Dynamically Coordinated Reception of Multiple Signals in Correlated Noise

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Abstract — **New methods for the exploitation of noise correlation in multiple signal receivers are proposed. In contrast to [2] the entire correlation functions are exploited. The optimal Rate and signal power distributions are given and the SNR gain by coordinated transmission is derived. An example for an application of such receivers is fast digital transmission over parallel twisted pair lines by multiple duplex transmission techniques (HDSL, VHDSL etc.; cf. [3, 4]).**

I. Introduction

In [2] techniques for coordinated transmission over two parallel channels are discussed, which exploit the correlation of simultaneous signal samples by means of linear combinations at the transmitter (precorrelation) and the receiver (decorrelation). This method provides two equivalent channels with uncorrelated noise and maximum SNR spread. It corresponds to a Karhunen-Loève-Transformation for correlated random variables. As these methods only exploit the correlation of simultaneous noise samples, we apply the denomination "static coordination".

For channels with intersymbol interference and/or coloured noise, static coordination is not exhaustive. Therefore, we propose several structures for a "dynamically coordinated" transmission over parallel channels which exploit the entire cross correlation functions of the noise processes.

II. Dynamic Coordination

Without loss of generality, we assume optimum linear zero forcing equalization and T spaced sampling for each of the D signals. Noise is characterized by discrete time auto- and cross-correlation functions $\varphi_{m\ell}(\kappa)$; $m, \ell \in \{1, 2, \ldots, D\}$, $\kappa \in \mathbb{Z}$. One possible version of dynamic coordination is a generalized noise prediction via decision feedback. Noise estimates are derived from previous noise measurements of <u>all</u> D channels via $D \times D$ prediction filters $\mathcal{P}_{m\ell}(z) = \sum_{\lambda=1}^{L} \rho_{m\ell}(\lambda) \cdot z^{-\lambda}$ with degree L. The noise measurements are determined by decision-feedback and subtracted from the receiver input samples in order to get white residual noise processes with minimum noise variances σ_m^2. The tap weights $\rho_{m\ell}(\lambda)$ of the prediction filters can be determined by generalized Yule-Walker-equations:

$$\sum_{m=1}^{D} \sum_{\lambda=1}^{L} \rho_{m\ell}(\lambda) \varphi_{im}(\lambda - k) = \varphi_{\ell i}(\lambda) \qquad (1)$$

$$\text{for} \quad i, \ell \in \{1, 2, \ldots, D\} \; ; \quad k \in \{1, \ldots, L\}$$

For a sufficient degree L and small error probabilities only correlations of simultaneous noise samples remain, expressed by a covariance matrix $\mathbf{N}$. Therefore, a static coordination procedure can be applied, e.g. by a precorrelation of the signals via a matrix $\mathbf{\Psi}$ at the transmitter and a decorrelation via a matrix $\mathbf{\Psi}^{-1} = \mathbf{\Psi}^{*T}$ at the receiver. These matrices are obtained from a eigenvalue decomposition of $\mathbf{N}$: $\mathbf{N} = \mathbf{\Psi}^{*T} \mathbf{diag}\,(\lambda_1, \ldots, \lambda_D)\, \mathbf{\Psi}$. The eigenvalues λ_m are the minimum noise variances of the equivalent channels with white, mutually uncorrelated noise processes and maximum SNR spreads. The static coordination can be incorporated into the dynamic coordination by the choise $\rho_{m\ell}(0) \neq 0$ for $\ell > m$. Again, the filter coefficients are given by eq. (1) for $k \in \{0, 1, \ldots, L\}$ for $i < \ell$. The receiver with coordinated noise prediction can be transformed into an equivalent structure with cross-coupled input filters and a multidimensional DFE as usual. The matrix of input filters may be interpreted as a "coordinated whitened matched filter", cf. [1]. A multidimensional Tomlinson-Harashima precoding is also possible.

III. Rate Distribution and Coordination gain

A maximization of the minimum Euclidean distance of the D-dimensional signal constellation yields the following optimal partitioning of the total transmission rate R_T into rates R_m for the individual channels:

$$R_m = \frac{1}{D} R_T + \frac{1}{2} \mathrm{ld} \left(\sqrt[D]{\prod_{\ell=1}^{D} \lambda_\ell} \Big/ \lambda_m \right) \qquad (2)$$

The total signal power S_T has to be equally distributed to those channels with transmission rate > 0. For an approach according to the MMSE-criterion, a power distribution corresponding to the well-known water filtering rule, $S_m + \lambda_m =$ const. with $\sum_{m=1}^{D} S_m = S_T$ is the optimum. The SNR gain G_c by coordinated transmission is well approximated by

$$G_c = \max_m \sigma_m^2 \Big/ \sqrt[D]{\prod_{\ell=1}^{D} \lambda_\ell} \qquad (3)$$

This formula exactly corresponds to the coordination gain derived from capacity arguments, cf. [2].

From a great amount of crosstalk measurements for twisted pair lines (AWG26) in the German Telekom network, we found gains up to 3 dB for dual-duplex HDSL (2.048 Mbit/s, quaternary signalling) by dynamic coordination.

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References

[1] W. van Etten: Maximum Likelihood Receiver for Multiple Channel Transmission Systems. *IEEE Tr. on Comm.*, vol. COM-24, pp. 276–283, Feb. 1976.

[2] J.W. Leichleider: Coordinated Transmission for Two-Pair Digital Subscriber Lines. *IEEE J. on Selected Areas in Comm.*, vol. JSAC-9, pp. 920–930, Aug. 1991.

[3] *IEEE J. on Selected Areas in Comm., Special Issue, "High Rate Digital Subscriber Lines"*, vol. JSAC-9, Aug. 1991.

[4] J. Huber: Distance-Gains by Multiple-Duplex Transmission, Coding and Shaping for HDSL. In *Proc. of the Int. Comm. Conference (ICC '93)*, Geneva, pp. 1820–1829, May 1993.