Comparison of High-Performance Codes on AWGN Channel with Erasures

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Abstract

This paper provides an overview of near Shannon-limit operating codes when transmitted over the additive white Gaussian noise (AWGN) channel with erasures. We compare the performance of standardized low-density parity-check (LDPC) codes and parallel-concatenated (turbo) codes to two progressive edge growth (PEG) optimized codes and a new design. The assumed channel, an AWGN channel with erasures, plays an important role in the field of satellite communications. The standardized codes we chose for our comparison purposes are the DVB-S2 LDPC code and a previously designed turbo code of 3GPP2. Furthermore, we use the PEG algorithm, which is improved by a novel method, to design better LDPC codes for this channel.

1 Introduction

The ambitious aim of approaching the Shannon limit has become more feasible after the discovery of turbo codes by Berrou et alii [1]. Another class of iteratively decodable codes, LDPC codes, was first introduced by Gallager in his Ph.D. thesis in 1963 [2] and experienced later a renaissance after its rediscovery by MacKay [3]. These two code classes and their variations (serially concatenated codes [4], repeat-accumulate (RA) codes [5], etc.) constitute the two main pillars of what we call high-performance codes, i.e. codes operating near Shannon limit.

LDPC codes are usually decoded by the belief propagation algorithm, which can be considered in the general framework of decoding on graphs. However, the existence of cycles in the Tanner graph of the LDPC code degrades the performance of the decoding algorithm. The progressive edge growth (PEG) algorithm [6] addresses this problem, and proposes a solution by maximizing the local girth with a greedy algorithm. This method is shown to be one of the best code construction methods, so we use this algorithm and improve its results further by modifying the parity-check matrix.

The advances in the area of coding have made the digital video broadcasting (DVB) standards body search for an efficient channel code for second generation satellite applications (DVB-S2), where they finally standardized a set of LDPC codes [7]. Similarly in the standardization of 3rd generation mobile services, turbo codes (3GPP2) have been deployed [8]. These codes were not originally intended to be applied to satellite communications, but will represent the turbo codes in this comparison due to their high performance and standardization. In this correspondence we give an overview of the performance achieved by today’s best-known low-rate codes (namely rate=1/4) transmitted over the AWGN channel with erasures.

This paper is organized as follows: We first describe the additive white Gaussian noise channel with erasures. Secondly, we state the codes that we have used for comparison purposes. Then, we present the bit error rate curves of these codes transmitted over AWGN channel and the performance on AWGN channel with erasures. Finally, we sum up our comparison for this particular channel.

2 Channel Model

For our comparison we assume the channel to be an AWGN channel with erasures. This channel was first used to model magnetic and optical recording channels [9]. Therefore, the codes used on this channel have been designed to have high rates. However, we are especially interested in low-rate codes for satellite communication purposes.

The discussed channel model is depicted in figure 1. The BPSK-encoded information symbols are transmitted over an AWGN channel with noise variance \( \sigma^2 \), with its output remaining unchanged with probability \( 1 - ER \) and erased with probability \( ER \), where \( ER \) is the erasure rate of the channel. Such a channel model is important for satellite communications, as well as for mobile and underwater communications, where the distribution of erasures can be considered as uniform within a codeword by using a random interleaver of sufficient length.
3 Compared Codes

3.1 Standardized Codes

The first code investigated is one of the LDPC codes standardized as DVB-S2 by the European Telecommunications Standards Institute (ETSI). This standard defines a set of codes of different rate and code length, the one we chose has an encoded word length of 64800, rate 1/4. Like all DVB-S2 codes, it is easily encodable because of its lower triangular parity-check matrix structure [7].

Since we want to compare the performance of these codes to standard turbo codes, we use the well-known 3GPP2 [8] turbo code as a benchmark. It is generated by two recursive systematic convolutional (RSC) encoders with transfer functions $Y_0(D) = \frac{1+D+D^2}{1+D+D^2}$ and $Y_1(D) = \frac{1+D+D^2}{1+D+D^2}$. This standard allows for different encoder block lengths between 378 and 20730 and also different rates with puncturing. We chose rate 1/4 and set the encoder block length to 12282 (encoded word length 49128).

3.2 Non-standardized Codes

We have designed three more LDPC codes for our comparison purposes. In all of our designs we used a powerful code design technique called progressive edge growth (PEG) [6]. This method is proven to perform very well, especially at short and moderate block lengths. Since we want to compare the codes to existing longer codes, we used PEG and the welding algorithm. The details of this algorithm will be given in a separate paper. The main underlying idea of this construction method is to combine the structure of the PEG designed code with randomly structured graph codes. In this case, the welding method yields a code with an encoded word length of 48000 bits.

Furthermore, we consider two PEG optimized codes of length 49152, applying no additional algorithm. For the first code we use an optimized degree distribution for the AWGN channel [10], whereas the second code is optimized for the binary erasure channel (BEC). When we consider the results obtained by density evolution [11], we see that the first code designed for AWGN channel has a threshold $10 \log_{10} (E_b/N_0) = -0.6234$ dB (which corresponds to noise variance $\sigma^2 = 2.2978$) and is only 0.19130 dB away from the capacity of the AWGN channel. Similarly, the second code, with BEC-optimized degree distribution, has a threshold of 0.7426760, which corresponds to a 2.84622% capacity gap. These codes have been chosen because of their suitable degree distributions for practical purposes (i.e. variable nodes have a degree less than 15), furthermore they have strictly concentrated\(^2\) or regular check-node degree distributions.

The progressive edge growth [6] algorithm works on the Tanner graph of an LDPC code. It optimizes the local girth of a node edge by edge in a greedy manner. However, the complexity of this algorithm has to be considered for the large length of the codes we are interested in. A better variant of the algorithm that resembles bit filling [12] is also described in [6], where the target girth of the graph is fixed. This variant is suggested for constructing low-rate long-length codes. We have used the latter algorithm for our code design purposes.

A further code used in our investigation is the asymmetric multiple turbo code described in [13]. This systematic rate 1/4 code is composed of 4 encoders with transfer functions $Y_0(D) = 1+D+D^2$, $Y_1(D) = 1+D+D^2$, $Y_2(D) = \frac{1+D+D^2}{1+D+D^2}$ and $Y_3(D) = \frac{1+D+D^2}{1+D+D^2}$. It is optimized for very large lengths in the order of 100000 input bits and performs very close to Shannon limit, but for comparison purposes was simulated with an encoded word length of 64800 bits.

4 Results

4.1 Standardized Codes

In this section we discuss our findings and compare the performances of the discussed codes. To begin with, the DVB-S2 code with its encoded word length of 64800 bits length is the longest code in our comparison. The 3GPP2 code has an encoded word length of 49128 bits, which is very close to the PEG-optimized codes we compare to. Figure 2 displays the comparison for these standardized codes, with the left part of the curves comparing the bit error rate performance and the right curves showing the tolerated erasure rates depicted on the right vertical axis. It can be seen that on pure AWGN channel, the turbo code (3GPP2 with 12 iterations) performs approximately 0.3 dB better than its LDPC counterpart (DVB-S2 with 50 iterations). Although the turbo code has a shorter block length, it can be seen that it tolerates up to 3% more erasures.

4.2 Non-standardized Codes

The code designed by welding and the PEG code optimized for AWGN channel were generated from the same degree distribution, therefore, their comparison would let us judge the effectiveness of the new design method. On AWGN channel it can be seen that the code designed by welding performs 0.2 dB better than the AWGN channel optimized PEG code. If we compare two codes on AWGN channel with erasures,

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\(\text{\footnote{\text{Capacity-Rate/Capacity}}}\)

\(\text{\footnote{\text{only two different degrees that are consecutive}}}\)
the code designed by welding tolerates 1.5% to 2% more erasures than its counterpart. These results show that the welding strategy yields better results on both of these channels.

A second comparison can be conducted between the PEG designed codes. Here we face an interesting result. The code based on the degree distribution optimized for BEC does not only perform 2-3% better on AWGN channel with erasures, but is also slightly (< 0.1 dB) better on pure AWGN channel. The explanation for these findings comes from degree distributions: Although both chosen codes have maximum variable node degree as 14, in the AWGN optimized code the degree 14 nodes have a weight (from edge perspective) 0.2449580, whereas in the BEC optimized code, they have a weight of 0.07649380. Consequently, the PEG algorithm working for the AWGN optimized code is challenged in fixing the local girth to the target girth. As a result, the BEC optimized code has a better local girth distribution which yields a better result with the same code length.

Furthermore, we compare a multiple turbo code which was optimized using advanced knowledge in information combining [14] with the previous codes. While its performance (after 12 iterations) matches the 3GPP2 code in the waterfall region of the AWGN channel curve, it outperforms the compared LDPC codes by approximately 0.3 dB. For the AWGN channel with erasures, the multiple turbo code follows the 3GPP2 code’s performance for low erasure rates and approximates the LDPC codes for erasure rates greater than 0.4. The tolerable erasure rate for high SNR’s approaches 0.68, which is clearly below the 3GPP2 turbo code. These findings are presented in figure 3.

Comparing the performance of the standardized codes to the codes optimized for AWGN and BEC channel, as well as the welding algorithm and the multiple turbo code, it has to be stated that the standardized codes still perform better for the AWGN channel with erasures, since they are exceeding a tolerated erasure rate of 70% for high signal-to-noise ratios. The improved performance of the DVB-S2 code compared to the PEG codes and the code constructed by the welding algorithm might be partially attributed to its larger length.

Fig. 2. Comparison of standardized codes, 3GPP2 code with 8 and 12 iterations, resp., DVB-S2 code with 50 iterations
5 Conclusions

We have compared several different high-performance codes on AWGN channel with erasures. The performance of standardized and non-standardized codes as well as our own design were lined up against each other. The standardized codes, especially the 3GPP2 code, are not outperformed at the moment. We are currently working on the welding algorithm and its application to PEG-optimized codes. We expect this to lead to further improvement of the algorithm and to allow us to outperform the standardized codes.

References


