Performance Estimation of Bit-Interleaved Coded Modulation Based On Information Processing Characteristics

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***Abstract*—Information processing characteristics (IPC) provide a general framework for the analysis of a coding scheme. In this paper, we utilize IPCs to obtain performance estimates for the end-to-end coded channel for bit-interleaved coded modulation (BICM) using a given modulation format and coding scheme, i.e., its bit error rate and capacity. The proposed method enables to fully decouple the analysis of the coding scheme from the analysis of the higher-order modulation scheme and delivers very good performance estimates. Exemplarily, this is validated by means of numerical simulations for BICM using convolutional- and LDPC-coded amplitude-shift keying.**

I. Introduction

Bit-interleaved coded modulation (BICM) is a pragmatic approach to coded modulation and, due to its flexibility, robustness, and simplicity, is the de-facto standard for modern coded digital transmission systems [1], [2]. It is thus of particular interest to estimate the performance of BICM in combination with a given modulation format and coding scheme, i.e., channel code, encoder, and corresponding decoder.

In case of hard-input channel decoding, typically bit error rate (BER) processing characteristics of the coding scheme are sufficient, i.e., based on the uncoded BER of the communication channel the BER of the coded channel can be estimated. However, coding schemes significantly benefit from reliability information on the decoder input symbols; soft-decisions should be fed to the channel decoder. In this case, more elaborate methods to predict the BER of the end-to-end (coded) channel are required, typically based on (refinements of) the union bound [1]–[6]. In this paper, we present a method, which is based on the principle of information processing characteristics (IPC) [7]–[9]. It exploits the mutual information of the communication channel and the characteristics how the channel decoder processes the available information at its input. Based on the BICM capacity of the communication channel between encoder and decoder, we derive an estimate for the capacity of the end-to-end channel from encoder input to decoder output. This capacity estimate is then translated into an estimate for the coded BER using two well-known bounds based on modeling the end-to-end channel as a binary symmetric memoryless channel (BSC) and a binary erasure channel (BEC) [10].

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The method is applicable for any BICM setup with soft- or hard-decision demapping. As opposed to [1]–[6], it does not require deep insight into the channel code properties, such as, e.g., its weight distribution spectrum, as it is based only on the IPC of the coding scheme, which in turn can be derived from its BER performance over the binary-input AWGN channel. Exemplarily, we consider BICM of amplitude-shift keying (ASK) transmitted over the AWGN and a log-normal fading channel using convolutional codes in combination with soft-input Viterbi decoding [11], and LDPC codes with belief propagation decoding [12].

Following a brief review of the principle of IPCs in Sec. II, the BICM-ASK setup and the proposed method of IPC-based performance estimation are presented in Sec. III. The effectiveness of our approach is compared to simulation results in Sec. IV; a discussion of the method concludes the paper.

II. Information Processing Characteristics

We consider the generic transmission system depicted in the top part of Fig. 1, consisting of a channel encoder, a binary-input soft-output communication channel, and a channel decoder. Without restriction of generality, we assume the information symbols[1] u at the encoder input as independent equiprobable binary symbols. The encoder of rate $R_\mathrm{c} = k/n$ encodes k information symbols into a codeword of n binary code symbols x, which is transmitted over the communication channel to be detailed later. Given the received symbols v, which represent estimates of the code symbols (reliability information or hard decisions), the decoder estimates the transmitted information. Here, we restrict to hard-output decoders, i.e., $\hat{u}$ are from the same (binary) alphabet as u.

The concept of IPCs provides a general framework for the analysis of the performance of this coding scheme [7], [9], [14]. As opposed to the commonly used end-to-end performance measure average bit error rate (i.e., $\mathrm{BER}_{\mathsf{code}} =$

[1]Bold letters denote vectors, upper-case letters random variables, and lower-case letters the particular realization. $I(\cdot;\cdot)$: mutual information, $\mathrm{E}\{\cdot\}$: expectation operator, $\Pr\{\cdot\}$: probability, $H(\cdot)$: entropy, $\mathrm{e}_2(p) = -p\log_2(p) - (1-p)\log_2(1-p)$: binary entropy function [13]. For compact notation, we omit explicit symbol/time indices, where it does not impair clarity.

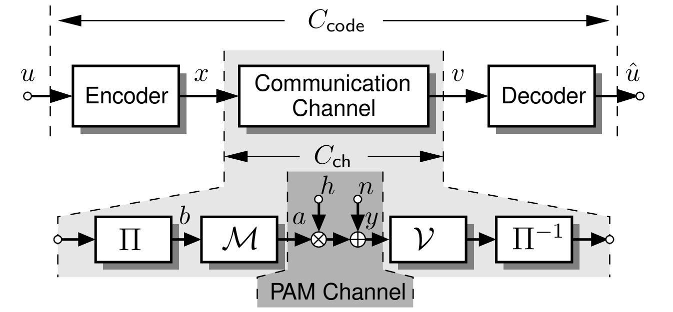

Fig. 1. System model of coded transmission. Top: generic model, bottom: BICM-ASK for the communication channel.

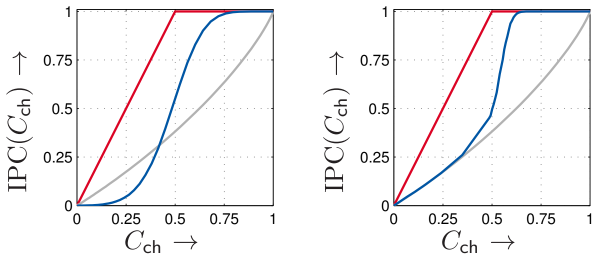

Fig. 2. IPCs of rate-1/2 coding schemes. Left: convolutional code (nonsystematic encoding, constraint length $\nu = 4$) with soft-input Viterbi decoding. Right: LDPC code (from WLAN-802.11n std., systematic encoding, codeword length 648) with BP decoding. Red: ideal coding scheme. Gray: uncoded.

$\mathrm{E}\{\Pr\{U \neq \hat{U}\}\}$) as a function of the signal-to-noise ratio (SNR), IPCs characterize the coding scheme with respect to mutual information as a function of the capacity of the underlying channel. More precisely for a given hard-output coding scheme code, we define

$$\text{IPC}_{\text{code}}(C_{\text{ch}}) = C_{\text{code}} = \frac{1}{k}\sum_{\kappa=1}^{k} I(U_\kappa; \hat{U}_\kappa) \qquad (1)$$

where C_{ch} denotes the capacity of the binary-input communication channel between encoder and decoder, i.e.,

$$C_{\text{ch}} = \frac{1}{n}\sum_{\nu=1}^{n} I(X_\nu; V_\nu) \ . \qquad (2)$$

An ideal encoder/decoder pair achieves $C_{\text{code}} = 1$, as long as the entropy of X, $H(X) = R_{\text{c}}$, does not exceed the capacity C_{ch} of the communication channel, and otherwise degrades linear [7], i.e.,

$$\text{IPC}_{\text{ideal}} = \min(C_{\text{ch}}/R_{\text{c}}, 1) \ . \qquad (3)$$

Since for most non-ideal coding schemes an analytical expression for the IPC does not exist, IPCs have to be obtained by simulation [7]. As IPCs depend only on the capacity of the underlying channel, this can be done by simulation of BPSK transmission over an AWGN channel. Depending on the investigated coding scheme, two options for the communication channel have to be considered. For soft-input channel decoding, v represent reliability information, and the capacity of the communication channel is given as the constellation-constraint capacity of BPSK transmitted over an AWGN channel. For hard-input channel decoding, v represent hard-decisions on the code symbols, and C_{ch} is given as the capacity of a BSC with equivalent $\text{BER} = \mathrm{E}\{\Pr\{X \neq V\}\}$. In both cases, assuming the errors introduced by the decoder follow a memoryless process[2], the mutual information of the overall channel can be estimated from the simulated BER_{code} via $C_{\text{code}} = 1 - \mathrm{e}_2(\text{BER}_{\text{code}})$.

Fig. 2 shows the IPCs of rate-1/2 coding schemes over the binary-input AWGN channel for a maximum-free distance convolutional code (constraint length $\nu = 4$, nonsystematic encoding) with soft-input Viterbi decoding [11] (left) and an LDPC code from the WLAN-802.11n standard [15] (systematic encoding, codeword length $n = 648$) with soft-input belief-propagation (BP) decoding [12] (right). The IPCs of an ideal coding scheme (red) and of uncoded transmission (BPSK over AWGN channel with hard decisions, gray) are also given. The systematically encoded LDPC code is always superior to uncoded transmission.

[2]This may be forced by ideal interleavers at the encoder input and a corresponding deinterleaver at the decoder output.

III. Performance Estimation of BICM

In this section, we show how accurate estimates for the performance of the end-to-end coded transmission chain are obtained using IPCs. Exemplarily we consider the case that the communication channel represents BICM of ASK [1], [2], but note that the proposed method can be transferred to any modulation format.

A. BICM for ASK-Transmission

In a BICM setup, as depicted in the bottom part of Fig. 1, the code symbols x are interleaved and partitioned into touples $\boldsymbol{b}$ of $\log_2(M)$ code symbols, which are mapped to amplitude coefficients a taken from an M-ary bipolar ASK signal constellation $\mathcal{A}$ with variance σ_a^2. The mapping is specified by a bijective binary labeling rule $\mathcal{M} : \boldsymbol{b} \in \{0; 1\}^{\log_2(M)} \mapsto a \in \mathcal{A}$ (e.g., Gray or set-partition labeling); the overall rate of the BICM setup is denoted as $R = R_{\text{c}} \log_2(M)$. After transmission using conventional digital pulse-amplitude modulation (PAM) [11], the receive symbols are given as the amplitude coefficient scaled by a fading coefficient h and corrupted by additive Gaussian noise n with variance σ_n^2, i.e.,

$$y = ha + n \ . \qquad (4)$$

As a measure for the quality of the communication channel, we consider the ratio of the transmitted energy per ASK symbol E_{s} over the one-sided noise power-spectral density N_0; thus, we have $E_{\text{s}}/N_0 = \sigma_a^2/(2\sigma_n^2)$. The receiver performs a demapping operation $\mathcal{V}$, which computes reliability information for the code symbols based on the received symbol, e.g., the probability $\Pr\{B = 0|Y\}$ $(= 1 - \Pr\{B = 1|Y\})$, or equivalent metrics, which are deinterleaved and fed to the decoder.

The capacity of the BICM-ASK transmission is given as the sum of the bit level capacities $I(B_\mu; Y)$ [1], [16], [17], i.e., in bit per PAM channel use we have

$$C_{\text{BICM}}(E_{\text{s}}/N_0) = \sum_{\mu=1}^{\log_2(M)} I(B_\mu; Y) \ . \qquad (5)$$

For brevity, the dependence on the applied binary labeling rule and E_s/N_0 is omitted where possible.

For comparison, we also consider the case when the communication channel represents BICM-ASK with a hard-decision demapper, i.e., $\mathcal{V}$ decides for the most likely transmitted symbol $\hat{a}$ and hence the corresponding bit label $\hat{\boldsymbol{b}}$. The capacity per use of the PAM channel calculates to [13]

$$C_{\mathsf{hard}} = \log_2(M) \cdot (1 - \mathrm{e}_2(\mathrm{BER}_{\mathsf{unc}})) \tag{6}$$

where $\mathrm{BER}_{\mathsf{unc}} = \mathrm{E}\{\Pr\{X \neq V|Y\}\}$ denotes the average bit error rate of the communication channel (which is easily calculated for M-ASK with given labeling rule).

B. Estimating the Capacity of the End-to-End Channel

Due to the interleaver in the BICM setup, the ideal BICM communication channel can be seen as a binary memoryless channel with average capacity $C_{\mathsf{ch}} = C_{\mathsf{BICM}}/\log_2(M)$ per communication channel use [7]. Thus, the capacity per use of the coding channel from the input of the encoder to the output of the (hard-output) decoder can be expressed as

$$C^{\mathsf{IPC}}_{\mathsf{code}}(E_s/N_0) = \mathrm{IPC}_{\mathsf{code}}\left(C_{\mathsf{ch}}(E_s/N_0)\right) \tag{7}$$

where $\mathrm{IPC}_{\mathsf{code}}$ is the IPC of the applied encoder/decoder-pair, as defined in (1). Multiplying the estimated capacity with the overall rate of the modulation R translates it into units of bit per use of the PAM channel and enables comparison to the BICM capacity of ASK, cf. (5).

Exemplarily, Fig. 3 demonstrates this procedure for Gray-labeled 4-ASK and the rate-$1/2$ convolutional code shown in Fig. 2. For each ratio E_s/N_0 (here 3 dB) the average BICM capacity is processed through the IPC of the convolutional code; the output is the estimated capacity of the coded channel per binary information symbol. For comparison, the IPC and the resulting capacity of an ideal coding scheme are shown.

C. Estimating the Bit Error Rate of the End-to-End Channel

Based on the estimated capacity of the coding channel, $C^{\mathsf{IPC}}_{\mathsf{code}}$, we employ two well-known bounds [10] to obtain an estimate for the bit error rate of the coding channel, $\mathrm{BER}_{\mathsf{code}}$. A lower bound is given by assuming that the errors at the decoder output are a memoryless process, such that the overall channel represents a BSC. The second bound is an upper bound and is based on the contrary assumption, i.e., the errors are concentrated in a single burst [10]. In summary, the BER of the overall channel of the BICM setup is bounded by

$$\mathrm{e}_2^{-1}\left(1 - C^{\mathsf{IPC}}_{\mathsf{code}}\right) \leq \mathrm{BER}^{\mathsf{IPC}}_{\mathsf{code}} \leq \left(1 - C^{\mathsf{IPC}}_{\mathsf{code}}\right)/2\,. \tag{8}$$

However, it has to be emphasized that the bounds are tight only in case of ideal interleaving. The performance can be better than the lower bound, if the interleaver size is finite, and thus the errors follow a process with arbitrary-structured memory. This argument is confirmed by the fact that memory increases capacity [13], [18].

Finally, we note that in case of hard-input decoding, i.e., using (6) in (7), the lower bound is equivalent to directly using BER processing characteristics.

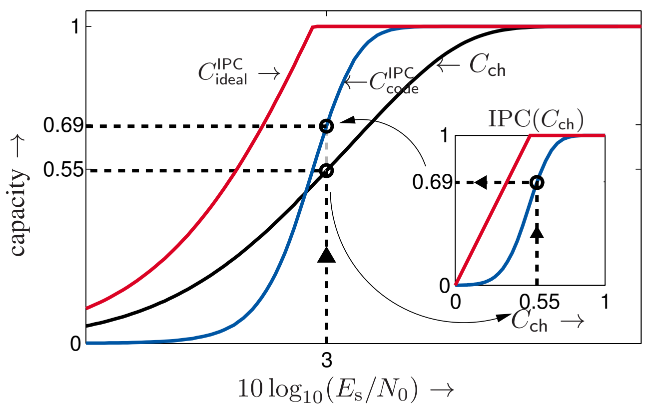

Fig. 3. Illustration of IPC-based estimation of the capacity of the end-to-end coded channel for BICM using Gray-labeled 4-ASK with the convolutional code and soft-input Viterbi decoding shown in Fig. 2 (left).

IV. Numerical Results

Fig. 4 depicts the estimated capacities of the coded channel in bit per PAM channel use, i.e., $C^{\mathsf{IPC}}_{\mathsf{code}} \cdot R$, for different coding schemes with 4-, 8-, and 16-ASK for an AWGN channel ($h = 1$). We spend one bit redundancy per ASK symbol ($R_c = 1/2$ for 4-, $2/3$ for 8-, and $3/4$ for 16-ASK) and consider the following coding schemes: 1) on the left hand side of Fig. 4: maximum-free-distance convolutional codes with constraint length $\nu = 4$ (interleaver size 10.000 symbols) in combination with soft-input Viterbi decoding (blue), 2) on the right hand side of Fig. 4: the LDPC code from WLAN-802.11n (codeword length and interleaver size $n = 648$) in combination with soft-input BP decoding (blue), and 3) in both plots the ideal coding scheme (red). For $R_c = 1/2$, the IPCs of the coding schemes are shown in Fig. 2. These results are compared to the capacity of uncoded transmission (gray) obtained from (6), scaled by a factor of R_c to enable comparison. It can be observed, that the capacity of an ideal coding scheme follows the BICM capacity up to the point where $C_{\mathsf{code}}R < R$ and then delivers the maximum possible mutual information of R bit per ASK symbol. The capacity of the convolutional coding schemes with soft-input decoding is superior to uncoded transmission at about 1 dB (10 dB, 17 dB) for 4-(8-, 16-)ASK; note that using this method with hard-input decoding estimates a loss of approx. 2 dB (not shown). The systematically encoded LDPC coding scheme always achieves better performance than uncoded transmission.

The $\mathrm{BER}_{\mathsf{code}}$-estimates obtained from the bounds given in (8) are shown in Fig. 5 for the setting considered above (solid/dashed: lower/upper bound, left/right hand side of (8)). For the ideal coding scheme, only the lower bound is given, which coincides with the rate-distortion-capacity bound. For comparison simulation results of BICM-ASK transmission are also shown (markers). Especially the lower bound agrees well with the simulation results and thus delivers quite accurate estimates for the BER performance of the BICM system Noteworthy, the results are accurate in the entire SNR regime, different from approaches based on the union bound, which are often useless at low SNR [5], [6]. Of course, relative

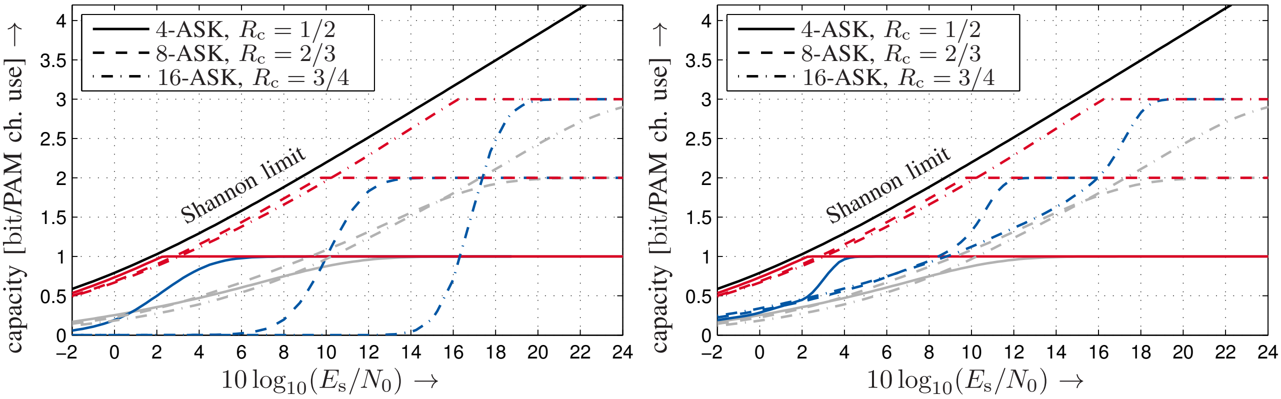

Fig. 4. Capacity estimates for Gray-labeled BICM-ASK (blue) using convolutional codes ($\nu = 4$) with soft-input Viterbi decoding (left) and LDPC codes with soft-input belief-propagation decoding (right). For comparison: Ideal coding scheme (red), uncoded transmission (gray, scaled by R_c).

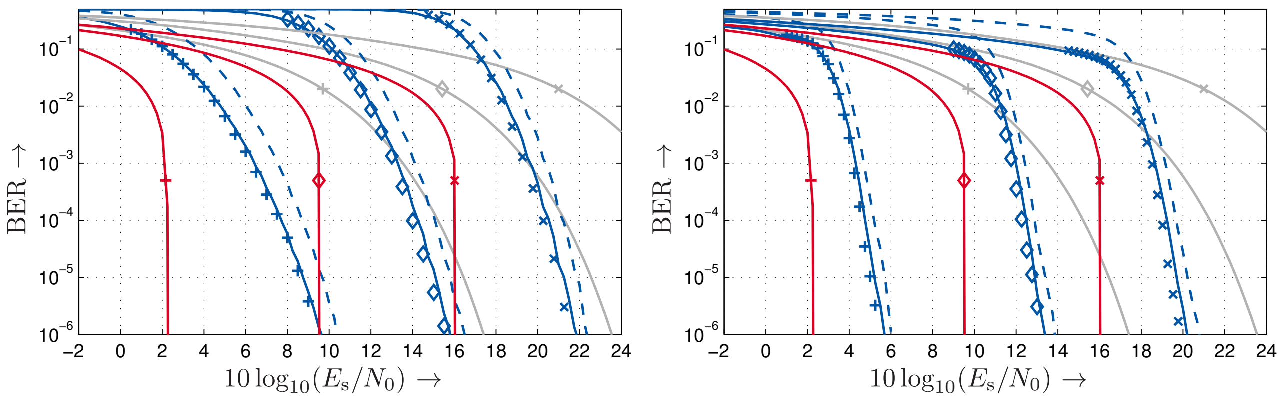

Fig. 5. BER estimates (solid/dashed: lower/upper bound) for Gray-labeled BICM-ASK (blue, left-to-right: 4- (+), 8- (◊), 16-ASK (x)) using convolutional codes ($\nu = 4$) with soft-input Viterbi decoding (left) and LDPC codes with soft-input belief-propagation decoding (right). For comparison: Simulation results of BICM-ASK with corresponding coding scheme (markers), ideal coding scheme/rate-distortion capacity bound (red), uncoded transmission (gray).

statements valid for the capacity translate into the same statements on the BER performance, since both bounds are monotonic functions. E.g., the use of convolutional codes is favorable over uncoded transmission for E_s/N_0 larger than 1 dB, 10 dB, and 17 dB for 4-, 8-, and 16-ASK, respectively.

For sake of completeness, results for 4-ASK subject to log-normal fading (parameterized such that $20\log_{10}(|H|)$ is independent, identical Gaussian distributed with variance 6 dB) are provided in Fig. 6. The BICM capacity has been averaged over the fading distribution prior to using IPCs.

Translating the capacity of the coding channel as a function of E_s/N_0 into a capacity-curve plotted vs. E_b/N_0, where $E_b = E_s/C_{\mathsf{code}}^{\mathsf{IPC}}$ is the energy per information bit, one obtains estimates for the power efficiency of the coded system. Fig. 7 depicts these curves for ASK transmission with Gray (left) and set-partition labeling (right) for the coding schemes considered above. Note that BICM-ASK with an ideal coding scheme is not wideband optimal with Gray-labeling, whereas it is first-order-optimal with set-partition labeling [19], [20]; for the non-ideal coding schemes, neither Gray, nor set-partition labeling are wideband optimal. In case of nonsystematically encoded convolutional codes, the minimum ratio E_b/N_0 is attained at non-zero rate (indicated with a marker). These points indicate an optimum operating point for an additional outer coding scheme.Selecting the outer code rate accordingly, the concatenated coding scheme operates with highest power-efficiency (basically, this is an explanation for the excellent performance of the concatenation of an inner rate-1/2 convolutional code and an outer high-rate Reed-Solomon code, e.g., in the NASA standard [21]). If set-partition labeling is applied, the optimum outer rate is slightly larger compared to Gray-labeling. For the systematically encoded LDPC codes this optimum is attained at zero rate. However, for 4-ASK,

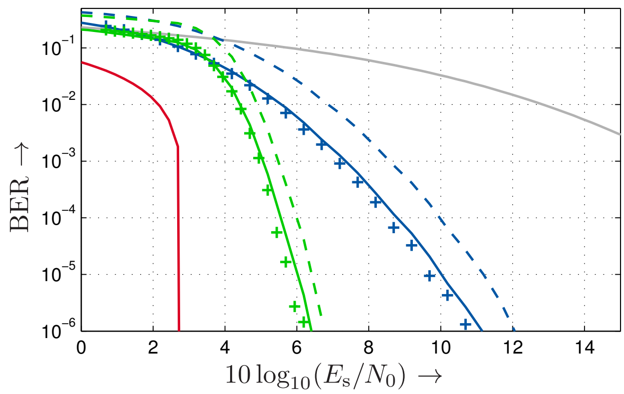

Fig. 6. BER estimates compared to simulation results for 4-ASK using a convolutional code ($R_c = 1/2$, $\nu = 4$) with soft-input Viterbi-decoding (blue), an LDPC code ($R_c = 1/2$, codeword length $n = 648$) with BP decoding (green), and uncoded transmission (gray) subject to log-normal fading.

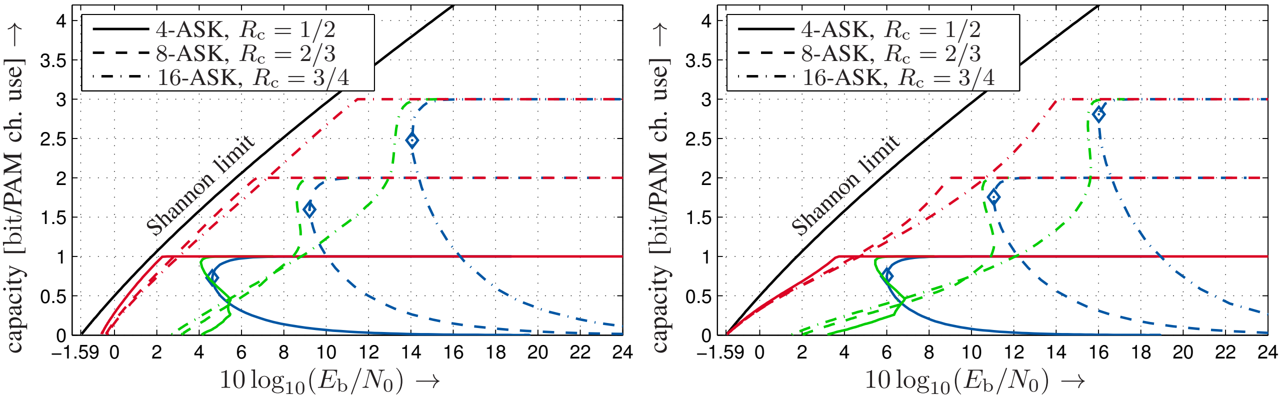

Fig. 7. Capacity estimates for BICM-ASK (left: Gray-labeling, right: set-partition labeling) using convolutional codes ($\nu = 4$) with soft-input Viterbi decoding (blue) and LDPC codes with soft-input BP decoding (green) plotted vs. E_b/N_0. For comparison: Ideal coding scheme (red).

rates close to one are favorable, in particular compared to medium rates, where the minimum achievable E_b/N_0 is larger.

V. Discussion and Conclusions

The intention of the proposed method is to provide good-matching estimates for the end-to-end performance of BICM transmission using a given modulation format (i.e., constellation, labeling rule, and channel model), and coding scheme (i.e., channel code, encoder, and corresponding decoder). To this end, we consider IPCs as a powerful tool for the analysis. The main benefit of the proposed method is the fact, that the analysis of the channel coding scheme and the analysis of BICM for higher-order modulation schemes can be fully decoupled. It is thus sufficient to evaluate two independent building blocks of BICM, the BICM capacity of the modulation scheme C_{ch} and the IPC of the coding scheme $\mathrm{IPC}_{\mathsf{code}}$. Modeling the end-to-end channel from encoder input to decoder output as a BSC, a very good BER estimate is given by combining these two modules according to

$$\mathrm{BER}_{\mathsf{code}}(E_s/N_0) = \mathrm{e}_2^{-1}\left(1 - \mathrm{IPC}_{\mathsf{code}}\left(C_{\mathsf{ch}}(E_s/N_0)\right)\right) . \quad (9)$$

Consequently, the vast amount of research on channel coding for the binary-input AWGN channel can directly be translated to analyze BICM of higher-order modulation. Especially for complex BICM setups, which may include fading scenarios, as well as different kinds of detectors, the proposed method leads to significant reduction in computational complexity, since assessing the coded system performance can be decoupled from simulation of the uncoded transmission chain.

The drawbacks of the proposed method are its non-closed-form results, the required accuracy for computing the IPC of the coding scheme and the BICM capacity (both directly related to the region of interest for the BER), as well as the fact that it relies on an ideal interleaver in the BICM setup. Optimizing of BICM using different kinds of interleavers (optimized or none at all), as, e.g., performed in [22], can not be covered. However, if BICM is applied in fading scenarios, as commonly envisioned, this drawback becomes irrelevant.

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