have a significantly lower density compared to the PEG codes of comparable length. This allows for faster decoding with the BP algorithm. If one considers not only the length but also the decoding speed as a system parameter, the standardized codes are comparable to the PEG-optimized codes discussed in this work. Detailed results in this direction can be found in [12].

V. Conclusions

We have shown that the multiple-bases approach is applicable to modern codes from the IEEE WiMAX standard. The multiple-bases concept works very well when codes with a special structural property in their parity-check matrix are considered. Codes of this type are frequently used in communications standards, as they allow for efficient storing of the parity-check matrix. For the LDPC codes defined in the WiMAX standard, a performance improvement of about

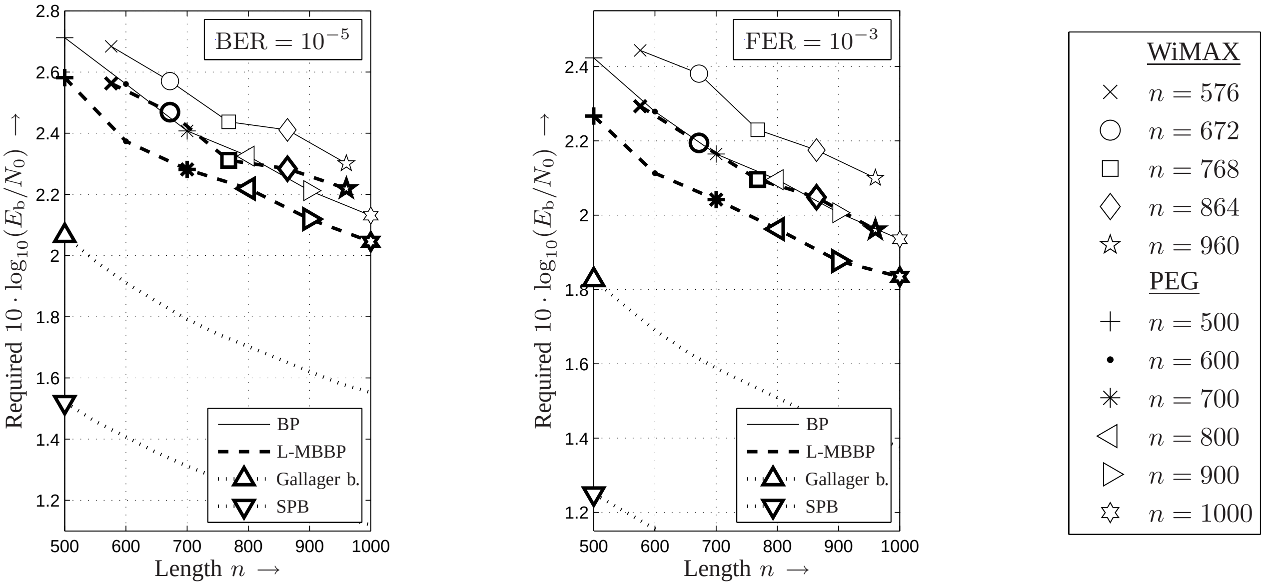

Fig. 3. Required SNR to meet given quality constraints $\mathrm{BER} = 10^{-5}$ and $\mathrm{FER} = 10^{-3}$, respectively, using WiMAX codes ($576 \leq n \leq 960$) and PEG-optimized codes ($500 \leq n \leq 1000$). For comparison, the Gallager bound and the sphere packing bound (SPB) are shown.

$0.15\,\mathrm{dB}$ is possible when using MBBP. The comparison to PEG codes showed the superiority of PEG codes in terms of decoding performance.

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