On the Achieveability of Maximum Capacity with Synchronous CDMA

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Abstract

The search of "optimum" spreading sequences for transmission over the additive white Gaussian noise multiple-access channel (AWGN-MAC) with synchronous CDMA in dependence of users' powers is considered. "Optimum" means that a set of users in a CDMA system can transmit reliably at its sum rate being arbitrarily close to the corresponding sum capacity of the AWGN-MAC. We will focus our attention to the situation that the number of users K is larger than the number of chips L. Using the insight given by an equivalent model for CDMA the design rule for "optimum" spreading sequences is derived for the case that only a part of all users has equal power and it is possible to map the other users into L distinct sets in such a way that the total power of the users in each such set is at most one Lth of all users' total power.

I. Introduction

In recent papers [3], [6] it was shown that the spectral efficiency of synchronous CDMA systems supporting an infinite number of users K which employ randomly chosen spreading sequences can approach the Shannon bound for transmission over the AWGN-MAC. This can be achieved by the application of interference cancellation at the receiver provided the rate of each single user is infinitesimal.

In contrast to this, for non-zero rates and a finite number of users each user's capacity employing CDMA is smaller than his/her maximum capacity determined by the AWGN-MAC. This loss is caused by the spreading of the users' data symbol sequences. Nonetheless, under certain prerequisites it is possible to choose spreading sequences in such a way that the sum capacity of a set of users in a synchronous CDMA system is equal to the corresponding sum capacity given by the AWGN-MAC.

We denote as *sum capacity* of a user set the maximum achievable sum rate allowing reliable transmission of these users data symbols. It is well-known [1] that the sum capacity of a user set for transmission over the AWGN-MAC equals the capacity of a single user AWGN channel with the same total input power.

The choice of spreading sequences for synchronous CDMA systems was studied by Rupf and Massey in [5]. Considering the sum capacity they derived conditions verifying whether users' spreading sequences are "optimum". "Optimum" is used in the sense that by means of these spreading sequences the corresponding sum capacity of the AWGN-MAC can be achieved for a set of users. Further, assuming that all users have same power, it was shown in [5] that their sum capacity equals that of the AWGN-MAC if the so called Welch-bound-equality sequence multisets are employed as spreading sequences.

We approach once again the problem of designing "optimum" spreading sequences with the help of a simple model for CDMA. This model well illustrates that a necessary condition for the existence of "optimum" spreading sequences is that the number of users is at least equal to the number of chips. Furthermore, it can be seen that for $K = L$ the sum capacity of CDMA can be equal to the corresponding capacity of the AWGN-MAC only if all users have the same power.

This restriction with respect to the users' powers can be weakened for $K > L$. We show that in this case "optimum" spreading sequences exist also under certain circumstances and a rule for their appropriate design is presented. It is sufficient if there are L equal power users in the CDMA system whereas the other users' powers fulfill a condition being derived in this paper.

Finally, we obtain that it is impossible to find "optimum" spreading sequences for all ratios of users' powers and all user sets even if they contain more than L users.

The paper is organised as follows. In Section 2 we describe the synchronous CDMA model. We introduce an equivalent model for CDMA with $L \geq K$ in Section 3, and constraints concerning the existence of "optimum" spreading sequences are deduced. The case $2 \leq L < K$ is studied in Section 4. The conditions having to be fulfilled by "optimum" spreading sequences are given. At the end, a design rule for the spreading sequences is obtained provided a particular prerequisite with respect to users' powers holds.

II. Synchronous Transmission with CDMA

The discrete-time model of the symbol and chip synchronous CDMA system we will use is given in Fig. 1. This model describes the transmission of independent code symbol sequences $X_k[\mu]$, $1 \leq k \leq K$, belonging to K users over an additive white Gaussian noise multiple-access channel. All code symbols $X_k[\mu]$ are drawn independent and identically distributed (i.i.d.) from a zero mean complex distribution with variance $\sigma^2_{X_k} = E\{|X_k|^2\}$, $1 \leq k \leq K$. The distribution itself is not fixed. Furthermore, we assume that the total input power $P = \sum_{k=1}^{K} \sigma^2_{X_k}$ is constrained.

Due to the synchronous transmission and the independent identical distribution of all $X_k[\mu]$, it is sufficient to consider one symbol interval (e.g., the first one) and the time index $[\mu]$ can be omitted.

The kth user's symbol X_k is spread by a complex sequence $\mathbf{s}_k^T = (s_{k1}, ..., s_{kL})$ having norm $\mathbf{s}_k^H \mathbf{s}_k = L$. The superscript T denotes the transposition and H stands for the complex conjugation and transposition.. This yields $\mathbf{T}_k = (T_{k1}, ..., T_{kL}) = X_k \mathbf{s}_k^T$, $1 \leq k \leq K$. Further, the average power, $\sigma^2_{T_k} = E\{|T_{kl}|^2\}$, is constrained by

$$\sigma^2_{T_k} = \frac{1}{L} \sum_{l=1}^{L} E\{|X_k|^2\} |s_{kl}|^2 = \sigma^2_{X_k}. \tag{1}$$

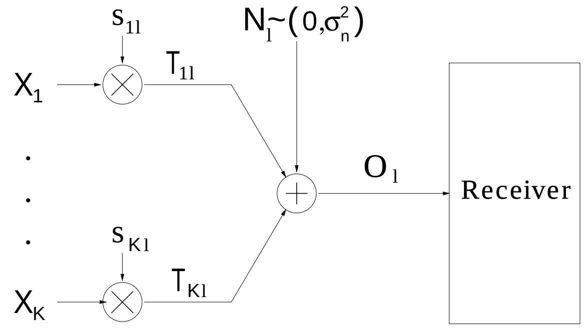

Fig. 1: Synchronous CDMA system

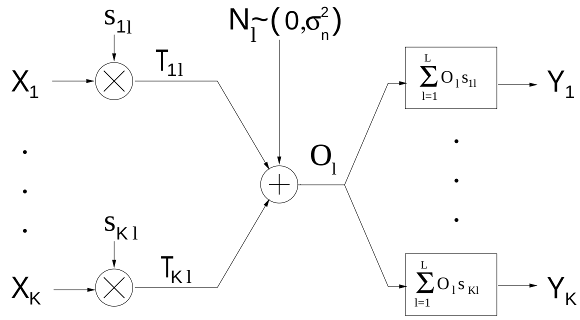

Fig. 2: CDMA system with $L \geq K$

The additive white Gaussian noise N_l, $1 \leq l \leq L$, is a proper complex random variable and has zero mean and power σ_n^2.

The output signal of the AWGN-MAC in one symbol interval is $\mathbf{O}^T = (O_1, ..., O_L)$, where $O_l = \sum_{k=1}^{K} T_{kl} + N_l$, $1 \leq l \leq L$. The transmission of one symbol for each user can also be described by

$$\mathbf{O} = \mathbf{SX} + \mathbf{N}, \tag{2}$$

where $\mathbf{S} = (\mathbf{s}_1, ..., \mathbf{s}_K)$, $\mathbf{X}^T = (X_1, ..., X_K)$, $\mathbf{N}^T = (N_1, ..., N_L)$. The users' rates R_k given in (bits/chip), $1 \leq k \leq K$, for reliable transmission are constrained by (see [1])

$$\begin{aligned} R_i &\leq C_{AWGN-MAC}^{\{i\}} = \log_2\left(1 + \frac{\sigma_{X_i}^2}{\sigma_n^2}\right) \\ R_i + R_j &\leq C_{AWGN-MAC}^{\{i,j\}} = \log_2\left(1 + \frac{\sigma_{X_i}^2 + \sigma_{X_j}^2}{\sigma_n^2}\right) \\ &\vdots \\ \sum_{k=1}^{K} R_k &\leq C_{AWGN-MAC}^{\{1...K\}} = \log_2\left(1 + \frac{\sum_{k=1}^{K}\sigma_{X_k}^2}{\sigma_n^2}\right). \end{aligned}$$

These bounds can only be reached if the symbols T_{kl}, belonging to user k, are drawn i.i.d. from a complex Gaussian distribution with power $\sigma_{X_k}^2$. A necessary condition for the independence of $T_{ki} = X_k s_{ki}$ and $T_{kj} = X_k s_{kj}$ is to avoid spreading at all, i.e., $L = 1$. Otherwise, under the condition of a non-interfered transmission, T_{kj} could be deduced from T_{ki} as $\mathbf{s}_k$ is assumed to be known.

III. Optimum Spreading for $L \geq K$

The number of chips per spreading sequence is assumed to be at least equal to the number of users ($L \geq K$). Hence, K linearly independent spreading sequences $\mathbf{s}_k$ can be used. In order to provide a set of sufficient statistics the receiver's front end may consist of a bank of K matched filters, as shown in Fig. 2.

The output of the matched filter bank $\mathbf{Y}^T = (Y_1, ..., Y_K)$ is given as

$$\mathbf{Y} = \mathbf{S}^H\mathbf{SX} + \mathbf{S}^H\mathbf{N} = \mathbf{MX} + \tilde{\mathbf{N}}, \tag{3}$$

where $\mathbf{M} = \mathbf{S}^H\mathbf{S}$ denotes the correlation matrix of the spreading sequences. The covariance of the noise $\tilde{\mathbf{N}} = \mathbf{S}^H\mathbf{N} = (\tilde{N}_1, ..., \tilde{N}_K)^T$ is solved as $E\{\tilde{\mathbf{N}}\tilde{\mathbf{N}}^H\} = \sigma_N^2\mathbf{M}$. The diagonal elements of $\mathbf{M}$ are $m_{kk} = \mathbf{s}_k^H\mathbf{s}_k = L$. In general, spreading sequences of two users $k \neq j$ are not orthogonal, i.e., $m_{kj} = \mathbf{s}_k^H\mathbf{s}_j \neq 0$.

The kth row of Eq. (3) can be written as

$$Y_k = LX_k + \sum_{j=1,\, j\neq k}^{K} m_{kj}X_j + \tilde{N}_j. \tag{4}$$

We see the output Y_k can be considered as the superposition of the input LX_k, the *Multiuser interference* $\sum_{\substack{j=1 \\ j\neq k}}^{K} m_{kj}X_j$ and the colored Gaussian noise $\tilde{N}_j$. From this follows that the transmission of $X_1, ..., X_K$ over the AWGN-MAC with CDMA is equivalent to the transmission of $LX_1, ..., LX_K$ over K correlated parallel channels, as shown in Fig. 3.

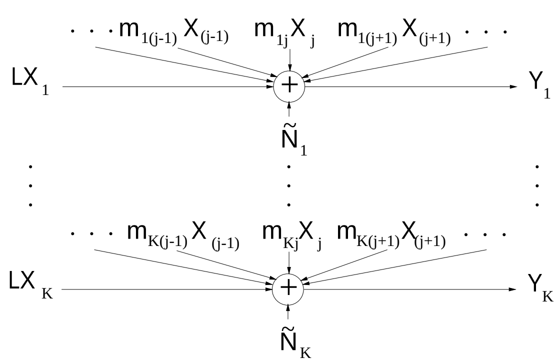

Fig. 3: Equivalent Model for CDMA with $L \geq K$

How must the spreading sequences be chosen as to achieve maximum sum capacity C_{CDMA}, which is defined as $C_{CDMA} := \max_{\mathbf{S}} C_{CDMA}(\mathbf{S})$, where

$$C_{CDMA}(\mathbf{S}) := \max \sum_{k=1}^{K} R_k, \tag{5}$$

for a given matrix $\mathbf{S}$? On the one hand we could use the formula for $C_{CDMA}(\mathbf{S})$. So we obtain for $C_{CDMA}(\mathbf{S})$ with the help of Eq. (3) (cf. [1])

$$\begin{aligned} &C_{CDMA}(\mathbf{S}) \\ &= \frac{1}{L}\log_2\left[\frac{\det\left(E\{\mathbf{YY}^H\}\right)}{\det\left(E\{\tilde{\mathbf{N}}\tilde{\mathbf{N}}^H\}\right)}\right] \\ &= \frac{1}{L}\log_2\left[\frac{\det\left(\mathbf{M}\,\mathrm{diag}(\sigma_{X_1}^2 ... \sigma_{X_K}^2)\,\mathbf{M} + \sigma_n^2\mathbf{M}\right)}{\det\left(\sigma_n^2\mathbf{M}\right)}\right] \\ &= \frac{1}{L}\log_2\left[\det\left(\frac{\mathrm{diag}(\sigma_{X_1}^2 ... \sigma_{X_K}^2)\,\mathbf{M}}{\sigma_n^2} + \mathbf{I}^K\right)\right]. \end{aligned} \tag{6}$$

$\mathbf{I}^K$ denotes the K by K identity matrix. Then, the maximum sum capacity C_{CDMA} can be found by maximizing (6) over all possible sets of spreading sequences $\mathbf{S}$.

A simpler approach is to use our equivalent model. Since the linear transformation performed by the bank of matched filters does not affect the maximum achievable sum rate of any set of users the system's capacity region is preserved.

First, as all code symbol sequences X_k, $1 \leq k \leq K$ are uncorrelated, it is optimum to transmit each user's code symbols over one independent channel being not interfered by other users. This leads to the condition that all spreading sequences have to be orthogonal. The equivalent model for this case is depicted in Fig. 4.

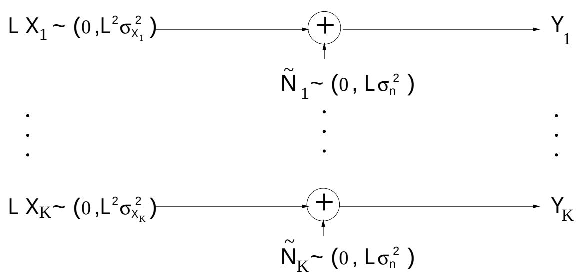

Fig. 4: Model for CDMA with orthogonal spreading sequences and $L \geq K$

Due to the orthogonality of the spreading sequences, $\mathbf{M}$ simplifies to $L\mathbf{I}^K$. Therefore, the Gaussian noise $\tilde{N}_k$ is now independent from channel to channel and its power is equal to $L\sigma_n^2$, $\forall k$. Thus, we have in Fig. 4 K parallel independent channels. So, the sum capacity given by (6) is the sum of the capacities of the K parallel channels.

Furthermore, it has to be considered that the input power $P_{sum} = \sum_{k=1}^{K} L^2\sigma_{X_k}^2$ is constrained by the total power L^2P. Hence, the sum capacity $C_{CDMA}(\mathbf{S})$ is maximum for equal powers $\sigma_{X_k}^2 = \sigma_X^2 = \frac{L^2P}{K}$, $\forall\, k$ (cf.[1]). With Eq. (6) the sum capacity $C_{CDMA}(\mathbf{S})$ is derived as

$$\begin{aligned} C_{CDMA}(\mathbf{S}) &= \frac{1}{L}\sum_{k=1}^{K}\log_2\left(1+\frac{L\sigma_X^2}{\sigma_N^2}\right) \\ &= \frac{K}{L}\log_2\left(1+\frac{L\sigma_X^2}{\sigma_N^2}\right). \end{aligned} \tag{7}$$

Provided the number of users K is fixed it can be shown that $C_{CDMA}(\mathbf{S})$ is maximized for $L = K$ and becomes

$$C_{CDMA} = \log_2\left(1+\frac{L\sigma_X^2}{\sigma_N^2}\right) = \log_2\left(1+\frac{K\sigma_X^2}{\sigma_N^2}\right). \tag{8}$$

The explanation for this result is: if L would be larger than K, L independent channels could be created - to use only K is suboptimum as resources (bandwidth) are wasted.

Further, it can be seen that only if $L = K$, the sum capacity $C_{CDMA}(\mathbf{S})$ may be the same as that of the corresponding AWGN-MAC. In contrast to this, when $K < L$, comparing $C_{CDMA}(\mathbf{S})$ (Eq. (7)) with the sum capacity of the AWGN-MAC for these users

$$\begin{aligned} C_{AWGN-MAC} &= \log_2\left(1+\frac{\sum_{k=1}^{K}\sigma_X^2}{\sigma_N^2}\right) \\ &= \log_2\left(1+\frac{K\sigma_X^2}{\sigma_N^2}\right) \end{aligned} \tag{9}$$

we obtain

$$C_{CDMA}(\mathbf{S}) \leq C_{AWGN-MAC} \tag{10}$$

as the arithmetic mean is at least equal to the geometric mean.

We see, in CDMA systems with $K \leq L$, only the sum capacity for the set with $K = L$ is equal to that of the corresponding AWGN-MAC provided all users have same power. For unequal-power users there is no set of users being able to transmit reliably at sum rate arbitrarily close to the bounds of the AWGN-MAC.

IV. Optimum Spreading for $L < K$

Now we address the case $2 \leq L < K$. As the number of chips L is smaller than the number of users K at most L linearly independent sequences can be found. For the convenience of further considerations and without loss of generality it can be assumed that these L sequences $(\mathbf{s}_l^O)^T = (s_{l1}^O, ..., s_{lL}^O)$ are orthogonal and have norm L, i.e.,

$$(\mathbf{s}_i^O)^H\mathbf{s}_j^O = 0, \qquad i \neq j \tag{11}$$

$$(\mathbf{s}_j^O)^H\mathbf{s}_j^O = L, \qquad 1 \leq j \leq L. \tag{12}$$

Then, the K users' spreading sequences

$$\mathbf{s}_k = \sum_{j=1}^{L} r_{kj}\mathbf{s}_j^O,\; r_{kj} \in \Im \tag{13}$$

can be defined as linear combinations of the L independent sequences $\mathbf{s}_l^O$ ($\Im$ denotes the set of complex numbers). Introducing the matrices

$$\mathbf{S} = (\mathbf{s}_1, ..., \mathbf{s}_K), \qquad \mathbf{S}^O = (\mathbf{s}_1^O, ..., \mathbf{s}_L^O),$$

$$\mathbf{R} = (\mathbf{r}_1, ..., \mathbf{r}_K) = \begin{pmatrix} r_{11} & \cdots & r_{K1} \\ \vdots & \vdots & \vdots \\ r_{1L} & \cdots & r_{KL} \end{pmatrix}$$

the generation of the spreading sequences can be expressed in compact matrix notation as $\mathbf{S} = \mathbf{S}^O\mathbf{R}$. Further, from $\mathbf{s}_k^H\mathbf{s}_k = L$ and Eq. (12) follows that $\mathbf{r}_k^H\mathbf{r}_k$ must be 1.

In order to provide a set of sufficient statistics a bank of only L matched filters is necessary at the receiver (see Fig.5).

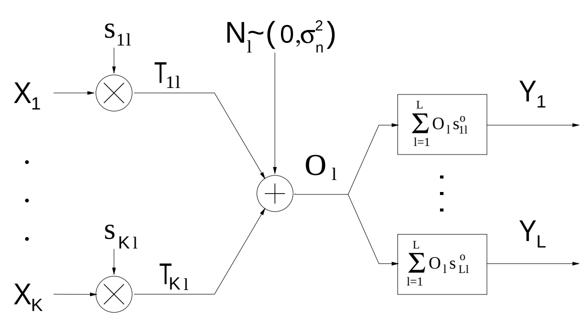

Fig. 5: CDMA system with $2 \leq L < K$

The matched filters' output $\mathbf{Y} = (Y_1, ..., Y_L)^T$ is

$$\mathbf{Y} = (\mathbf{S}^O)^H(\mathbf{S}^O\mathbf{R}\mathbf{X} + \mathbf{N}) = L\mathbf{R}\mathbf{X} + \tilde{\mathbf{N}} = \mathbf{Z} + \tilde{\mathbf{N}}, \tag{14}$$

where $\mathbf{Z} = L\mathbf{R}\mathbf{X}$. The lth element $Z_l = \sum_{k=1}^{K} L r_{kl} X_k$ is a superposition of the input symbols $X_1, ..., X_K$. The Gaussian noise $\tilde{\mathbf{N}} = (\mathbf{S}^O)^H \mathbf{N}$ has zero mean and variance $L\sigma_n^2 \mathbf{I}^L$. That is, all noise samples are independent.

Like in the previous section, it is possible to obtain from Eq. (14) an equivalent model for the CDMA system which is given in Fig. 6. The lth channel represents the lth row of Eq. (14). This model well illustrates that for $L < K$ only L independent channels can be created.

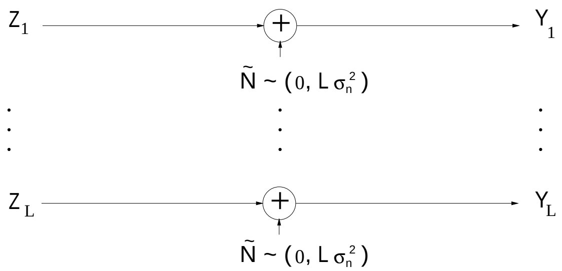

Fig. 6: Equivalent Model for CDMA with $2 \le L < K$

Consider the set $\mathcal{G}$ comprising all K users. How have the spreading sequences to be chosen to achieve $C_{CDMA}(\mathbf{S}) = C_{AWGN-MAC}$?

As the total input power $P_{sum} = \sum_{l=1}^{L} E\{|Z_l|^2\}$ is constrained by $L^2 P$ and the Gaussian noise is independent from channel to channel and has equal power it follows immediately (cf.[1]) that $C_{CDMA}(\mathbf{S}) = C_{AWGN-MAC}$ if and only if

- each input sequence $Z_l[\mu]$, $1 \le l \le L$, consists of independent and identically Gaussian distributed samples,
- all Z_l are mutually independent from each other,
- all powers $E\{|Z_l|^2\}$ have to be equal, i.e., $E\{|Z_l|^2\} = \sigma_Z^2 = \frac{P_{sum}}{L}$, $1 \le l \le L$.

The first condition is fulfilled by drawing the sequence $X_k[\mu]$, $1 \le k \le K$, i.i.d. from a Gaussian distribution. The second and third conditions can be expressed mathematically

$$E\{\mathbf{Z}\mathbf{Z}^H\} = \frac{P_{sum}}{L}\mathbf{I}^L. \tag{15}$$

Using $\mathbf{Z} = L\mathbf{R}\mathbf{X}$ and taking into account (11) and (12), we solve that the linear coefficients r_{kj} have to be designed according to

$$\mathbf{R}\text{diag}(\sigma_{X_1}^2, ..., \sigma_{X_K}^2)\mathbf{R}^H = \frac{P_{sum}}{L^3}\mathbf{I}^L \tag{16}$$

$$\mathbf{r}_k^H \mathbf{r}_k = 1. \tag{17}$$

Equivalent conditions for complex spreading sequences were derived by M.Rupf and J.Massey [5] by maximization of the capacity $C_{CDMA}(\mathbf{S})$.

Considering the model given in Fig. 6 we realize: if it is possible to map all K users into L distinct subsets $\mathcal{L}_1, ..., \mathcal{L}_L$ with equal sum power, then, $C_{CDMA}(\mathbf{S}) = C_{AWGN-MAC}$ is fulfilled by allocating one orthogonal spreading sequence to each subset (see Fig. (7)). In contrast to the case $L = K$, it is not necessary that all users have equal power as to transmit reliably at sum rate $R_{sum} = \sum_{k=1}^{K} R_k$ arbitrarily close to sum capacity of the AWGN-MAC.

Let us now turn to the situation that it is impossible to divide the whole set of users into distinct subsets with equal sum powers.

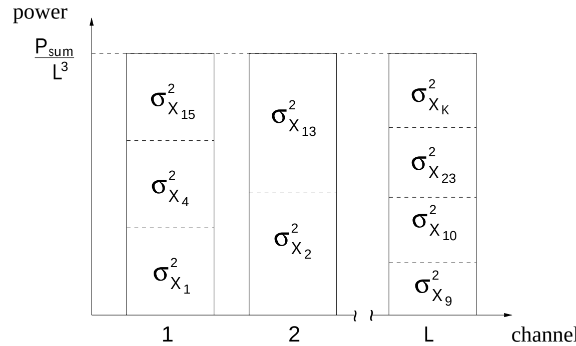

Fig. 7: Allocation of users to distinct subsets with equal power

Provided the powers of all users are equal to σ_X^2 Eq. (16) simplifies to

$$\sigma_X^2 \mathbf{R}\mathbf{R}^H = \frac{P_{sum}}{L^3}\mathbf{I}^L \tag{18}$$

and maximum sum capacity can be reached by employing so called Welch-bound equality sequence multisets [5].

However, it is not necessary that all users transmit with equal power in order to find “optimum” spreading sequences. We can give a design rule for “optimum” spreading sequences if only L users, for example user $1 \le k \le L$, have equal power $\sigma_{X_k}^2 = \sigma_{eq}^2$, and the unequal powers of the other $K - L$ users fulfill a certain prerequisite.

Allocating the users $(L+1), ..., K$ to L distinct sets $\mathcal{L}_l'$ we can define $Z_l' = L \sum_{k \in \mathcal{L}_l'} X_k$, $1 \le l \le L$. Hence, all Z_l' are independent of each other, but their powers $\sigma_{Z_l'}^2 = E\{|Z_l'|^2\} = L^2 \sum_{k \in \mathcal{L}_l'} \sigma_{X_k}^2$ are different. As to be able to assign to each channel in the end the same power $\frac{P_{sum}}{L}$, the allocation of the users $(L+1), ..., K$ to the sets $\mathcal{L}_l'$ has to be carried out in such a way that

$$\frac{P_{sum}}{L} \ge \sigma_{Z_l'}^2, \; 1 \le l \le L. \tag{19}$$

This is the necessary prerequisite. If (19) is fulfilled, we can solve for the L equal power users

$$r_{kl} = \sqrt{\frac{\frac{P_{sum}}{L} - \sigma_{Z_l'}^2}{L^3 \sigma_{eq}^2}} \exp\left(j2\pi\frac{kl}{L}\right), \text{ for } 1 \le k,l \le L. \tag{20}$$

Knowing that $r_{kl} r_{kl}^H = \frac{\frac{P_{sum}}{L} - \sigma_{Z_l'}^2}{L^3 \sigma_{eq}^2}$, for $1 \le k,l \le L$, we see that Eq. (16) holds since (see Fig. 8)

$$\begin{aligned} E\{Z_l Z_l^H\} &= \sigma_{Z_l'}^2 + E\left\{ L\sum_{k=1}^{L} r_{kl} X_k \left(L\sum_{k=1}^{L} r_{kl} X_k \right)^H \right\} \\ &= \frac{P_{sum}}{L}. \end{aligned}$$

and

$$E\{Z_l Z_j^H\} = E\{Z_l'(Z_j')^H\} + L^2 \sum_{k=1}^{L} r_{kl} r_{kj}^H E\{|X_k|^2\}$$

$$
\begin{aligned}
&= \sqrt{\frac{\left(\frac{P_{sum}}{L} - \sigma^2_{Z'_l}\right)\left(\frac{P_{sum}}{L} - \sigma^2_{Z'_j}\right)}{L^2}} \\
&\quad \times \sum_{k=1}^{L} \exp\left(j2\pi \frac{k(l-j)}{L}\right) \\
&= 0.
\end{aligned}
$$

Finally, it can be shown that condition (17) is fulfilled for all users, too.

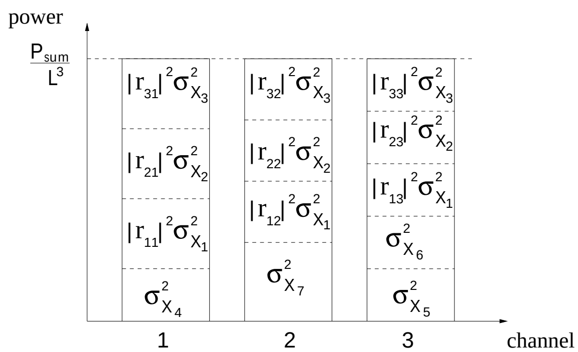

Fig. 8: Example for optimum choice of linear coefficients for $K = 7$ and $L = 3$

We would like to mention that the case of equal-power users is included in this more general solution.

Finally, it can be realized from the model that there are power distributions where no linear coefficients can be found as to get $C_{CDMA}(\mathbf{S}) = C_{AWGN-MAC}$ even if $K > L$.

In order to illustrate this we consider the sum capacity for $K = 3$ and $L = 2$. The conditions given in Eq. (16) become

$$
\begin{aligned}
0 &= r_{11}^* r_{12}\sigma^2_{X_1} + r_{21}^* r_{22}\sigma^2_{X_2} + r_{31}^* r_{32}\sigma^2_{X_3} && (21)\\
0.5P &= |r_{11}|^2\sigma^2_{X_1} + |r_{21}|^2\sigma^2_{X_2} + |r_{31}|^2\sigma^2_{X_3} && (22)\\
0.5P &= |r_{12}|^2\sigma^2_{X_1} + |r_{22}|^2\sigma^2_{X_2} + |r_{32}|^2\sigma^2_{X_3}, && (23)
\end{aligned}
$$

where $P = \sigma^2_{X_1} + \sigma^2_{X_2} + \sigma^2_{X_3}$ and the superscript $*$ denotes complex conjugation. Eq. (21) stands for the independence of Z_1 and Z_2 whereas Eq. (22), (23) represents the fact that the powers of Z_1 and Z_2 have to be equal. Further, from Eq. (17) follows

$$
|r_{11}|^2 + |r_{12}|^2 = 1,\ |r_{21}|^2 + |r_{22}|^2 = 1,\ |r_{31}|^2 + |r_{32}|^2 = 1. \quad (24)
$$

Let us assume that the power of user 1 (e.g. $\sigma^2_{X_1} = 10^2$) is considerably larger than those of user 2 and 3 (e.g. $\sigma^2_{X_2} = \sigma^2_{X_3} = 10^{-2}$). Then, it is impossible to create spreading sequences for these users in such a way that all conditions can be satisfied. On the one hand, condition (21) leads to $|r_{11}| \approx 0$ or $|r_{12}| \approx 0$. On the other hand, $|r_{11}|^2 \approx |r_{12}|^2 \approx 0.5$ is demanded by Eqs. (22) and (23).

It can be obtained that the fulfillment of the condition

$$
\frac{\sigma^2_{X_2} + \sigma^2_{X_3}}{\sigma^2_{X_1}} > 0.5, \quad (25)
$$

is a necessary prerequisite for the existence of optimum spreading sequences in this case. This means the power of the strongest user must be less than twice the sum of the other two users' powers.

V. Summary

In the previous sections it was derived that it is possible to design optimum spreading sequences not only for CDMA systems with equal-power users but, under certain conditions, for unequal-power users, too.

Nonetheless, it is often impossible to find optimum spreading sequences due to the relation of users' powers. Especially for small numbers of users unequal power distributions may lead to this situation. Therefore, it should be interesting to find stronger constraints for the existence of optimum or near optimum spreading sequences depending on the users powers.

Finally, it follows that all transmission systems fulfilling $L \geq K$, can achieve maximum sum rate only for equal-power users as well as orthogonal spreading sequences with $L = K$. That is, a lot of interesting receivers for real systems, e.g. the decorrelating decision-feedback receiver [2], being applied to achieve low symbol error rates for unequal power users while minimizing the interfering noise by choosing $L \gg K$ are suboptimal from an information-theoretic viewpoint in this case.

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