

Signal Combining for Relay Transmission with Rateless Codes

Azad Ravanshid, Lutz Lampe*, and Johannes Huber

Lehrstuhl für Informationsübertragung, Universität Erlangen–Nürnberg, Germany

*Department of Electrical and Computer Engineering, University of British Columbia, Canada.

Email: {azad@LNT.de, Lampe@ece.ubc.ca, huber@LNT.de}

Abstract—The invention of practical rateless codes in the form of Luby transform and Raptor codes has facilitated the implementation of decode-and-forward relaying schemes which permit the relay to autonomously switch between listening and collaboration phase. Considering the classical three-node relay network employing such a flexible decode-and-forward mechanism, in this paper we investigate signal combining strategies for the destination node. In particular, we compare information and energy combining considered previously in the literature and introduce a new, so-called mixed combining scheme, which is a hybrid of the two former strategies. Assuming general finite-size signal constellations we show that mixed combining is advantageous over the pure combining schemes in terms of achievable rate given the same total transmit energy. A comparison of the associated constellation-constrained capacities with simulated rates achieved for relay transmission with moderate-length Raptor codes underscores (i) the relevance of the capacity-based analysis and (ii) the suitability of rateless codes for relay transmission.

I. INTRODUCTION

In the wake of advances in multiple-antenna transmission and with the ongoing evolution towards network communication theory, the study of relay assisted communication schemes [1] has experienced a recent revival (e.g. [2]–[6]). The fundamental building block of such collaborative transmission schemes consists of a source, a relay, and a destination node, and a number of protocols have been developed in the literature by which the relay is enabled to assist the source-to-destination communication link. One very popular protocol is the decode-and-forward (DF) protocol, in which the transmission interval is divided into a listening phase, during which the relay only receives, and a collaboration phase, during which the relay transmits the successfully decoded source message. The duration of the listening phase can be predetermined [3], [4] or it can be adapted to the actual quality of the source-to-relay channel [5]. In case of the latter, and in the absence of channel state information (CSI) at the sender, the relay would decide on its own when to switch from listening to collaborating.

Recently, the application of rateless codes, in particular Luby transform (LT) and Raptor codes [7], [8], has been advocated for in [9], [10] in order to accomplish DF transmission with a flexible duration of the listening phase. While [9] considered three-node relay channel with non-orthogonal source-to-destination and relay-to-destination channels, relay-

ing with multiple relay nodes and orthogonal subchannels were studied in [10]. The assumption of orthogonal channels enables the use of two different signal combining schemes at the destination, which are referred to as energy combining (EC) and information combining (IC) in [10].

In this paper, we are concerned with the signal combining at the destination node for DF relaying based on rateless code in the three-node relay channel. We formulate EC and IC in a common framework and introduce a new combining scheme, which is a hybrid of EC and IC and which we refer to as mixed combining (MC). The three schemes are compared based on the achievable rate for the associated relay channel, for which, different from [10], we consider transmission with finite-size constellations. We suggest an adaptive optimization of MC, which requires only a one-bit feedback from the destination to the source. Such a feedback requirement is similar to the acknowledgment sent from the destination to source and relay to terminate the transmission with rateless codes. Numerical results for achievable rates illustrate the advantage of the new MC over IC and EC, and simulation results using medium-length Raptor codes confirm that the predicted gains can be realized with practical coding schemes.

The remainder of this paper is organized as follows. In Section II we introduce the relay-system setup and the combining schemes. In Section III, we derive the achievable rates for the combining schemes and optimize MC. Numerical and simulation results are presented in Section IV and conclusions are given in Section V.

II. RELAY TRANSMISSION AND SIGNAL COMBINING

In this section, we first briefly introduce the considered relay transmission system and then describe the signal combining schemes applied at the destination node.

A. System Setup

We consider the three-node wireless relay network shown in Figure 1 consisting of a source node S , a relay node R , and a destination node D , in which the source wishes to communicate a message to the destination. For simplicity, we assume that all nodes employ a single antenna. The channels between different nodes are modeled as frequency-flat fading additive white Gaussian noise (AWGN) channels which remain constant during the transmission of a least one

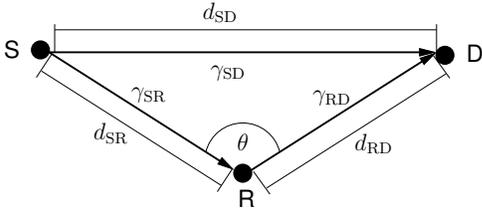


Fig. 1. Three-node relay channel model with distances d_{XY} between nodes X and Y . The node-to-node channels are frequency flat fading AWGN channels with instantaneous signal-to-noise power ratios γ_{XY} between nodes X and Y .

message. The instantaneous signal-to-noise power ratio (SNR) for the source-to-destination (SD), source-to-relay (SR), and relay-to-destination (RD) is denoted by γ_{SD} , γ_{SR} , and γ_{RD} , respectively. The relay node operates in the half-duplex mode and applies the DF paradigm to assist the source. That is, the transmission process is organized in two phases. In the first phase (listening phase), the source broadcasts its message, and destination and relay node receive. Both receiving nodes try to decode the source message. In the case that the relay successfully decodes before the destination, the second phase (collaboration phase) starts where both source and relay transmit the message and the destination continues receiving. As in [9], [10], we assume the application of rateless codes to protect the message against channel errors together with an error detection mechanism (e.g. cyclic-redundancy check code) to terminate decoding, and that CSI is only available at the receiver sides of the respective link. The use of rateless codes enables the practically seamless adaptation of the duration of the two communication phases to the SR channel quality without the need for CSI at the source node [9], [10]. Furthermore, source and relay node transmit with the same average power during the respective transmission phases [5], [9], [10].

Finally, we assume that orthogonal resources are available for the SD and RD channels. For example, this could be accomplished by spread-spectrum communication with orthogonal spreading sequences assigned to source and relay, which would also allow the destination to identify the respective senders. Such a setup was considered in [10] for retransmission with multiple relays. We extend the scheme from [10] in that we allow the relay to re-use the spreading sequence allocated to the source. That is, the relay is enabled to transmit using both the source and its own spreading sequence. This is a reasonable extension as (i) both resources are occupied during the collaboration phase anyhow, i.e., this extension does not drain any additional network resources, and (ii) the self-adaptivity of the transmission scheme is maintained as no assumptions about the successful decoding at the relay are made and no feedback from relay to source node is necessary. Note that this would not be the case if also the source was allowed to change its mode of transmission.

B. Signal Combining

Let us define the length- k source message vector \mathbf{m} . Furthermore, let us denote the two orthogonal signals resources (e.g. spreading sequences) establishing orthogonal SD and RD channels by \mathbf{s}_1 and \mathbf{s}_2 . In the listening phase, the source transmits the length- n_L code vector $\mathbf{c}_L = \mathbf{m}\mathbf{G}_L$ using \mathbf{s}_1 , where \mathbf{G}_L is the $k \times n_L$ part of the generator matrix of the rateless code that is used until the relay has decoded \mathbf{m} .

We now consider the collaboration phase only, i.e., the relay successfully decoded the source message before the destination. Let us define the length $(n - n_L)$ code vectors

$$\mathbf{c}_C^1 = \mathbf{m}\mathbf{G}_C^1, \quad (1)$$

$$\mathbf{c}_C^2 = \mathbf{m}\mathbf{G}_C^2, \quad (2)$$

where $n > n_L$ is the number of coded bits received by the destination until successful decoding, and \mathbf{G}_C^1 and \mathbf{G}_C^2 are $k \times (n - n_L)$ generator matrices of the rateless codes during the collaboration phase. While the source always transmits the vector \mathbf{c}_C^1 using \mathbf{s}_1 , the availability of two orthogonal channels enables different signal combining schemes for the collaboration phase.

- 1) *Energy combining (EC)*: The relay node transmits \mathbf{c}_C^1 using \mathbf{s}_2 , and the SD and RD received signals are maximal-ratio (i.e., energy) combined at the destination. Decoding is done based on the total generator matrix $[\mathbf{G}_L \ \mathbf{G}_C^1]$.
- 2) *Information combining (IC)*: The relay node transmits \mathbf{c}_C^2 using \mathbf{s}_2 , and the SD and RD received signals are jointly decoded (i.e., information combined) based on the total generator matrix $[\mathbf{G}_L \ \mathbf{G}_C^1 \ \mathbf{G}_C^2]$ at the destination.
- 3) *Mixed combining (MC)*: The relay node transmits \mathbf{c}_C^1 using \mathbf{s}_1 with a fraction r , $r \in [0, 1]$, of its transmit power and \mathbf{c}_C^2 using \mathbf{s}_2 with fraction $1 - r$ of its transmit power. Thus, the total transmit power at the relay is kept constant. The SD and RD signals that are transmitted with \mathbf{s}_1 are energy combined, while the RD signal transmitted with \mathbf{s}_2 is used for joint decoding based on $[\mathbf{G}_L \ \mathbf{G}_C^1 \ \mathbf{G}_C^2]$. Energy combining of \mathbf{s}_1 -signals is accomplished through the use of Alamouti's space-time block code (STBC) [11]. Note that the source node does not alter its transmission mode when entering the collaboration phase.

Energy and information combining have been investigated for transmission with Gaussian signals in [10]. Mixed combining is a generalization of these pure combining schemes. Note that regardless of the combining scheme, the relay sends a parity vector of length $(n - n_L)$, i.e., \mathbf{c}_C^1 or \mathbf{c}_C^2 , that is different from the vector of n_L parity symbols transmitted during the listening phase. Hence, the considered rateless coded relay scheme always perform information combining across the two phases, which is also referred to as code combining [12] and coded cooperation [3] in the context of relaying using fixed-rate codes.

III. ANALYSIS OF COMBINING SCHEMES

In this section, we compare the three combining schemes in terms of achievable rates. To this end, we assume that the rateless codes are capacity-approaching for the SD, SR, and RD channels, where, throughout this paper and in slight abuse of denotation, capacity is defined as mutual information for a given signal constellation, which is also known as constellation-constrained capacity [13, Section 3.5].

A. Achievable Rate for Relaying

Let us denote the capacities of the SR and SD channel by C_{SR} and C_{SD} respectively, and the capacity of the joint SD and RD channel during the collaboration phase by C_{Comb} . We have that $C_{\text{SR}} = C(\gamma_{\text{SR}})$ and $C_{\text{SD}} = C(\gamma_{\text{SD}})$, where $C(\gamma)$ is the constellation-constrained capacity given the, e.g., quadrature-amplitude modulation (QAM) or phase-shift keying (PSK) constellation at SNR γ . The capacity C_{Comb} depends on the applied combining scheme. The capacity unit is bit per channel use. Then, following the arguments from [5], [9], arbitrarily low error rate is achievable with a code of rate $R = k/n = R_{\text{max}} - \delta$ for any $\delta > 0$, where R_{max} is given by

$$R_{\text{max}} = \begin{cases} \frac{C_{\text{Comb}} C_{\text{SR}}}{C_{\text{SR}} - C_{\text{SD}} + C_{\text{Comb}}}, & \text{if } C_{\text{SR}} > C_{\text{SD}} \\ C_{\text{SD}}, & \text{else.} \end{cases} \quad (3)$$

For the case that $C_{\text{SR}} > C_{\text{SD}}$ and thus a collaboration phase is reached, (3) intuitively follows from assuming equality in the two constraints

$$nR < n_L C_{\text{SD}} + (n - n_L) C_{\text{Comb}} \quad (4)$$

and

$$nR < n_L C_{\text{SR}} \quad (5)$$

for $R = R_{\text{max}}$.

R_{max} is an important measure as, in principal, the application of rateless codes provides the possibility of self-adaptation of the actual code rate arbitrarily close to R_{max} . For brevity, we refer to R_{max} from (3) as the achievable rate in the following.

B. Comparison of Combining Schemes

Since EC benefits from SNR combining and IC uses parallel channels, the capacities C_{Comb} for the three combining schemes from Section II-B are given by

$$C_{\text{Comb}}^{\text{EC}} = C(\gamma_{\text{SD}} + \gamma_{\text{RD}}) \quad (6)$$

$$C_{\text{Comb}}^{\text{IC}} = C(\gamma_{\text{SD}}) + C(\gamma_{\text{RD}}) \quad (7)$$

$$C_{\text{Comb}}^{\text{MC}} = C(\gamma_{\text{SD}} + r\gamma_{\text{RD}}) + C((1-r)\gamma_{\text{RD}}). \quad (8)$$

Furthermore, since R_{max} in (3) is strictly monotonically increasing in C_{Comb} , we wish to maximize C_{Comb} for given γ_{SD} and γ_{RD} . While for general constellations, e.g., QAM and PSK, the constellation-constrained capacity $C(\gamma)$ cannot be expressed in closed form [13, Section 3.5], we know from the relation between mutual information and the minimum mean-square error (MMSE) when estimating the transmitted signal point from the received signal and the monotonicity of the

MMSE with respect to the SNR γ [14] that $C(\gamma)$ is a concave function of γ . Hence, for SNRs $a, b, c \geq 0$, we have

$$C(a+b) + C(c) \geq C(c+b) + C(a) \quad (9)$$

if and only if $(c \geq a \vee b = 0)$,

This immediately leads to the following result.

Lemma 1: For general finite-size constellations, the capacities of the combining channels are related by

$$C_{\text{Comb}}^{\text{EC}} \leq \left\{ \begin{array}{l} C_{\text{Comb}}^{\text{IC}} \\ C_{\text{Comb}}^{\text{MC}} \end{array} \right\} \text{ for all pairs } (\gamma_{\text{SD}}, \gamma_{\text{RD}}), \quad (10)$$

$$C_{\text{Comb}}^{\text{IC}} \leq C_{\text{Comb}}^{\text{MC}} \quad (11)$$

if and only if $[(1-r)\gamma_{\text{RD}} \geq \gamma_{\text{SD}} \vee r = 0]$.

From this Lemma we conclude that mixed combining can be superior to information combining if and only if $\gamma_{\text{RD}} > \gamma_{\text{SD}}$. More specifically, any choice of $r \in (0, 1 - \gamma_{\text{SD}}/\gamma_{\text{RD}})$ will lead to $C_{\text{Comb}}^{\text{MC}} > C_{\text{Comb}}^{\text{IC}}$.

C. Optimization of Mixed Combining

We are now interested in the power ratio $r = r_{\text{opt}}$ that maximizes the capacity $C_{\text{Comb}}^{\text{MC}}$ in (8). Using again that $C(\gamma)$ is concave, we obtain

$$r_{\text{opt}} = \max\{0, (1 - \gamma_{\text{SD}}/\gamma_{\text{RD}})/2\}. \quad (12)$$

If $\gamma_{\text{RD}} < \gamma_{\text{SD}}$, then $r = 0$ and MC becomes identical to IC. Otherwise, the effect of MC is to equalize the effective SNRs $\gamma_{\text{SD}} + r\gamma_{\text{RD}}$ and $(1-r)\gamma_{\text{RD}}$ as both become $(\gamma_{\text{SD}} + \gamma_{\text{RD}})/2$ for $r = r_{\text{opt}}$. Furthermore, if $\gamma_{\text{SD}} = 0$, then $r = r_{\text{opt}} = 0.5$ and the relay fully re-uses the orthogonal channel from the SD link.

Since (12) can only be evaluated at the destination node, where the ratio $\gamma_{\text{SD}}/\gamma_{\text{RD}}$ is known, a low-rate feedback channel from the destination which conveys a quantized version of r_{opt} to the relay would be required. In particular, for the numerical results in the next section, we apply a 1-bit feedback from the destination to adjust the parameter r , where the size-2 codebook is designed such that the mean-square error with respect to r_{opt} is minimized. This kind of feedback is alike the acknowledgment signal sent by the destination to terminate the transmission after successful decoding [9], [10], [15].

IV. NUMERICAL RESULTS

In this section, we present illustrative numerical results to compare the three combining schemes. We assume binary phase-shift keying (BPSK) modulation at the source and the relay. The SR, SD, and RD channels are modeled as flat Rayleigh fading with instantaneous SNRs $\gamma_{\text{SR}} = G_{\text{S}}\gamma_{\text{SD}}$ and $\gamma_{\text{RD}} = G_{\text{D}}\gamma_{\text{SD}}$, where G_{S} and G_{D} denote the SNR gains of the SR and RD channel with respect to the SD channel. For this purpose, we adopt the log-distance path-loss model presented in [16] for two-phase relay transmission, so that

$$G_{\text{S}} = [1 + \xi^2 - 2\xi \cos(\theta)]^{\alpha/2}, \quad (13)$$

$$G_{\text{D}} = G_{\text{S}}/\xi^{\alpha}, \quad (14)$$

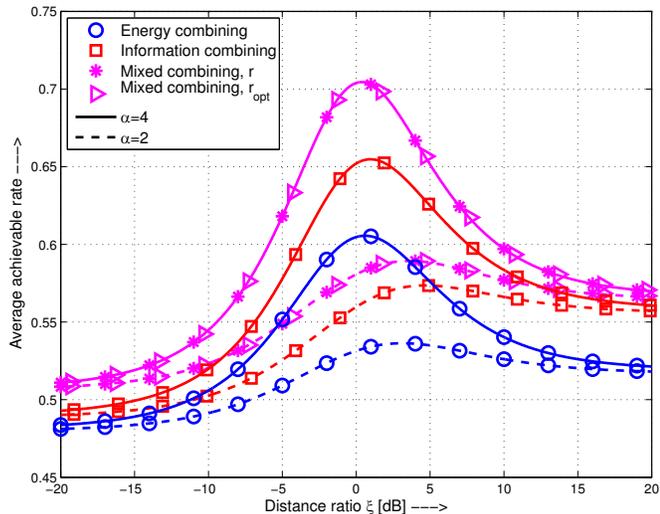


Fig. 2. Average achievable rate \bar{R}_{\max} with R_{\max} from (3) as function of the ratio ξ of RD-distance to SR-distance and for path-loss exponent $\alpha = 4$ and $\alpha = 2$. BPSK transmission and different combining schemes.

where

$$\xi = \frac{d_{RD}}{d_{SR}} \quad (15)$$

is the ratio of RD-distance d_{RD} to SR-distance d_{SR} , θ is the angle of the line connecting source, relay, and destination, and α denotes the path-loss exponent (see Figure 1). For simplicity (cf. also [16]), we adopt $\theta = \pi$ for the following results, which means that the relay is located on the line between source and destination and thus $d_{SD} = d_{SR} + d_{RD}$.

First, we evaluate the achievable rate R_{\max} from (3) for the different combining schemes. Figure 2 shows the average rate $\bar{R}_{\max} = \mathcal{E}\{R_{\max}\}$, where averaging with respect to channel fading is done by means of Monte Carlo integration, as function of the distance ratio ξ assuming path-loss exponents $\alpha = 2$ and $\alpha = 4$, respectively. The average SNR $\bar{\gamma}_{SD} = \mathcal{E}\{\gamma_{SD}\} = 0$ dB is adjusted for the SD channel. Since the relay moves closer to the source as ξ increases, the percentage of collaboration increases with growing ξ . We observe that MC consistently achieves the best performance, i.e., highest rate, followed by IC. This has been expected from the analysis in Section III-B. The gain of MC over IC is a function of both ξ and α , and fairly notable for not-too-large ξ , where often $\gamma_{RD} > \gamma_{SD}$. It disappears for $\xi \rightarrow \infty$, because MC converges towards IC as $\gamma_{RD} \rightarrow \gamma_{SD}$. The gain for pure IC over EC is monotonically increasing with the length of the collaboration phase. Note that the total transmit energy is independent of the combining scheme, since the duration of the collaboration phase only depends on the SR channel.

For the following results, we assume $G_S = 15$ dB and $G_D = 10$ dB as an exemplary scenario, which corresponds to $\alpha \approx 4$ and $\xi \approx 4/3$. Figure 3 shows the average achievable rate \bar{R}_{\max} (solid lines) as a function of the average SD channel SNR $\bar{\gamma}_{SD}$ for the three combining schemes. The consistent advantage of MC over IC and EC is confirmed,

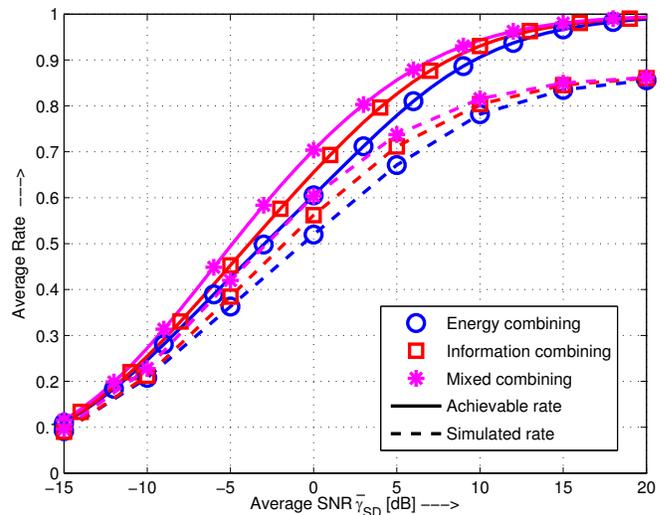


Fig. 3. Average achievable rate \bar{R}_{\max} with R_{\max} from (3) (solid lines) and simulated rate (dashed lines) as function of the average SD SNR $\bar{\gamma}_{SD}$. BPSK transmission. $G_S = 15$ dB and $G_D = 10$ dB

and the gains in terms of average SNR can be on the order of 3 dB compared to EC and 1 dB with respect to IC. Also included in Figure 3 are the averages of simulated rates using Raptor codes (dashed lines). The particular Raptor code consists of a rate-0.95 regular low-density parity-check (LDPC) outer code and an LT inner code generated using the degree distribution from [8, Table I]. The input-word length is chosen as $k = 950$ bits, which means that we apply fairly moderate-length codes. It can be seen from Figure 3 that the relative simulated performance among the combining schemes is as predicted from the capacity results. Furthermore, a closer comparison of simulated and achievable rates shows that the Raptor codes require an “overhead factor” [8], [10] of about $(1 + \epsilon) = 1.15$ throughout the entire SNR range, which is quite remarkable considering the length of the codes. We note that this overhead is practically independent of the combining scheme, which indicates that Raptor codes are able to make use of the available mutual information regardless of the type of combining channel through which it is provided.

The performance improvement due to MC in terms of rate-outage probability, i.e., the probability that a certain rate is not supported by the relay channel, can be inferred from Figure 4, where the numerically evaluated cumulative density functions (CDFs) for R_{\max} (lines) with different combining schemes are shown for $\bar{\gamma}_{SD} = 0$ dB. Notable gains of MC over IC (and thus also EC) are observed for outage rates between 10 % and 80 %. Also included in this figure are the CDFs for the simulated rates using Raptor codes (markers), where the measured rates are multiplied with the factor $(1 + \epsilon) = 1.15$. The perfect match of CDF for the shifted simulated rate and the CDF for the achievable rate confirms that Raptor codes are a practical way to harvest the capacity-gains due to improved signal combining for DF relaying.

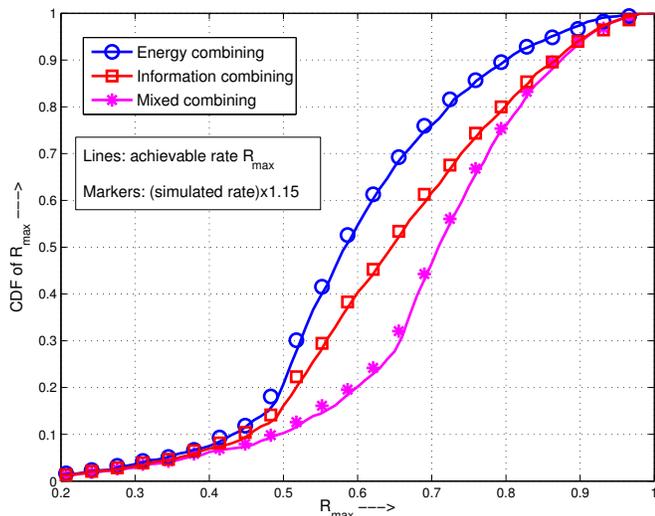


Fig. 4. CDF of achievable rate R_{\max} from (3) (lines) and CDF for the simulated rate multiplied with a factor 1.15 (markers). $\bar{\gamma}_{SD} = 0$ dB, $G_S = 15$ dB, and $G_D = 10$ dB.

Finally, Figure 5 presents a 3-dimensional scatter plot of the combining capacities C_{Comb} from (6)-(8) for 2,000 random channel realizations. It can be seen from the 2-dimensional projection of the points onto the $(C_{\text{Comb}}^{\text{MC}}, C_{\text{Comb}}^{\text{IC}})$ -plane that MC always achieves a higher or the same rate as IC. While this is clear for optimally adjusted r , we note that the results were obtained with 1-bit quantization of r . Likewise, the projection onto the $(C_{\text{Comb}}^{\text{IC}}, C_{\text{Comb}}^{\text{EC}})$ -plane confirms the superiority of IC over EC. Hence, MC is the scheme of choice not only on average, but for every realization of the relay channel.

V. CONCLUSIONS

In this paper, we have investigated different signal combining schemes for decode-and-forward relay transmission with variable duration of the listening phase. In particular, we have assumed the application of rateless codes such that the relay autonomously switches from listening to collaboration phase. As in the related literature, we have assumed the availability of orthogonal channels for source-to-destination and relay-to-destination communication, for which information combining is possible at the destination node. We have proposed a new hybrid combining scheme, and we have established that this new combining scheme outperforms known energy and information combining for general finite-size signal constellations. The results from the capacity-based analysis have been confirmed by simulations for relay transmission with moderate-length Raptor codes, which require a redundancy overhead of about 15 % compared to capacity.

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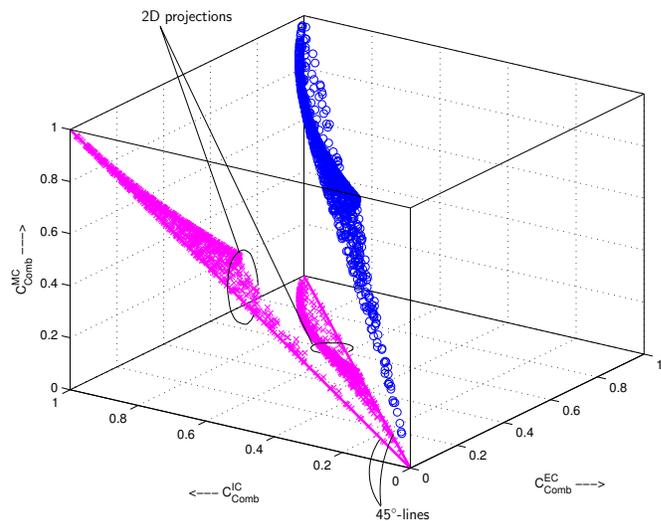


Fig. 5. Scatter plot of the combining capacities C_{Comb} from (6)-(8) for 2,000 channel realizations. $\bar{\gamma}_{SD} = 0$ dB, $G_S = 15$ dB, and $G_D = 10$ dB.

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