Equalization with Widely Linear Filtering

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Abstract — **The concept of widely linear estimation is applied to the equalization problem. Widely linear equalizers and decision–feedback equalizers with widely linear feedforward filtering are presented. For mobile communication channels significant performance gains can be achieved compared to conventional equalizers.**

I. Introduction

Recently, the concept of *Widely Linear Estimation* (WLE) has been introduced in the signal processing literature [1]. E.g. in [2] WLE has been employed to improve demodulation of direct–sequence code–division multiple–access (DS–CDMA) signals. In this paper, it is shown that WLE is also beneficial for equalization of intersymbol interference (ISI) channels. The presented concept is applicable if symbols from a real–valued alphabet are transmitted over a channel with complex–valued taps in equivalent baseband representation. Such a scenario arises e.g. in the GSM (Global System for Mobile Communication) system.

II. System Model

For transmission with linear modulation over an ISI channel the discrete–time received signal after symbol–spaced sampling is given by $r[k] = \sum_\kappa h[\kappa]\, a[k-\kappa] + n[k]$. The transmitted data symbols $a[\cdot]$ are taken from a real–valued symbol alphabet such as $\{\pm 1\}$ (BPSK). $h[\cdot]$ and $n[\cdot]$ are the complex–valued discrete–time overall impulse response and wide–sense stationary complex additive white Gaussian noise, respectively.

In the receiver, widely linear processing is applied, i.e., an estimate $y[k]$ for the transmitted symbol $a[k]$ is generated by filtering $r[\cdot]$ as well as the complex–conjugated received sequence $r^*[\cdot]$ with filters $f_1[\cdot]$ and $f_2[\cdot]$, respectively, and combining the results, $y[k] = \sum_\kappa f_1[\kappa]\, r[k-\kappa] + \sum_\kappa f_2[\kappa]\, r^*[k-\kappa]$. The overall system may be represented by a two–branch filter bank with subchannel impulse responses $h[k]$ and $h^*[k]$.

III. Filter Optimization

The same overall system model arises for conventional fractionally–spaced equalization (FSE) with sampling rate $2/T$ (T: symbol interval). Hence, results from FSE may be used for optimization of the widely linear structure. The WLE filters (of finite or infinite length) minimizing the mean–squared error (MSE) are given by the polyphase components of an MMSE–FSE filter for a channel with impulse response $\tilde{h}[k_1] = h[k]$, $k_1 = 2\,k$, $\tilde{h}[k_1] = h^*[k]$, $k_1 = 2\,k+1$ (k_1 denotes discrete time with respect to fractionally–spaced sampling). Since the underlying system model also arises in a diversity reception context, results from equalization for diversity signals may be used alternatively for calculation of the optimum filter settings. The mean–squared error for MMSE–WLE with infinite–length filters is $\sigma^2_{e,\mathrm{MMSE-WLE}} = \sigma_n^2\, T \int\limits_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{1}{|H(\mathrm{e}^{j2\pi fT})|^2 + |H(\mathrm{e}^{-j2\pi fT})|^2 + \sigma_n^2/\sigma_a^2}\, \mathrm{d}f$. Instead of the MMSE criterion, the zero–forcing (ZF) criterion may be applied as well. ZF equalization is even possible with FIR filters, if the transfer functions $H(z)$ and $H^*(z^*)$ of both subchannels have disjoint roots, i.e., if $H(z)$ has no real roots and no conjugate pairs of roots.

Performance of WLE can be further improved by employing additional noise prediction, or equivalently decision feedback. For filter optimization results on fractionally–spaced decision–feedback equalization (DFE) or on diversity combining DFE can be exploited. Here we only give an expression for the optimum feedback filter of an MMSE–WDFE with infinite–length filters, whose transfer function is $B(z)-1$, where the causal and monic polynomial $B(z)$ is the solution of $H(z)\,H^*(1/z^*) + H^*(z^*)\,H(1/z) + \sigma_n^2/\sigma_a^2 = C\,B(z)\,B^*(1/z^*)$ (C: constant factor). The corresponding MMSE is

$$\sigma^2_{e,\mathrm{MMSE-WDFE}} = \sigma_n^2 \exp\left(-T \int\limits_{-\frac{1}{2T}}^{\frac{1}{2T}} \ln\left[\,|H(\mathrm{e}^{j2\pi fT})|^2 + |H(\mathrm{e}^{-j2\pi fT})|^2 + \sigma_n^2/\sigma_a^2\right] \mathrm{d}f\right)$$

IV. Performance

As an example, we consider a channel with one zero on the unit circle, i.e., $H(z) = \frac{1}{\sqrt{2}}(1 - \mathrm{e}^{j\varphi}\, z^{-1})$, $\varphi \in [0,\, 2\pi)$. For widely linear equalization, $\sigma^2_{e,\mathrm{ZF-WLE}} = \sigma_n^2/(2\,|\sin\varphi|)$, $\sigma^2_{e,\mathrm{MMSE-WLE}} = \sigma_n^2/(2\,\sqrt{(1+\sigma_n^2/2)^2 - \cos^2\varphi})$ is valid. Obviously, ZF–WLE is also applicable for zeros on the unit circle if $\varphi \notin \{0;\pi\}$. This is in contrast to conventional ZF–LE, which produces an infinite noise enhancement in this case. For widely linear receivers with decision feedback, one obtains $\sigma^2_{e,\mathrm{ZF-WDFE}} = \sigma_n^2/(1+|\sin\varphi|)$, $\sigma^2_{e,\mathrm{MMSE-WDFE}} = \sigma_n^2/(1+\sigma_n^2/2+\sqrt{(1+\sigma_n^2/2)^2-\cos^2\varphi})$, whereas for conventional ZF–DFE $\sigma^2_{e,\mathrm{ZF-DFE}} = \sigma_n^2$ is valid. Hence, the gain G of ZF–WDFE compared to ZF–DFE is given by $G = 1 + |\sin\varphi|$, which attains its maximum $G = 2$ for a channel with one zero at $z = \pm j$.

Finally, we compare the novel and conventional MMSE receivers for a stochastic test channel of order 6 with mutually uncorrelated complex Gaussian coefficients of equal mean power. Simulations show that MMSE–WLE and MMSE–WDFE gain more than 5 dB and about 0.7 dB at BER $= 10^{-4}$ compared to MMSE–LE and MMSE–DFE, respectively.

In a practical system like GSM, the novel schemes have a clear advantage over the conventional ones, which have been considered in most previous publications on low–complexity equalization for GSM.

References

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