On CDMA Transmission over Mismatched Fading Channels employing MMSE–Receivers and Successive Cancellation

Alexander Lampe
Universität Erlangen–Nürnberg, Cauerstraße 7/NT, 91058 Erlangen, Germany
Phone: +49-9131-85 27114, Fax: +49-9131-85 28919, Email: alampe@LNT.de

Abstract — **Recently, a closed solution to the capacity of synchronous CDMA for fading channels supposing exact channel knowledge at the receiver and infinite spreading factor, i.e., $N\to\infty$ has been derived by Shamai and Verdú [1]. On the other hand, considering imperfect channel state information Evans and Tse [2] derived analytical solutions to the signal to noise ratio provided by linear multiuser receivers and could also give results for the error variance resulting from linear channel estimation. Here, lower and upper bounds on the users' SIR reachable by means of a nonlinear MMSE–receiver applying successive cancellation while assuming a certain channel estimation accuracy are given. Note that due to imperfect channel estimation the interference from previously decoded users cannot be cancelled completely even if no decoding errors occur.**

I. Lower and Upper Bound on SIR

We consider the synchronous transmission of K users over frequency selective fading channels to a common receiver and model the shifted replicas of the kth user's random spreading sequence arriving over the L resolved paths as L independently chosen random spreading sequences $\boldsymbol{s}_{k,1}[\mu],\dots,\boldsymbol{s}_{k,L}[\mu], \forall k$, with spreading factor N [2] (Note, this model holds exactly for the equivalent case of K users transmitting with L antennas over flat fading channels to a single receiver). Assuming imperfect channel knowledge at receiver site, the path weights are modeled as sum of a Gaussian distributed MMSE path weight estimate $\hat{h}_{k,l}[\mu]$ with variance $1/L-J$ and orthogonal estimation error $\hat{n}_{k,l}[\mu]$ with power J. For sake of simplicity, we suppose that channel estimation is performed with equal accuracy for all users $1\le k\le K$, paths $1\le l\le L$ and time slots. Now, considering user k the signal employed for subsequent processing in a specific time interval $[\mu]$ after cancellation of user $k+1,\dots,K$ based on their perfectly decoded symbols is

$$\begin{aligned}\boldsymbol{y}_k[\mu] &= \sum_{\kappa=1}^{k}\sum_{l=1}^{L}\boldsymbol{s}_{\kappa,l}[\mu](\hat{h}_{\kappa,l}[\mu]+\hat{n}_{\kappa,l}[\mu])x_\kappa[\mu]\\ &\quad+\sum_{\kappa=k+1}^{K}\sum_{l=1}^{L}\boldsymbol{s}_{\kappa,l}[\mu]\hat{n}_{\kappa,l}[\mu]x^d_{\kappa,l}[\mu]+\boldsymbol{n}[\mu],\end{aligned}$$

where the N dimensional vector $\boldsymbol{n}$ represents the additive channel noise. The i.i.d. components of $\boldsymbol{n}$ are zero mean complex Gaussian with variance σ_n^2. Further, the users' channel symbols are denoted as $x_k\in\mathcal{X}$, $1\le k\le K$, having equal power σ_x^2 and the superscript d marks the already known channel symbols of previously decoded users. Assuming uncorrelated channel estimation errors for the different users as well as paths and time slots, a lower bound on the resulting signal to interference ratio $\mathrm{SIR}_k[\mu]$ at the output of an MMSE filter extracting the signal of user k from $\boldsymbol{y}_k[\mu]$ can be solved for $N\to\infty$ with $|\hat{\boldsymbol{h}}_k[\mu]|^2 \triangleq \sum_{l=1}^{L}|\hat{h}_{k,l}[\mu]|^2$ as

Theorem: For arbitrarily long spreading sequences and constant load β the normalized signal to interference ratio $\mathrm{SIR}(\alpha=k/N,\beta=K/N)\triangleq\mathrm{SIR}_k[\mu]/|\hat{\boldsymbol{h}}_k[\mu]|^2$ is in probability lower bounded by

$$\mathrm{SIR}(\alpha,\beta)[\mu]\ge\frac{\gamma^{\mathrm{L}}(\alpha,\beta)}{1+J\gamma^{\mathrm{L}}(\alpha,\beta)},$$

where

$$\begin{aligned}\gamma^{\mathrm{L}}(\alpha,\beta) &= \left(\frac{\sigma_n^2}{\sigma_x^2}+\frac{\alpha(L-1)J}{1+\gamma^{\mathrm{L}}(\alpha,\beta)J}+\right.\\ &\left.+(\beta-\alpha)L\int_0^\infty\frac{\zeta f_{J|X|^2}(\zeta)}{\sigma_x^2+\gamma^{\mathrm{L}}(\alpha,\beta)\zeta}\mathrm{d}\zeta+\alpha\int_0^\infty\frac{\zeta f_{|\hat{h}|^2+J}(\zeta)}{1+\gamma^{\mathrm{L}}(\alpha,\beta)\zeta}\mathrm{d}\zeta\right)^{-1}.\end{aligned}$$

Here, $f_{|\hat{h}|^2+J}(\zeta)$ as well as $f_{a|X|^2}(\zeta)$ denote the pdf of $|\hat{\boldsymbol{h}}|^2+J$, as well as the pdf of the squared absolute value of the transmit symbols $x\in\mathcal{X}$ multiplied by a, respectively.

In addition, for single path fading channels an upper bound can be given resulting from the assumption of correlated channel estimation errors.

Lemma: For a single path fading channel $\mathrm{SIR}(\alpha,\beta)$ is for $N\to\infty$ upper bounded by

$$\mathrm{SIR}(\alpha,\beta)\le\frac{\gamma^{\mathrm{U}}(\alpha,\beta)}{1+J\gamma^{\mathrm{U}}(\alpha,\beta)},$$

where $\gamma^{\mathrm{U}}(\alpha,\beta)=\left(\frac{\sigma_n^2}{\sigma_x^2}+\alpha\int_0^\infty\frac{\zeta f_{|\hat{h}|^2+J}(\zeta)}{1+\gamma^{\mathrm{U}}(\alpha,\beta)\zeta}\mathrm{d}\zeta\right)^{-1}$.

Based on the above formulas as well as results on iterative channel estimation we can show that successive cancellation yields considerable gains compared to linear interference suppression even if the channel state is not know exactly at receiver site. It turns out that this advantage depends heavily on the system load as well as number of propagation paths. Moreover, we can show that also in this case nonorthogonal multiple access can reach a higher spectral efficiency than orthogonal schemes. Finally, it is worth noting that other nonlinear receivers, systems with multiple transmit and receive antennas as well as the problem of imperfectly known reference symbols for channel estimation can be treated analytically in the same way.

References

[1] S. Shamai (Shitz) and S. Verdú,"The effect of frequency–flat fading on the spectral efficiency of CDMA", submitted to IEEE Trans. on IT, Nov. 1999.

[2] J. S. Evans and D. N.C. Tse,"Linear Multiuser Receivers for Multipath Fading Channels", submitted to IEEE Trans. on IT, 1999.