# A New Stopping Criterion for the Sphere Decoder in UWB Impulse–Radio Multiple–Symbol Differential Detection


Andreas Schenk*, Robert F.H. Fischer*, and Lutz Lampe†

*Institute for Information Transmission, University Erlangen–Nuremberg, Erlangen, Germany

†Department of Electrical and Computer Engineering, University of British Columbia, Vancouver, Canada

Email: {schenk,fischer}@lnt.de, lampe@ece.ubc.ca



***Abstract*—Multiple-symbol differential detection (MSDD) is a powerful technique for power-efficient low-complexity auto-correlation receivers in ultra-wideband impulse-radio (UWB-IR) systems. Since MSDD can be represented as a tree search problem, it is efficiently implemented by the Sphere Decoder (SD) algorithm. In this paper, we introduce a novel stopping criterion for the SD, which is based on a lower bound on the packing radius of lattices and enables early termination of the search process. As a result, the application of this stopping criterion reduces the number of search steps and thus lowers the complexity of MSDD for UWB-IR. Numerical results show that the proposed stopping criterion achieves optimal MSDD performance at reduced SD complexity.**


## I. Introduction

Ultra-wideband impulse-radio (UWB-IR) is an attractive technology for short-range wireless communications. Its advantages include multiple-access capability for a large number of simultaneous users, robustness in dense multipath environments, and fine timing resolution [1]. Among several versions of UWB-IR, transmitted reference (TR) signaling in combination with auto-correlation receivers (ACR) have recently received considerable attention, as they accomplish to capture the energy of the UWB-IR received signal even in challenging multipath scenarios without the need for explicit channel estimation [2]. To overcome the major conceptual drawback of TR, the waste of transmit power and bandwidth due to the transmission of non-information bearing reference pulses, differential encoding at the transmitter and differential detection (DD) at the receiver side have been introduced for TR-based UWB-IR [2], [3]. This differential transmitted reference (DTR) transmission preserves the simplicity of the TR approach at an improved performance

The power efficiency, i.e., the required received signal-to-noise ratio (SNR) per data bit to achieve a certain error rate, of DTR can be further improved by replacing DD with multiple-symbol differential detection (MSDD) [4]. MSDD performs joint decision for a block of $L$ symbols. It has been shown in [5] that even large block-lengths $L$ become applicable by the use of the Sphere Decoder (SD) [6] to solve the underlying tree search problem.

In this paper, we present and discuss a new stopping rule for the search performed by the SD with the aim of further reducing MSDD complexity. To this end, we first review a stopping criterion for general lattice decoding based on a lower bound on the packing radius of lattices. We show that this criterion preserves the optimality of the decoder output while reducing the tree search complexity. It is thus directly applicable for a broad class of detection problems such as signal detection in multi-antenna systems [7], [8] or MSDD of differential phase-shift keying [9], [10]. Then, we transfer this approach to MSDD in UWB-IR DTR and obtain the new stopping criterion for the SD. We present simulative evidence for the complexity reduction that is achieved due to early termination of the SD, which at high SNR becomes approximately a factor of two and comes without loss in error-rate performance.

This paper is organized as follows. Section II summarizes the SD principle and introduces the use of the packing radius as a stopping criterion. In Section III this stopping criterion is adapted for MSDD in UWB-IR DTR systems. Error-rate performance and complexity of the SD with the proposed early termination are illustrated by means of numerical results in Section IV. The paper concludes with final remarks in Section V.

## II. Sphere Decoder and Early Termination

In this section, we first briefly summarize the tree search process of the SD according to the Schnorr–Euchner strategy [6]. Then, we introduce the notion of stopping criteria for the SD. Finally, the use of the packing radius as stopping criterion for the SD is presented.

### A. Sphere Decoder

As a depth-first tree search algorithm [11], the SD starts at the root of the tree and first generates a best-estimate by extending a single path always in the direction of the smallest branch metric, until the end of the tree is reached. This path and its path metric are stored as the current best path and search radius, respectively. Note that terminating the search process at this point is equivalent to decision-feedback (DF) detection, also known as Babai's nearest-plane algorithm [12]. To guarantee optimality, the SD back-traces the so far obtained best path and iteratively checks in every depth of the tree the next-best branch, extending it if its path metric up to this point is smaller than the current search radius. If a leaf of the tree is reached, it has to have the smallest path metric so far, and thus the search radius is updated and the path is


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stored as the new preliminary result. The search process stops if no path can be extended any further without exceeding the search radius, i.e., back-tracing leads back to the root. A good measure for decoding complexity, which is independent of the specific implementation of the SD, is given by counting the number of nodes examined during the tree search [13]. In this paper, we adopt this complexity measure.

### B. Stopping Criteria

The above description of the search process shows that the SD finds a preliminary result (the DF path) as fast as possible and then continuously tries to improve it. Hence, the search process can be terminated at any time, still preserving a possibly non-optimal but maybe sufficiently good solution. This motivates the following two stopping criteria for the SD which enable a trade-off between performance and complexity.

First, the search process may be terminated, if the path metric of any preliminary result is below a given threshold, or geometrically if any examined signal point lies inside a sphere of radius $R_{\text{stop}}$ centered at the received point. Note that setting the stopping radius arbitrarily large results in DF detection, as in this case decoding is terminated for the first leaf reached.

Second, to limit the maximum decoder complexity and thus the worst-case number of operations, the maximum number of considered nodes during a tree search may be limited to $C_{\max}$. Again, DF detection can be emulated with this stopping criterion, by setting $C_{\max}$ equal to the tree depth, the minimum number of considered nodes to generate a "useful" decoder output.

### C. Optimum Stopping Radius

We now present a specific threshold-based stopping criterion, which does not alter the optimality of the final output of the SD.

*1) Packing Radius:* The minimization problems solved by the SD can be described as a closest-point problem, i.e., for a given $\boldsymbol{x} \in \mathbb{R}^L$

$$\hat{\boldsymbol{\lambda}} = \underset{\boldsymbol{\lambda}\in\boldsymbol{\Lambda}}{\operatorname{argmin}} \|\boldsymbol{x} - \boldsymbol{\lambda}\|^2 \tag{1}$$

or as a shortest-vector problem

$$\hat{\boldsymbol{\lambda}} = \underset{\boldsymbol{\lambda}\in\boldsymbol{\Lambda}}{\operatorname{argmin}} \|\boldsymbol{\lambda}\|^2 \tag{2}$$

in an $L$-dimensional discrete set $\boldsymbol{\Lambda} = \{\boldsymbol{\lambda} = \boldsymbol{z}\boldsymbol{G} |\, \boldsymbol{z} \in \mathcal{Z}^L\}$ with generator matrix $\boldsymbol{G}$, index set $\mathcal{Z}$, and $\|\cdot\|$ is the Euclidean norm. Most prominent examples are the closest-lattice-point problem of lattices [6], where $\mathcal{Z} = \mathbb{Z}$, signal detection in multi-antenna systems [7], or MSDD of DPSK signals transmitted over flat fading channels [9]. Subsequently, we consider a given, arbitrary index set $\mathcal{Z}$.

The minimum squared Euclidean distance of $\boldsymbol{\Lambda}$ is

$$d^2_{\min}(\boldsymbol{\Lambda}) \overset{\text{def}}{=} \min_{\substack{\boldsymbol{\lambda}\neq\boldsymbol{\lambda}'\\ \boldsymbol{\lambda},\boldsymbol{\lambda}'\in\boldsymbol{\Lambda}}} \|\boldsymbol{\lambda}' - \boldsymbol{\lambda}\|^2. \tag{3}$$

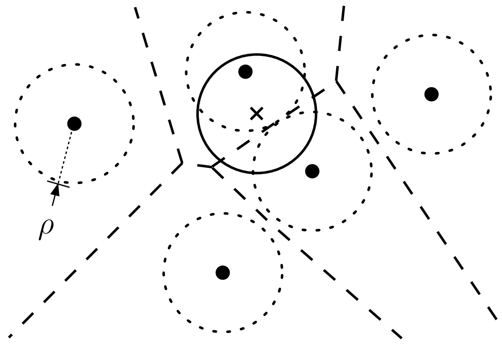


Fig. 1. Sketch of an arbitrary set $\boldsymbol{\Lambda}$ with its Voronoi regions to illustrate the use of the packing radius $\rho$ for decoding problems.

Spheres of radius equal to the *packing radius*

$$\rho \overset{\text{def}}{=} \frac{1}{2}\sqrt{d^2_{\min}(\boldsymbol{\Lambda})} \tag{4}$$

centered at each point of $\boldsymbol{\Lambda}$ are at most inspheres of the *Voronoi regions*, the set of all points in $\mathbb{R}^L$ that are closer to $\boldsymbol{\lambda}$ than to any other member of $\boldsymbol{\Lambda}$, namely

$$\mathcal{R}_V(\boldsymbol{\lambda}) \overset{\text{def}}{=} \left\{\boldsymbol{x} \in \mathbb{R}^L \big|\, \|\boldsymbol{x} - \boldsymbol{\lambda}\| \le \|\boldsymbol{x} - \boldsymbol{\lambda}'\|, \forall \boldsymbol{\lambda}' \in \boldsymbol{\Lambda}\right\} \tag{5}$$

as illustrated in Figure 1 (dotted circles). Consequently at most one point of $\boldsymbol{\Lambda}$ can be in the interior[1] of a sphere of radius $\rho$ centered at *any* point in $\mathbb{R}^L$ (cf. Figure 1 (solid circle)). Hence, any $\boldsymbol{\lambda}$ with $\|\boldsymbol{x} - \boldsymbol{\lambda}\|^2 \le \rho^2$ is the optimal solution to (1).

Considering problem (2), one can be sure that any examined $\boldsymbol{\lambda}$ is a shortest vector, if it fulfills $\|\boldsymbol{\lambda}\|^2 \le \rho^2$, as it must be the only point in a sphere of radius $\rho$ centered at the origin. Note that the converse does *not* hold. A search process for the shortest vector can hence safely be terminated without loosing optimality of the decoder output. This statement is trivial for lattices as it is only satisfied by $\boldsymbol{\lambda} = \boldsymbol{0}$.

*2) A Lower Bound on the Packing Radius:* The packing radius is usually not known for arbitrary $\boldsymbol{\Lambda}$, but a lower bound is known for lattices, which in turn may serve as a stopping radius for the SD. From the derivation of lattice reduction [14] we have: let $\boldsymbol{B}$ be the Gram–Schmidt orthogonal basis of $\boldsymbol{G}$ [15], i.e.,

$$\underbrace{\begin{bmatrix} \boldsymbol{g}_1 \\ \vdots \\ \boldsymbol{g}_L \end{bmatrix}}_{\boldsymbol{G}} = \underbrace{\begin{bmatrix} 1 & & \boldsymbol{0} \\ & \ddots & \\ m_{i,l} & & 1 \end{bmatrix}}_{\boldsymbol{M}} \cdot \underbrace{\begin{bmatrix} \boldsymbol{b}_1 \\ \vdots \\ \boldsymbol{b}_L \end{bmatrix}}_{\boldsymbol{B}} \tag{6}$$

with $\boldsymbol{b}_i\boldsymbol{b}_l^{\mathsf{T}} = 0$ for $i \neq l$. With $\boldsymbol{\delta} = \boldsymbol{z}' - \boldsymbol{z}$, $\boldsymbol{z}, \boldsymbol{z}' \in \mathcal{Z}^L$, $\boldsymbol{z}' \neq \boldsymbol{z}$, the distance of any two points of $\boldsymbol{\Lambda}$ is

$$\begin{aligned} \|\boldsymbol{\lambda}' - \boldsymbol{\lambda}\|^2 &= \|\boldsymbol{\delta}\boldsymbol{M}\boldsymbol{B}\|^2 \\ &= |\delta_L|^2\|\boldsymbol{b}_L\|^2 + \cdots \\ &\quad + |m_{L,1}\delta_L + \cdots + \delta_1|^2\, \|\boldsymbol{b}_1\|^2. \end{aligned} \tag{7}$$

[1]There can be more than one point on the boundary of the sphere, but for decoding problems these can be considered as equivalent.

Let $\boldsymbol{b}_j$ denote the basis vector with minimum length, i.e., $\|\boldsymbol{b}_j\|^2 \leq \|\boldsymbol{b}_i\|^2$, $i = 1, ..., L$, and $\boldsymbol{z}$ and $\boldsymbol{z}'$ differ only at this position $j$, from (7) we have

$$\|\boldsymbol{\lambda}' - \boldsymbol{\lambda}\|^2 \geq |\delta_j|^2 \|\boldsymbol{b}_j\|^2 + \cdots + |m_{j,1}\delta_j|^2 \|\boldsymbol{b}_1\|^2. \quad (8)$$

Since all addends are positive, truncating the summation and defining the minimum distance of the index set

$$d^2_{\min}(\mathcal{Z}) \stackrel{\text{def}}{=} \min_{\substack{z \neq z' \\ z, z' \in \mathcal{Z}}} |z' - z|^2 \quad (9)$$

similar to (3), yields a lower bound on the distance of two points of $\boldsymbol{\Lambda}$

$$\|\boldsymbol{\lambda}' - \boldsymbol{\lambda}\|^2 \geq d^2_{\min}(\mathcal{Z}) \cdot \min_{i=1,\ldots,L} \|\boldsymbol{b}_i\|^2 \quad (10)$$

and consequently the packing radius satisfies

$$\rho \geq R \stackrel{\text{def}}{=} \frac{1}{2}\sqrt{d^2_{\min}(\mathcal{Z})} \cdot \min_{i=1,\ldots,L} \|\boldsymbol{b}_i\|. \quad (11)$$

Equation (11) combines the minimum distance of the index set with properties of the orthogonal basis of the generator matrix, to obtain a lower bound on the packing radius. For lattices we have $\mathcal{Z} = \mathbb{Z}$ and $d^2_{\min}(\mathbb{Z}) = 1$ and the well-known bound from literature [14] results. Noteworthy, given a generator matrix with triangular structure, (11) simplifies to

$$R = \frac{1}{2}\sqrt{d^2_{\min}(\mathcal{Z})} \cdot \min_{i=1,\ldots,L} |g_{i,i}|. \quad (12)$$

The stated observations regarding the number of points in a sphere of radius $\rho$ as depicted in Figure 1 are clearly preserved for spheres of radius $R$ smaller than $\rho$. Hence, (12) provides an easy to compute stopping radius for the SD, which can be integrated into any problem of the form of (1) or (2) at no additional cost, yet preserving optimality of the decoder output.

## III. MSDD for UWB Impulse-Radio

We now turn to the SD for MSDD in UWB-IR DTR systems as described in [5] and adapt the previous derivations to develop a new stopping criterion.

### A. Signal Model

The transmit signal of UWB-IR DTR is given as

$$s(t) = \sum_{i=0}^{+\infty} b_i p^{\text{TX}}(t - iT) \quad (13)$$

where $b_i$ are the differentially encoded information symbols $a_i$ taking values $\pm 1$ with equal probability, such that

$$b_i = b_0 \prod_{k=1}^{i} a_k, \quad b_0 \stackrel{\text{def}}{=} 1 \quad (14)$$

$T$ is the symbol duration, and $p^{\text{TX}}(t)$ is the transmit pulse, whose duration is on the order of nanoseconds. The pulse energy is normalized to one and thus the energy per bit is given by $E_{\text{b}} = 1$.

The transmit signal passes through a multipath channel with impulse response $h^{\text{CH}}(t)$ and a lowpass receiver filter with bandwidth $B$ and impulse response $h^{\text{RX}}(t)$. For the case of no multiple-access interference the received signal can be written as

$$r(t) = h^{\text{CH}}(t) * h^{\text{RX}}(t) * s(t) + n(t) \quad (15)$$

where $n(t) = h^{\text{RX}}(t) * n_0(t)$, and $n_0(t)$ is white Gaussian noise with two-sided power-spectral density $N_0/2$. The effective pulse shape of the received signal is given by $p(t) \stackrel{\text{def}}{=} p^{\text{TX}}(t) * h^{\text{CH}}(t) * h^{\text{RX}}(t)$ of duration $T_p$, which satisfies $T_p < T$ to avoid inter-symbol interference.

For the sake of clarity, repetition coding and time hopping are omitted, but can be introduced straightforwardly for differential encoding at symbol rate and averaging of the received pulses comprising an information symbol prior to further signal processing at the receiver, which requires the channel to be constant in an increased interval [2], [5].

### B. MSDD Decision Metric

At the receiver, the joint decision of $L$ information symbols is based on the received signal in an observation window of length $(L+1)T$, in which the channel is assumed to be constant. In [4], [5], it is shown that the MSDD problem for UWB-IR DTR in a single-user scenario with additive Gaussian noise is of the form

$$\hat{\boldsymbol{a}} = \operatorname*{argmin}_{\tilde{\boldsymbol{a}} \in \{\pm 1\}^L} \Gamma\left(r(t)|\tilde{\boldsymbol{a}}\right) \quad (16)$$

where we have defined the decision metric

$$\Gamma\left(r(t)|\tilde{\boldsymbol{a}}\right) \stackrel{\text{def}}{=} \sum_{i=1}^{L}\sum_{l=0}^{i-1}\left(|Z_{l,i}| \cdot \left(1 - \varphi_{l,i} \cdot \prod_{k=l+1}^{i} \tilde{a}_k\right)\right) \quad (17)$$

with $\varphi_{l,i} \stackrel{\text{def}}{=} \operatorname{sign}(Z_{l,i})$, and

$$\begin{aligned} Z_{l,i} &\stackrel{\text{def}}{=} \int_0^{T_{\text{I}}} r(t + iT) \cdot r(t + lT) \, \mathrm{d}t \\ &= b_i b_l \int_0^{T_{\text{I}}} p^2(t) \, \mathrm{d}t + \eta_{l,i} \end{aligned} \quad (18)$$

is the output of an $L$-branch ACR with delays being multiples of $T$ and the integration interval $T_{\text{I}}$ is of the order of $T_p$. $Z_{l,i}$ represents the phase transition from $b_l$ to $b_i$ superposed by an "information $\times$ noise" and "noise $\times$ noise" term

$$\begin{aligned} \eta_{l,i} \stackrel{\text{def}}{=}\ & b_i \int_0^{T_{\text{I}}} p(t) n(t + lT) \, \mathrm{d}t + b_l \int_0^{T_{\text{I}}} p(t) n(t + iT) \, \mathrm{d}t \\ &+ \int_0^{T_{\text{I}}} n(t + lT) n(t + iT) \, \mathrm{d}t. \end{aligned} \quad (19)$$

The addends of the outer sum in (17) are always non-negative and depend solely on the first $i$ (preliminary) decisions of information symbols $\tilde{a}_k$, $k = 1, ..., i$. This allows to check the decision metric componentwise, and thus (16) can be solved using the SD operating on an $L$-dimensional binary tree as outlined in Section II-A (see also [5] for further details).

### C. Stopping Radius for the SD of UWB-IR MSDD

To obtain a stopping radius for the SD similar to that derived in Section II-B, we consider the difference of the metric of the transmitted sequences

$$d(\boldsymbol{a},\boldsymbol{a}') \stackrel{\text{def}}{=} \Gamma(r(t)|\boldsymbol{a}') - \Gamma(r(t)|\boldsymbol{a}) = \sum_{i=1}^{L}\sum_{l=0}^{i-1} |Z_{l,i}|\varphi_{l,i}\left(\prod_{k=l+1}^{i} a_k - \prod_{k=l+1}^{i} a'_k\right). \quad (20)$$

Denoting by $|Z_{\min}|$ the minimum over all $|Z_{l,i}|$, $i = 1, ..., L$, $l = 0, ..., i-1$, and since $b_l b_i = \prod_{k=l+1}^{i} a_k$, we have

$$d(\boldsymbol{a},\boldsymbol{a}') \geq |Z_{\min}| \cdot \sum_{i=1}^{L}\sum_{l=0}^{i-1} \varphi_{l,i}\left(b_l b_i - b'_l b'_i\right). \quad (21)$$

Assuming moderate to high SNR, it can readily be seen from (18) that $\varphi_{l,i} = \operatorname{sign}(Z_{l,i}) = b_l b_i$. Since $b_i \in \{\pm 1\}$, $b_l b_i (b_l b_i - b'_l b'_i) = \frac{1}{2}(b_l b_i - b'_l b'_i)^2$ holds, and we arrive at

$$\begin{aligned} d(\boldsymbol{a},\boldsymbol{a}') &\geq \frac{1}{2}|Z_{\min}| \cdot \sum_{i=1}^{L}\sum_{l=0}^{i-1}\left(b_l b_i - b'_l b'_i\right)^2 \\ &= \frac{1}{2}|Z_{\min}| \cdot \sum_{i=1}^{L}\sum_{l=0}^{i-1}\left(\prod_{k=l+1}^{i} a_k - \prod_{k=l+1}^{i} a'_k\right)^2 \\ &= \frac{1}{2}|Z_{\min}| \cdot d_{\mathrm{E}}^2(\boldsymbol{a},\boldsymbol{a}'). \end{aligned} \quad (22)$$

In (22) the definition of the Euclidean distance of two information sequences

$$d_{\mathrm{E}}^2(\boldsymbol{a},\boldsymbol{a}') \stackrel{\text{def}}{=} \sum_{i=1}^{L}\sum_{l=0}^{i-1}\left(\prod_{k=l+1}^{i} a_k - \prod_{k=l+1}^{i} \tilde{a}_k\right)^2 \quad (23)$$

has been applied [4]. Its minimum is found to be

$$\min_{\substack{\boldsymbol{a}' \neq \boldsymbol{a} \\ \boldsymbol{a},\boldsymbol{a}' \in \{\pm 1\}^L}} d_{\mathrm{E}}^2(\boldsymbol{a},\boldsymbol{a}') = 4L. \quad (24)$$

Combining (22) and (24) gives a lower bound on the "packing radius of the MSDD problem",

$$\frac{1}{2} \cdot \min_{\tilde{\boldsymbol{a}} \neq \boldsymbol{a}} d(\boldsymbol{a},\tilde{\boldsymbol{a}}) \geq L \cdot \min_{\substack{i=1,\dots,L \\ l=0,\dots,i-1}} |Z_{l,i}|. \quad (25)$$

Similar to (12) the lower bound depends on the minimum distance of "indices", here $\tilde{\boldsymbol{a}}$, and a scaling factor given by the specific problem structure. Here the scaling factor is the minimum of all $|Z_{l,i}|$, since, as there is no "vector × matrix"-structure as in Section II-B, not only the "diagonal" entries $Z_{i-1,i}$ count.

Following the reasoning from Section II-C, a lower bound on the packing radius can be used as a stopping radius for the SD. Hence, we define

$$R_{\text{stop}} \stackrel{\text{def}}{=} L \cdot \min_{\substack{i=1,\dots,L \\ l=0,\dots,i-1}} |Z_{l,i}| \quad (26)$$

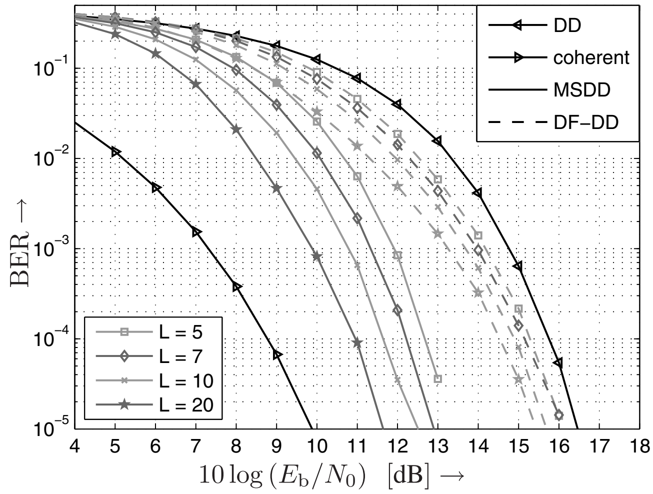


Fig. 2. BER vs. $10\log(E_{\mathrm{b}}/N_0)$ in dB of MSDD and DF-DD in UWB-IR DTR systems for channel model IEEE CM 2 and $T_{\mathrm{I}} = 30$ ns. Also shown: coherent detection with perfect channel knowledge and DD.

and propose that the SD is terminated for any preliminary result $\tilde{\boldsymbol{a}}$, that satisfies

$$\Gamma(r(t)|\tilde{\boldsymbol{a}}) \leq \alpha \cdot R_{\text{stop}}. \quad (27)$$

For $\alpha = 1$ the optimal SD output is obtained. Relaxation such that $\alpha > 1$ allows for a performance-complexity trade-off, as will be illustrated in the next section. In particular, $\alpha = 0$ is equivalent to the SD without stopping radius and $\alpha \to \infty$ results in DF-DD. We finally note that the proposed criterion, in particular the stopping radius $R_{\text{stop}}$, can be computed with negligible computational cost as function of the ACR outputs.

## IV. Numerical Results

In this section, we present numerical results to demonstrate the performance-complexity trade-off achieved with the new stopping criterion. We make the following standard assumptions: $T = 100$ ns, $p^{\mathrm{TX}}(t)$ is as a Gaussian 2nd derivative monocycle with 10 dB-bandwidth of 3.3 GHz, center frequency of 2.25 GHz, the receive filter bandwidth is $B = 3.5$ GHz, the ACR integration interval is $T_{\mathrm{I}} = 30$ ns, and $h^{\mathrm{CH}}(t)$ is generated according to the IEEE channel model CM 2 [16], where each realization is normalized to unit energy.

First, we recapitulate the benefits of MSDD with the performance curves in Figure 2, which show the bit error rate (BER) vs. SNR for MSDD with different $L$ (using the SD), symbol-by-symbol DD (i.e., $L = 1$, corresponding to evaluating only the 1st branch of ACR), DF-DD (i.e., SD with $R_{\text{stop}}^2 \to +\infty$, dashed-dotted lines with markers), and coherent detection with ideal channel knowledge. We observe that MSDD is clearly superior to DD and DF-DD and bridges the gap to detection with perfect channel state information for increasing $L$.

Next, Figure 3 compares the BER of MSDD via the traditional SD (SD, solid gray lines) and the SD with stopping criterion according to (27) with $\alpha = 1$ (Stop-SD, dash-dotted lines with markers). As expected, the SD with stopping radius

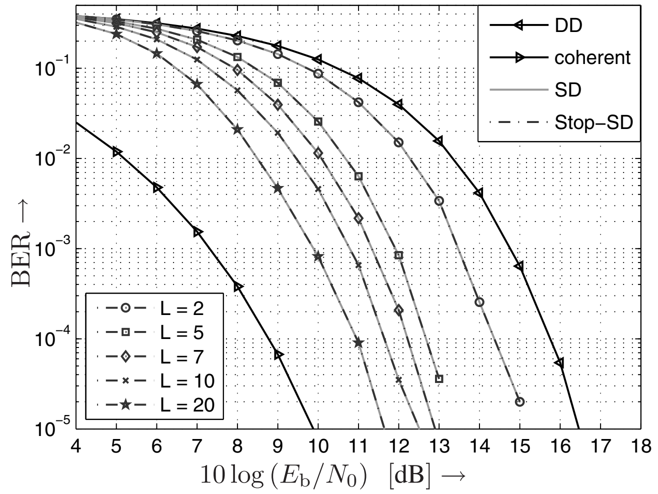


Fig. 3. BER vs. $10 \log (E_\mathrm{b}/N_0)$ in dB of MSDD via traditional SD and SD with stopping radius according to (26) in UWB-R DTR systems for channel model IEEE CM 2. Also shown: coherent detection with perfect channel knowledge and DD.

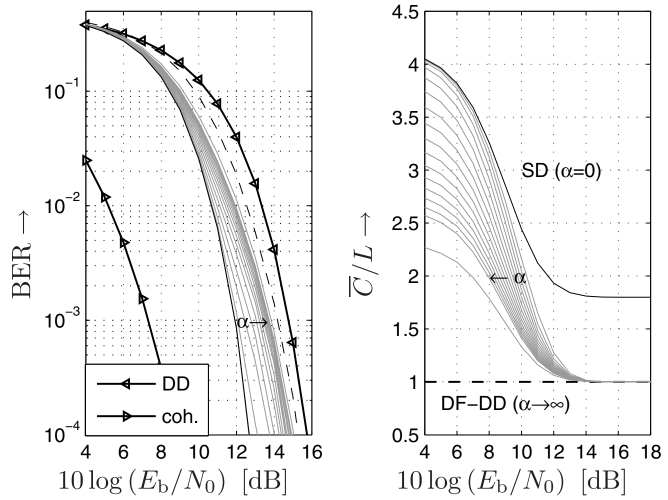


Fig. 5. BER and average complexity vs. $10 \log (E_\mathrm{b}/N_0)$ in dB of MSDD with $L = 5$ using the SD with scaled stopping radius for different $\alpha$ in comparison to the benchmark cases traditional SD (black solid line) and DF-DD (dashed).

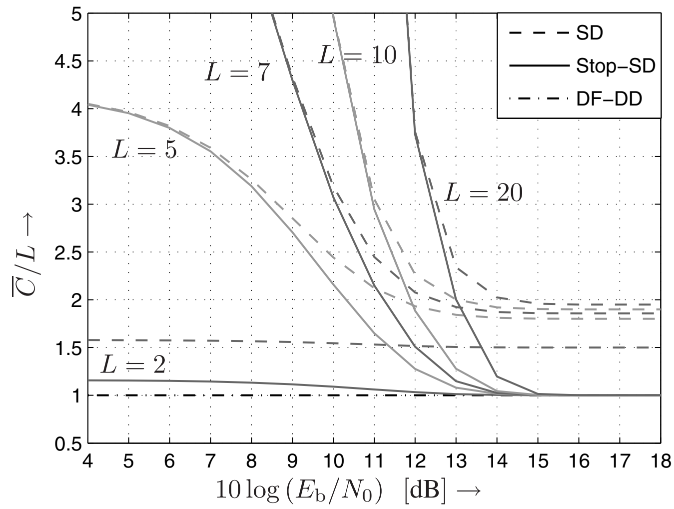


Fig. 4. Average complexity of MSDD via traditional SD and SD with stopping radius according to (26) vs. $10 \log (E_\mathrm{b}/N_0)$ in dB in UWB-IR DTR systems for channel model IEEE CM 2.

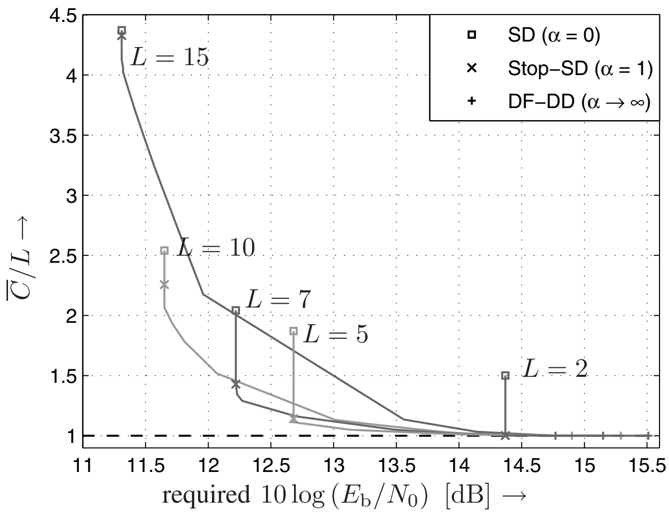


Fig. 6. Average complexity of the SD with scaled stopping radius vs. required $10 \log (E_\mathrm{b}/N_0)$ to achieve $\mathrm{BER} = 10^{-4}$. Curves are parameterized by $\alpha$ (see (27)).

$R_\mathrm{stop}$ (26) performs identical to the traditional SD in terms of BER.

However, using the new stopping criterion yields a clear complexity advantage. To see this, Figure 4 shows the average SD complexity $\overline{C}$ measured as the average number of nodes visited during tree search decoding, normalized to $L$. We observe that gains are achieved especially at large SNR. In this regime, the dominant event in the search process of the SD without stopping radius is to (i) find the DF-DD result after $L$ nodes, (ii) back-trace the DF-DD path, (iii) check and reject only a single alternative at every depth. The DF-DD point involves the visit of $L$ nodes, and the backtracing contributes another $L-1$ nodes. Hence, the total complexity is $2L-1$. With the proposed stopping radius according to (26), Stop-SD often terminates after the DF-DD has been found, and thus $\overline{C} \approx L$. This leads to MSDD performance at (almost) DF-DD complexity.

We now consider the relaxed stopping criterion (27) with $\alpha \geq 1$. Exemplarily for $L = 5$ Figure 5 depicts the BER and the average complexity for different $\alpha = 1, 2, ..., 20$ (gray lines) in comparison to MSDD using the traditional SD with no stopping radius ($\alpha = 0$, solid line), DF-DD ($\alpha \to \infty$, dashed), DD, and coherent detection. Increasing the stopping radius with $\alpha \geq 1$ reduces the average complexity in the low SNR regime as well at an continous trade-off between power efficiency of MSDD and DF-DD.

Figure 6 illustrates this performance-complexity trade-off for various $L$. The average complexity of the SD with early termination versus the SNR required to achieve a BER of $10^{-4}$ is shown. It can be seen that (i) the complexity of the SD with

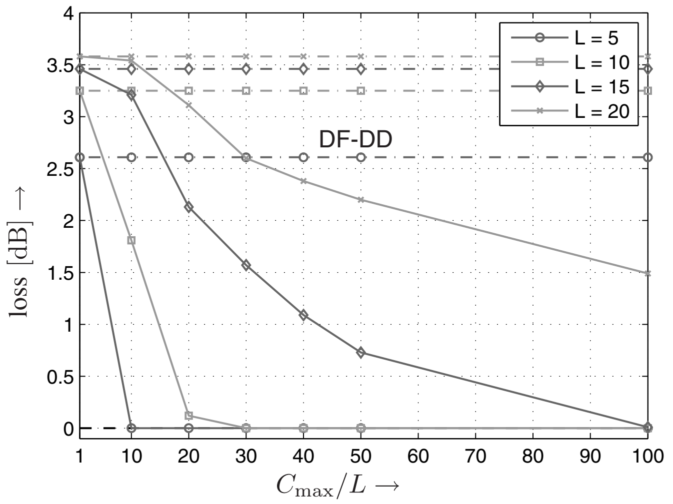


Fig. 7. Performance loss of SD due to maximum complexity limit $C_{\max}$ at $\mathrm{BER} = 10^{-4}$ compared to DF-DD ($C_{\max} = L$).

stopping radius drops close to that of DF-DD with no loss in performance for $\alpha = 1$ ('x' markers) especially for small to moderate $L$ and (ii) the parameter $\alpha \geq 1$ enables a trade-off between power efficiency and decoding complexity of MSDD and DF-DD.

The proposed stopping radius with $\alpha = 1$ mainly strikes at high SNR, but does little to reduce the complexity of the SD at low SNR (cf. Figure 4). Increasing the stopping radius with $\alpha > 1$, a significant complexity reduction in this regime is only achieved at notable performance degradation (cf. Figure 5). Hence for low SNR, limiting the maximum decoder complexity is an appealing approach. Figure 7 shows the performance loss in terms of the SNR required to achieve $\mathrm{BER} = 10^{-4}$ incurred when the maximum complexity is limited to $C_{\max}$. As mentioned in Section II-A, $C_{\max} = L$ is equivalent to DF-DD. We observe that bounding the maximum complexity at about $10L$ and $20L$ is appropriate for MSDD block-lengths of $L = 5$ and $L = 10$, respectively. For larger block-lengths $L$, however, limitation of maximum complexity results in notable performance losses even for large $C_{\max}$. Hence, in light of the performance curves in Figure 2, the use of such MSDD with moderate block-lengths $L \lesssim 10$ and the complexity-reduced SD is preferable.

## V. Conclusions

In this paper, we have presented a method to reduce the complexity of the SD for MSDD in UWB-IR DTR transmission. For this purpose, we have first derived a stopping radius for the SD based on a lower bound on the packing radius of discrete sets, whose application preserves the optimality of the decoder output but reduces the search complexity. We have then adapted this approach to the SD for MSDD in UWB-IR DTR systems. In this context, we have also introduced a relaxation factor, which allows us to trade-off decoding complexity and performance. We have presented a number of numerical results to demonstrate the advantages of the proposed stopping criterion. At high SNR, a factor of two in complexity reduction is gained with no degradation in error-rate performance.

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