ON HIERARCHICAL SIGNAL CONSTELLATIONS FOR THE GAUSSIAN BROADCAST CHANNEL

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ABSTRACT

With the introduction of digital broadcasting techniques to radio and television, it becomes desirable to have an efficient possibility to reach receivers operating under varying degrees of received signal strength. Cover [2] showed that it is not efficient to do time sharing, that is transmitting a robust but low quality signal time multiplexed with a signal more susceptible to perturbations on the channel but having higher quality of service. Therefore various techniques exist to implement data streams of differing importance into one coded modulation scheme. This is done by adapting signal constellation, mapping and code design. In this paper we examine the performances that can be obtained and the tradeoffs that have to be made for some of the most often used constellations namely amplitude shift keying (ASK).

I. INTRODUCTION

The introduction of hierarchical modulation to digital broadcasting systems is driven by two major problems concerning the receivers. Firstly, highly complex systems, such as high definition digital television (HDTV) [4,6], call for the possibility to have receivers at reduced cost that implement only a subset of the maximum performance in terms of screen size and resolution, thus allowing for portable equipment with antennas of reduced aperture e.g. Secondly, digital radio broadcasting applications have to overcome the annoying effect of the sudden drop out of the program at the receiver due to the fact that the entity of digital modulation and forward error correction performs almost perfect at elevated signal to noise ratios and then degrades to no possible service within a few tenth of a decibel of received signal strength. In such an environment the source decoder is not able to provide graceful degradation and therefore the user has to live with sudden and possibly repeated drop outs. The latter scenario is for example encountered in a mobile reception environment or in digital short wave transmissions over the ionosphere that are currently under investigation by Digital Radio Mondiale (DRM), an international consortium developing digital radio broadcasting to replace analog AM below 30MHz [3].

For both scenarios, hierarchical modulation offers a possibility to provide more flexibility to the receivers, flexibility that was perfectly provided by the old analog systems. The recently increasing interest in such techniques is mainly due to the fact that source coding is now able to support such capabilities efficiently, the currently evolving MPEG 4 standard e.g. [10].

II. MULTILEVEL CODING

Multilevel Coding (MLC) of digital modulation schemes is based on a binary set partitioning of the signal constellation. Introduced by Imai [9], it provides for flexibility in the choice of code rates as well as the class of component forward error correction codes used at the individual levels of mapping. An additional degree of freedom is given by the strategy of set partitioning implemented. Although asymptotically the channel capacity is independent of the partitioning strategy chosen, important differences exist with respect to practical implementations of finite block length [8,13]. Usually, a set partitioning according to Ungerböck [12] for maximum intra subset minimum Euclidean distance is applied. Nevertheless we will focus in this work on the opposite strategy, the so called block partitioning

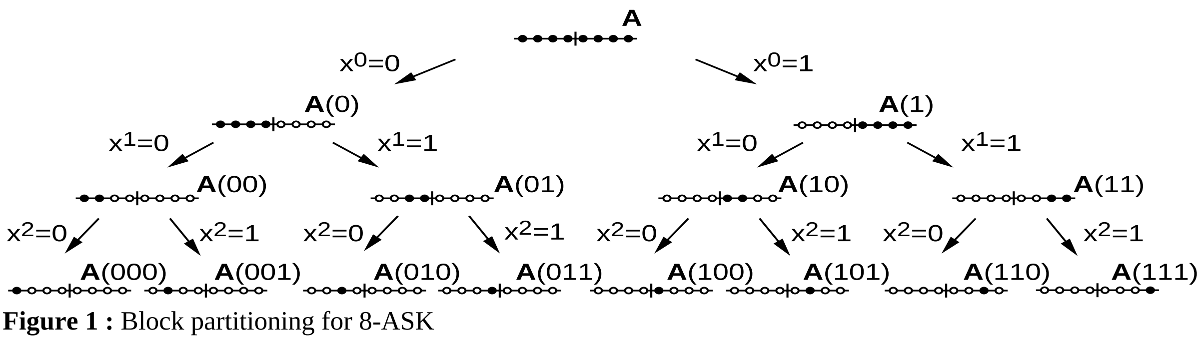

Figure 1 : Block partitioning for 8-ASK

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(BP), that minimizes the intra subset minimum Euclidean distance. Being less power efficient for finite block length than Ungerböck partitioning (UP), BP in return offers the possibility for graceful degradation and does not suffer from error propagation in a multi-stage-decoder realization. Clearly, it is possible to mix these two partitioning strategies in order to match the mapping to the desired number of steps in a softly degrading

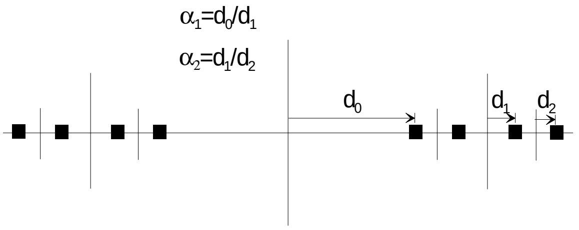

Figure 2 : Unequal distribution of signal points

system.

Figure 1 gives an example for BP of an 8-ASK constellation A. The individual x^i represent the labels associated with the i^{th} level of the MLC scheme, thus the second point from the left of the constellation has the label $\mathbf{x} = (x^0\ x^1\ x^2) = (0\ 0\ 1)$ e.g.

For UP the constellation is designed such as the capacities of all levels of the MLC scheme exceed the chosen rates of the component codes of the individual levels from a single signal to noise ratio (SNR) onwards. In addition to this single working point, the hierarchical levels of a BP scheme can be chosen to start functioning at different signal to noise ratios, hence allowing for an additional degree of freedom. For such a choice, the lowest level 0 can be received at the lowest SNR whereas higher levels need increasingly higher SNR's to be decodable. Examples for the implementation of graceful degradation by BP will be given in section 3.

Figure 1 shows a constellation with equally spaced signal points. By allowing the points to be placed at arbitrary locations on the axes one obtains additional degrees of freedom. Below we will limit ourselves to constellations maintaining a maximum degree of symmetry in order to keep decoding as simple as possible. Following the classification of signal constellations as given by [5,6,11], this results for level i of a MLC to be represented by a set of 2^i identical sub-constellations each of which is symmetrical with respect to a single decision threshold at its center but the minimum distance of the nearest points to the threshold being a free system parameter. Taking 8-ASK as an example, figure 2 defines the parameters α_1 and α_2 to be the degrees of deviation from a constellation with equally spaced signal points ($\alpha_1 = \alpha_2 = 1$). α_{i+1} is defined as the ratio of the Euclidean distances of the closest signal point to the decision threshold of level i over the closest signal point to threshold i+1. To make any comparison possible, all constellations are normalized to the same average energy for one symbol and all N signal points I_k are assumed to be equally probable ($p_k = 1/N$):

$$\sum_k p_k I_k^2 = \frac{1}{N}\sum_k I_k^2 \overset{!}{=} 1 \qquad (1)$$

III. ONE DIMENSIONAL CONSTELLATIONS

A. 4-ASK

The easiest and most instructive example is 4-ASK, having just one degree of freedom α_1. Figure 3 demonstrates the capacities of the two levels obtained from a 4-ASK constellation by block partitioning as described above with α_1 set to 2. As it will be done for all consecutive figures, the capacities of the equal distribution ($\alpha_i = \alpha^e = 1\ \forall i$) are included for reference. Note that the abscissa (E_s/N_0) is measured in average energy per symbol over average energy of the noise for the additive white gaussian noise channel (AWGN). Additionally the figures give a plot of the signal point spacing for the varying α's.

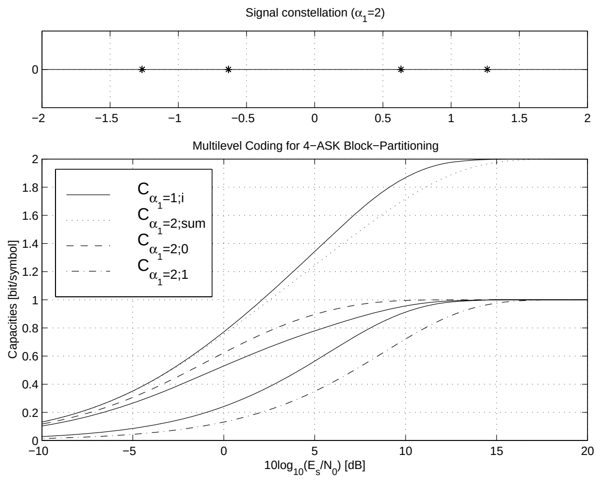

Figure 3 : Block partitioned 4-ASK with α_1=2

From figure 3, it can be seen that for a given rate R, the lower level with its corresponding capacity line $C_{\alpha;0}$ can be used at a considerably lower E_s/N_0 than the higher level $C_{\alpha;1}$, with the difference being 8.8dB for R=0.6 and α_1=2 e.g. The top curves represent the capacity limit $C_{\alpha;sum}$ for 4-ASK and a particular choice of α. They are independent of the partitioning strategy and can be obtained as the sum over the capacities of the sub-levels $C_{\alpha;sum} = C_{\alpha;0} + C_{\alpha;1}$.

From figure 3 we can see that a clustering of points away from the origin (α_1=2) results in the capacity line for level 0 to be shifted to the left with respect to an equally spaced distribution of signal points, towards lower values of required E_s/N_0. This improvement is opposed by a degradation of the upper level, which is shifted to the right. The magnitude of this effect depends on the degree of unequal spacing of signal points. In figure 4 this effect is amplified, due to an increase in α_1. Unfortunately the degradation of the upper level is not matched by an improvement of the same magnitude on the lower level for increasing α_1. Equation 2 therefore defines the overall performance $Loss_{total}$ to be the sum of

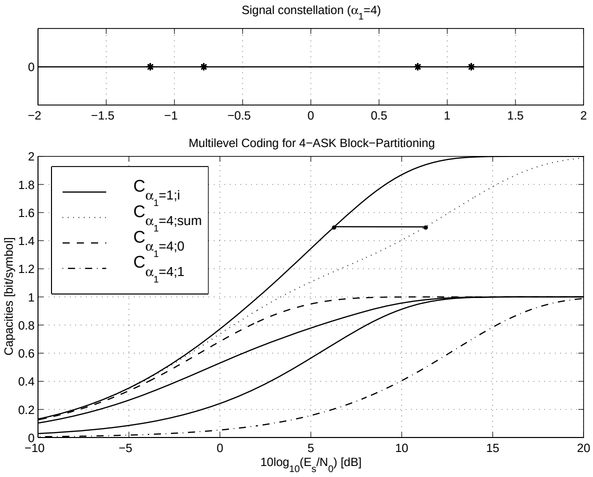

Figure 4 : Block partitioned 4-ASK with α_1=4

the degradation on the M individual levels L_i, as compared to α^e=1.

$$\text{Loss}_{\text{total}}\ [\text{dB}] = \sum_{i=0}^{M-1} \text{L}_i\ [\text{dB}] \qquad (2)$$

The loss due to a clustering of signal points with $\alpha > 1$ can also be seen from the severe degradation of the dotted overall capacity curve $C_{\alpha=4;sum}$ in figure 4, which becomes roughly 5dB at an overall capacity of 1.5 bit/symbol.

By following the opposite strategy and therefore moving the inner points closer to the origin, we obtain figures 5 and 6 for α_1=3/4 and α_1=1/2 respectively.

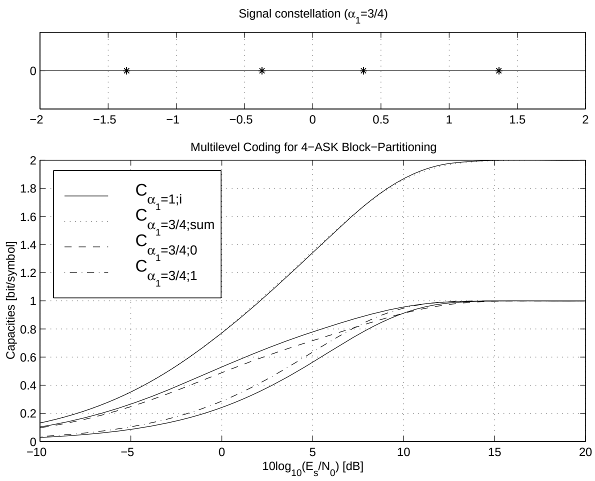

Figure 5 : Block partitioned 4-ASK with α_1=3/4

Here the capacities of the individual levels are shifted to the opposite sides of the equally spaced constellation with the magnitude of the effect increasing, as the inner points are placed closer to the origin. This leads to an increase in the intra subset minimum Euclidean distance of the upper level and therefore possibly to a more power efficient transmission.

Considering figures 3 to 6, we observe that a very flexible exchange between the signal energy necessary to operate the two levels can be obtained. With increasing α the lower level becomes more and more

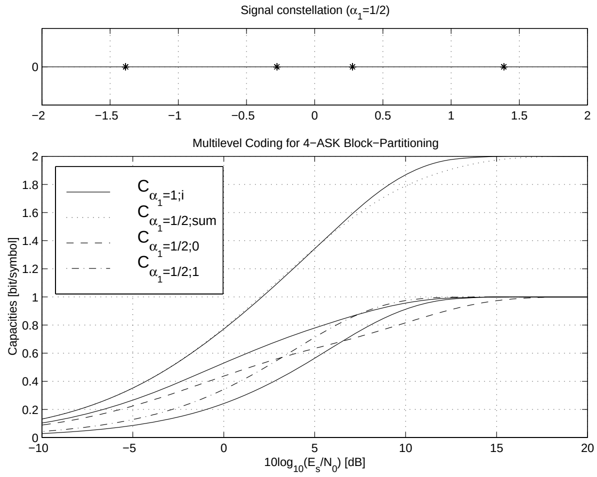

Figure 6 : Block partitioned 4-ASK with α_1=1/2

robust whereas the upper level requires increasingly higher signal strength. Therefore a tradeoff in performance between the two levels has to be made. Note that one has to keep in mind the differing strength of the effect on the two levels. Increasing α, level 0 becomes slowly more robust whereas level 1 is degrading rapidly. This can be demonstrated by choosing the following rates for the coding of the two levels : R_0 = ¼, R_1 = 7/8. Following the capacity design rule of [7] these rates can be asymptotically achieved where the capacities of the sub-levels have just this value. Table 1 gives the E_s/N_0 values required at minimum in order to get the desired capacities.

This simple example shows the great care that has to be taken in designing a hierarchical system. From the last column in table 1 it can be seen that the difference in performance of the two levels is not an appropriate measure, because its magnitude is mainly influenced by

4-ASK BP	level 0 C_0=1/4	level 1 C_1=7/8	difference
α_1=1/4	-4.3dB	7.4dB	11.7dB
α_1=1/2	-4.9dB	8.4dB	13.3dB
α_1=1	-5.3dB	9.3dB	14.6dB
α_1=2	-6.1dB	12.3dB	18.4dB
α_1=4	-6.5dB	16.4dB	22.9dB

Table 1 : E_s/N_o required to reached C_0=1/4, C_1=7/8

the large degradation of the upper level. This is true for all designs, where the lower level is operated at a relatively low rate due to good forward error correction, whereas the upper level has only a moderate degree of protection. Obviously this is the case one most often encounters in real systems.

The above results indicate the existence of an optimal value of α_1 for every combination of rates R_0 and R_1. Figure 7 gives the calculation of this optimal value for

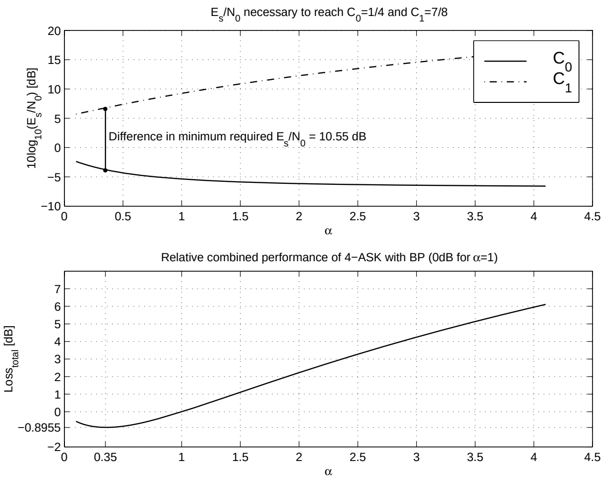

Figure 7 : Optimization of α_1 for $R_0 = 1/4$, $R_1 = 7/8$

the chosen example of $R_0 = 1/4$, $R_1 = 7/8$ and equation 2 being the evaluation criterion used.

From the lower plot in figure 7 the optimal value is found to be $\alpha_{1,opt}$=0.35. For this value of α_1 the "wake-up points" of the two levels of the MLC are separated by roughly 10.5dB. Any increase in the separation of the two levels has mainly to be paid for by a degradation of the upper level. Therefore higher separation can only be effectively achieved by increasing the amount of FEC on the lower level.

At the global optimum $\alpha_{1,opt}$ two opposing effects interact in the best possible way. Firstly a decrease in α_1 results in an increase of the minimum Euclidean distance of signal points within level 1 of the constellation. Secondly, the same decrease in α_1 leads to an increasing amount of demodulation errors being made between the two inner points of the constellation. As it was pointed put by [1], the latter is efficiently dealt with by the high degree of coding applied to level 0.

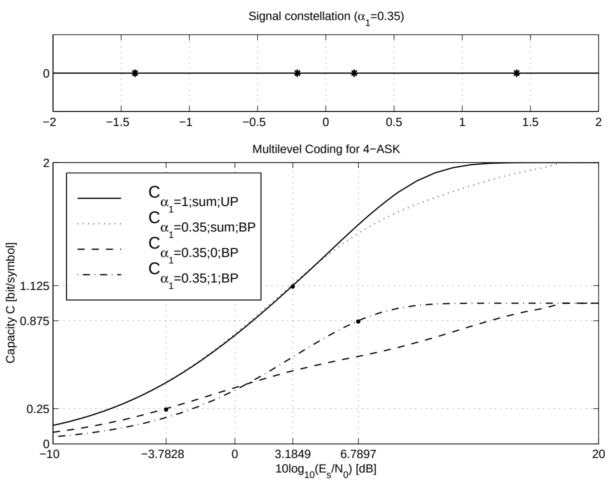

Figure 8 : Comparison of BP vs. UP for α_1=0.35, R_0=1/4, R_1=7/8

Having found the optimum constellation for given coding rates on the two levels, we have to compare this result with a MLC scheme using Ungerböck partitioning. Figure 8 draws a comparison between an equally spaced signal constellation for an UP and the optimum BP derived above. The UP scheme achieves the same overall capacity as the BP for a E_S/N_0 3.6dB lower but in return the BP scheme can transmit a reduced capacity of 0.25 at an E_S/N_0 approximately 7dB lower than the UP scheme. Hence it is up to the system designer to decide on a suitable trade-off.

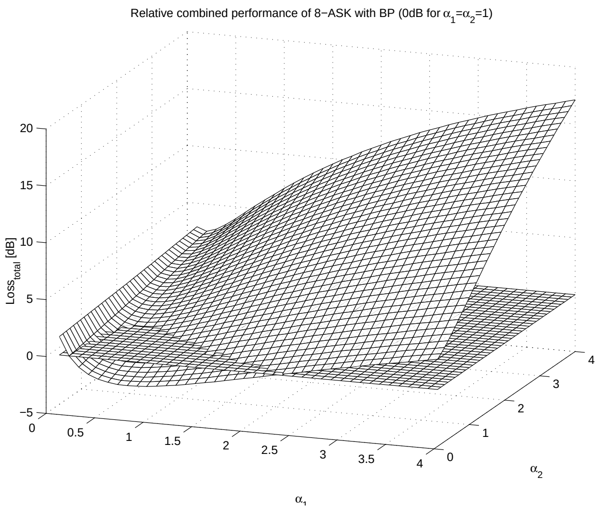

Figure 9 : Optimization of α_1 and α_2 for $R_0 = 1/4$, $R_1 = ½$; R_2=7/8

B. 8-ASK

Increasing the spectral efficiency of the modulation scheme by profiting from a 8-ASK instead of a 4-ASK signal constellation, we have two degrees of freedom, as shown in figure 2.

The loss of the non-uniform constellation with respect to the uniform constellation (α_1=α_2=1) in case of BP can be seen from figure 9. Note that the plane of zero loss is plotted for reference. Once again, it becomes evident that a clustering of the signal points around the decision thresholds of the two lower levels is beneficial and hence α_1 as well as α_2 should be chosen inferior to one. As figure 9 might suggest, the value for α_2 should be selected as small as possible, but one has to bear in mind that for decreasing α_2 the difference in performance of the two upper levels diminishes. Therefore, figure 10 gives an example of 8-ASK BP with α_1 and α_2 set to 0.7 and 0.5 respectively. Employing equation 2, the overall gain as compared to α_1=α_2=1 is 1dB. The different levels of the constellation span a range of minimum required SNR of 17.4dB. As for 4-ASK the large difference in required SNR for the above choice of the code rates is due to the high amount of FEC on the lowest level 0.

IV. QAM

Higher dimensional constellations, such as M-QAM e.g., are not separately investigated, as for block codes they can be efficiently replaced by parallel implementation of the one-dimensional ASK constellations. For a constant delay of the overall system due to channel coding, this yields the possibility of

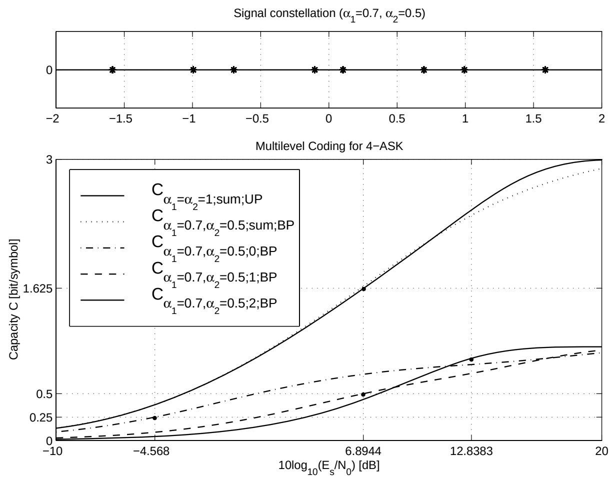

Figure 10 : Comparison of BP vs. UP for α_1=0.7, α_2=0.5,R_0 = 1/4, R_1 = ½, R_2=7/8

implementing longer block codes by multiplexing the individual bit streams of the one-dimensional constellations onto the same encoder and decoder respectively [7].

V. SUMMARY

In this paper we have examined the influence on performance of unequal spacing of signal points of a MLC-scheme with block partitioning. The attention was focused on the minimum SNR required to decode the individual sub-levels of such a graceful degrading scheme. It was found that for every specific choice of code rates an optimal constellation can be determined. For choices of code rates encountered in practise this leads to signal constellations which significantly differ from those non-uniform ones that are standardized in [4]. Furthermore, the optimal values derived for the asymptotic behaviour of the AWGN-channel can be used as inital values for a search by simulation of optima for other types of channels, the Rayleigh fading channel e.g.

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