Coded OFDM by Unique Word Prefix

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***Abstract*—In this paper, we propose a novel transmit signal structure for OFDM (orthogonal frequency division multiplexing). Instead of the conventional cyclic prefix (CP), we use a deterministic sequence, which we call unique word (UW), as guard interval. We show how unique words, which are already well investigated for single carrier systems with frequency domain equalization (SC/FDE), can also be introduced in OFDM symbols. Since unique words represent known sequences, they can advantageously be used for synchronization and channel estimation purposes. Furthermore, the proposed approach introduces a complex number Reed-Solomon (RS-) code structure within the sequence of subcarriers. This either allows for algebraic RS decoding or for applying a highly efficient Wiener smoother succeeding a zero forcing stage at the receiver to further improve the bit error ratio behavior of the system. These beneficial properties are achieved while additionally featuring around the same bandwidth efficiency as conventional CP-OFDM. We present simulation results in an indoor multipath environment to highlight the advantageous properties of the proposed scheme.**

I. Introduction

In conventional OFDM signaling, subsequent symbols are separated by guard intervals, which are usually implemented as cyclic prefixes (CPs) [1]. By this, the linear convolution of the signal with the channel impulse response is transformed into a cyclic convolution, which allows for a low complex equalization in frequency domain. In this paper, we propose to use known sequences, which we call unique words (UWs), instead of cyclic prefixes. The technique of using UWs has already been investigated in-depth for SC/FDE systems, where the introduction of unique words in time domain is straight forward [2], since the data symbols are also defined in time domain. In this work, we will show how unique words can be introduced in OFDM time domain symbols, even though the data QAM (quadrature amplitude modulation) symbols are defined in frequency domain. Furthermore, we will present two different receiver concepts adjusted to the novel transmit signal structure.

Fig. 1 compares the transmit data structure of CP- and UW-based transmission in time domain [3]. Both structures make sure that the linear convolution of an OFDM symbol with the impulse response of a dispersive (e.g. multipath) channel appears as a cyclic convolution at the receiver side. Nevertheless, there are also some fundamental differences between CP- and UW-based transmission:

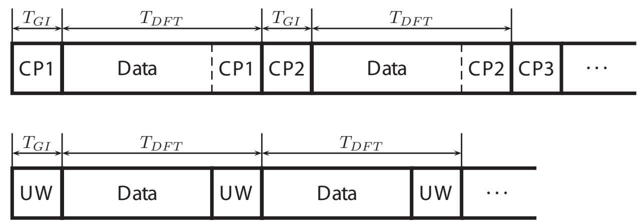

Fig. 1: Transmit data structure using CPs (above) or UWs (below).

- The UW is part of the DFT (discrete Fourier transform)-interval, whereas the CP is not. Due to that and in contrast to previous attempts of applying UW to OFDM [4], our UW-OFDM approach achieves an almost identical bandwidth efficiency as conventional CP-OFDM.
- The CP is random, whereas the UW is a known deterministic sequence. Hence, the UW can advantageously be utilized for synchronization [5] and channel estimation purposes [6].

Both statements hold for OFDM- as well as for SC/FDE-systems. However, in OFDM - different to SC/FDE - the introduction of UWs in time domain leads to another fundamental and beneficial signal property: A UW in time domain generates a word of a complex number RS (Reed Solomon)-code (cf. e. g. [7]) in the OFDM frequency domain symbol vector. Therefore, the UW could be exploited for algebraic error correction or (more appropriately) for erasure correction for highly attenuated subcarriers. However, as it turns out and as we will show in this paper, algebraic RS decoding leads to solving a very ill-conditioned system of equations and thus cannot achieve a reasonable solution, as soon as even only little noise is present in the system.

Another interpretation of the introduction of UWs in time domain is that it leads to correlations along the subcarriers. Therefore, a receiver based on a Bayesian estimation is obvious, too. We will show that a receiver based on a Bayesian estimation will in fact significantly improve the BER behavior by exploiting the covariance matrix of the subcarrier symbols.

The rest of the paper is organized as follows: In section II, we describe our approach of how to introduce unique words in OFDM symbols. Section III introduces an algebraic RS decoding and an LMMSE (linear minimum mean square

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error) receiver approach that exploits the a-priory knowledge introduced at the transmitter side. In section IV, the novel UW-OFDM concept is compared to the classical CP-OFDM by means of simulation results for both receiver concepts. For this, the IEEE 802.11a WLAN (wireless local area networks) standard serves as reference system.

Notation

Lower-case bold face variables ($\mathbf{a}, \mathbf{b}$,...) indicate vectors, and upper-case bold face variables ($\mathbf{A}, \mathbf{B}$,...) indicate matrices. To distinguish between time and frequency domain variables, we use a tilde to express frequency domain vectors and matrices ($\tilde{\mathbf{a}}, \tilde{\mathbf{A}}$,...), respectively. We further use $\mathbb{C}$ to denote the set of complex numbers, $\mathbf{I}$ to denote the identity matrix, $(\cdot)^T$ to denote transposition, $(\cdot)^H$ to denote conjugate transposition, $(\cdot)^\dagger$ to denote the Pseudo-Inverse, and $E[\cdot]$ to denote expectation.

II. Generation of Unique Words in OFDM Symbols

In conventional CP-OFDM, the data vector $\tilde{\mathbf{d}} \in \mathbb{C}^{N_d \times 1}$ is defined in frequency domain. Typically, zero subcarriers are inserted at the band edges and at the DC subcarrier position, which can mathematically be described by a matrix operation $\tilde{\mathbf{x}} = \mathbf{B}\tilde{\mathbf{d}}$ with $\tilde{\mathbf{x}} \in \mathbb{C}^{N \times 1}$ and $\mathbf{B} \in \mathbb{C}^{N \times N_d}$. $\mathbf{B}$ consists of zero-rows at the positions of the zero subcarriers, and of appropriate unit row vectors at the positions of data subcarriers. The vector $\tilde{\mathbf{x}}$ denotes the OFDM symbol in frequency domain. The vector of time domain samples $\mathbf{x} \in \mathbb{C}^{N \times 1}$ is calculated via an IDFT (inverse DFT) operation, which can conveniently be formulated in matrix notation by $\mathbf{x} = \mathbf{F}_N^{-1}\tilde{\mathbf{x}}$. Here, $\mathbf{F}_N$ is the N-point-DFT matrix defined by $\mathbf{F}_N = (f_{mn})$ with $f_{mn} = w^{mn}$ for $m = 0, 1, ..., N-1$, $n = 0, 1, ..., N-1$, and with $w = \mathrm{e}^{-j2\pi/N}$.

We now modify this conventional approach by introducing a pre-defined sequence $\mathbf{x}_u$, which we call unique word, and which shall form the tail of the time domain vector $\mathbf{x}$. Hence, $\mathbf{x}$ consists of two parts and is given by $\mathbf{x} = \begin{bmatrix}\mathbf{x}_d^T & \mathbf{x}_u^T\end{bmatrix}^T$, where $\mathbf{x}_d \in \mathbb{C}^{(N-N_u)\times 1}$ and $\mathbf{x}_u \in \mathbb{C}^{N_u \times 1}$. The vector $\mathbf{x}_u$ represents the UW of length N_u, and thus only $\mathbf{x}_d$ is random and affected by the data. In the following, but w.l.o.g., we will generate a zero UW $\mathbf{x} = \begin{bmatrix}\mathbf{x}_d^T & \mathbf{0}^T\end{bmatrix}^T$ such that $\mathbf{x} = \mathbf{F}_N^{-1}\tilde{\mathbf{x}}$ (note that our approach of introducing the zero word as guard interval is fundamentally different to zero padded (ZP) OFDM, as the zero word is now part of the DFT interval). This linear system of equations can only be fulfilled by reducing the number N_d of data subcarriers, and by introducing a set of redundant subcarriers instead. We let the redundant subcarriers form the vector $\tilde{\mathbf{r}} \in \mathbb{C}^{N_u \times 1}$, further introduce a permutation matrix $\mathbf{P} \in \mathbb{C}^{(N_d+N_u)\times(N_d+N_u)}$, and form an OFDM symbol (containing $N - N_d - N_u$ zero subcarriers) in frequency domain by

$$\tilde{\mathbf{x}} = \mathbf{B}\mathbf{P}\begin{bmatrix}\tilde{\mathbf{d}} \\ \tilde{\mathbf{r}}\end{bmatrix}. \tag{1}$$

$\mathbf{B}$ again inserts the zero subcarriers, but has now the dimensions $\mathbf{B} \in \mathbb{C}^{N \times (N_d+N_u)}$. We will detail the reason for the introduction of the permutation matrix and its specific construction shortly below.

The relation between the time and the frequency domain representation of the OFDM symbol can now be written as

$$\mathbf{F}_N^{-1}\mathbf{B}\mathbf{P}\begin{bmatrix}\tilde{\mathbf{d}} \\ \tilde{\mathbf{r}}\end{bmatrix} = \begin{bmatrix}\mathbf{x}_d \\ \mathbf{0}\end{bmatrix}. \tag{2}$$

With

$$\mathbf{M} = \mathbf{F}_N^{-1}\mathbf{B}\mathbf{P} = \begin{bmatrix}\mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22}\end{bmatrix}, \tag{3}$$

where $\mathbf{M}_{ij}$ are appropriate sized sub-matrices, it follows that $\mathbf{M}_{21}\tilde{\mathbf{d}} + \mathbf{M}_{22}\tilde{\mathbf{r}} = \mathbf{0}$, and hence $\tilde{\mathbf{r}} = -\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\tilde{\mathbf{d}}$. With the matrix $\mathbf{T} = -\mathbf{M}_{22}^{-1}\mathbf{M}_{21}$ ($\mathbf{T} \in \mathbb{C}^{N_u \times N_d}$), the vector of redundant subcarriers can thus be determined by the linear mapping

$$\tilde{\mathbf{r}} = \mathbf{T}\tilde{\mathbf{d}}, \tag{4}$$

which corresponds to a complex number RS-code construction along the subcarrier symbols (i.e. N_u subsequent zeros in the transform domain). Alternatively, (4) can be interpreted as introducing correlations in the vector $\tilde{\mathbf{x}}$ of frequency domain samples of an OFDM symbol.

We notice that the construction of $\mathbf{T}$, and thus also the variances of the redundant subcarriers, highly depend on the positions of the redundant subcarriers within the entire frequency domain vector $\tilde{\mathbf{x}}$. Hence, the permutation matrix $\mathbf{P}$ has to be chosen carefully. Simple trials show that the energy on the redundant subcarriers varies significantly with $\mathbf{P}$. The choice $\mathbf{P} = \mathbf{I}$ e.g. results in extremely high energy values. We thus select $\mathbf{P}$ such that $\mathrm{trace}\left(\mathbf{T}\mathbf{T}^H\right)$ becomes minimum [3]. One can show that this provides minimum energy on the redundant subcarriers on average (when averaging over all possible data vectors $\tilde{\mathbf{d}}$). In section IV, we will specify the permutation matrix $\mathbf{P}$ for our simulated system setup.

In the following, we use the notation $\tilde{\mathbf{c}}$ with

$$\tilde{\mathbf{c}} = \mathbf{P}\begin{bmatrix}\tilde{\mathbf{d}} \\ \tilde{\mathbf{r}}\end{bmatrix} = \mathbf{P}\begin{bmatrix}\mathbf{I} \\ \mathbf{T}\end{bmatrix}\tilde{\mathbf{d}} = \mathbf{G}\tilde{\mathbf{d}}, \tag{5}$$

($\tilde{\mathbf{c}} \in \mathbb{C}^{(N_d+N_u)\times 1}, \mathbf{G} \in \mathbb{C}^{(N_d+N_u)\times N_d}$) for the non-zero part of $\tilde{\mathbf{x}}$, such that $\tilde{\mathbf{x}} = \mathbf{B}\tilde{\mathbf{c}}$. From (5), we immediatley notice that an OFDM symbol based on our novel UW-OFDM approach can be interpreted as a systematic code in the frequency domain generated by the code generator matrix $\mathbf{G}$. This is illustrated in detail in Fig. 2. Based on the input data vector $\tilde{\mathbf{d}}$, the redundant subcarrier symbols $\tilde{\mathbf{r}}$ are generated by applying $\mathbf{T}$, and after a permutation by $\mathbf{P}$, we result at the desired code word $\tilde{\mathbf{c}}$.

III. Receiver

After the transmission over a multipath channel and after the common DFT operation, the non-zero part $\tilde{\mathbf{y}} \in \mathbb{C}^{(N_d+N_u)\times 1}$ of a received OFDM frequency domain symbol can be modeled as

$$\tilde{\mathbf{y}} = \mathbf{B}^T\mathbf{F}_N\mathbf{H}\mathbf{F}_N^{-1}\mathbf{B}\tilde{\mathbf{c}} + \mathbf{B}^T\mathbf{F}_N\mathbf{n}, \tag{6}$$

where $\mathbf{H}$ denotes a cyclic convolution matrix with $\mathbf{H} \in \mathbb{C}^{N \times N}$, and $\mathbf{n} \in \mathbb{C}^{N \times 1}$ represents a noise vector with the covariance matrix $\sigma_n^2\mathbf{I}$. The multiplication with $\mathbf{B}^T$ excludes the zero subcarriers from further operation. The matrix $\mathbf{F}_N\mathbf{H}\mathbf{F}_N^{-1}$

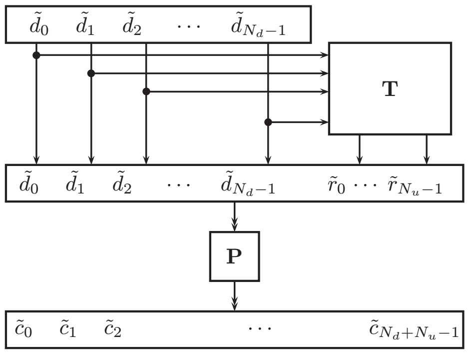

Fig. 2: Code word generator

is diagonal and contains the sampled channel frequency response on its main diagonal. $\tilde{\mathbf{H}} = \mathbf{B}^T\mathbf{F}_N\mathbf{H}\mathbf{F}_N^{-1}\mathbf{B}$ with $\tilde{\mathbf{H}} \in \mathbb{C}^{(N_d+N_u)\times(N_d+N_u)}$ is a down-sized version of the latter excluding the entries corresponding to the zero subcarriers. The received symbol can therefore also be written as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{c}} + \mathbf{B}^T\mathbf{F}_N\mathbf{n}. \tag{7}$$

As usual for conventional OFDM, we propose to apply a zero forcing (ZF) equalization by multiplying with $\tilde{\mathbf{H}}^{-1}$ from the left at first. This results in

$$\tilde{\mathbf{y}}^{'} = \tilde{\mathbf{H}}^{-1}\tilde{\mathbf{y}} = \tilde{\mathbf{c}} + \tilde{\mathbf{v}} \tag{8}$$

with the noise vector $\tilde{\mathbf{v}} = \tilde{\mathbf{H}}^{-1}\mathbf{B}^T\mathbf{F}_N\mathbf{n}$. In the following, we will present two different receiver strategies, whereas the first one uses algebraic RS decoding and the latter a Bayesian approach to decode (8).

A. Algebraic RS Decoding based UW-OFDM Receiver

In case of algebraic RS decoding, we apply the same principle as at the transmitter side to generate the redundant subcarriers $\tilde{\mathbf{r}}$, but now we aim at recovering highly attenuated subcarrier symbols $\tilde{\mathbf{c}}_b \in \mathbb{C}^{(m\times1)}$ from well received subcarrier symbols $\tilde{\mathbf{c}}_g \in \mathbb{C}^{(N_d+N_u-m)\times1}$ ($m \le N_u$). Let $\mathbf{P}^{'} \in \mathbb{C}^{(N_d+N_u)\times(N_d+N_u)}$ be a permutation matrix that splits up $\tilde{\mathbf{c}}$ into "good" and "bad" subcarrier symbols such that

$$\tilde{\mathbf{c}}^{'} = \begin{bmatrix}\tilde{\mathbf{c}}_g\\ \tilde{\mathbf{c}}_b\end{bmatrix} = \mathbf{P}^{'}\tilde{\mathbf{c}}, \tag{9}$$

and let

$$\mathbf{M}^{'} = \mathbf{F}_N^{-1}\mathbf{B}\mathbf{P}\mathbf{P}^{'-1} = \begin{bmatrix}\mathbf{M}_{11}^{'} & \mathbf{M}_{12}^{'}\\ \mathbf{M}_{21}^{'} & \mathbf{M}_{22}^{'}\end{bmatrix}, \tag{10}$$

where $\mathbf{M}_{ij}^{'}$ are appropriate sized sub-matrices. It follows that $\mathbf{M}_{21}^{'}\tilde{\mathbf{c}}_g + \mathbf{M}_{22}^{'}\tilde{\mathbf{c}}_b = \mathbf{0}$ (note that we applied a zero UW $\mathbf{0}$), and hence $\tilde{\mathbf{c}}_b = -\mathbf{M}_{22}^{'\dagger}\mathbf{M}_{21}^{'}\tilde{\mathbf{c}}_g$. With $\mathbf{T}^{'} = -\mathbf{M}_{22}^{'\dagger}\mathbf{M}_{21}^{'}$, we obtain

$$\tilde{\mathbf{c}} = \mathbf{P}^{'-1}\begin{bmatrix}\tilde{\mathbf{c}}_g\\ \tilde{\mathbf{c}}_b\end{bmatrix} = \mathbf{P}^{'-1}\begin{bmatrix}\mathbf{I}\\ \mathbf{T}^{'}\end{bmatrix}\tilde{\mathbf{c}}_g = \mathbf{G}^{'}\tilde{\mathbf{c}}_g. \tag{11}$$

With $\mathbf{P}^{'} = \begin{bmatrix}\mathbf{P}_1^{'T} & \mathbf{P}_2^{'T}\end{bmatrix}^T$ and $\mathbf{P}_1^{'} \in \mathbb{C}^{(N_d+N_u-m)\times(N_d+N_u)}$, we finally obtain the estimator

$$\widehat{\tilde{\mathbf{c}}} = \mathbf{G}^{'}\mathbf{P}_1^{'}\tilde{\mathbf{H}}^{-1}\tilde{\mathbf{y}}. \tag{12}$$

Note that since we have a zero UW of length N_u in the time domain, we are able to recover up to N_u erased subcarrier symbols. In case of $m = N_u$, this system of equations has a uniquely defined solution (i.e. the Pseudo-Inverse turns into an Inverse). In case of $m < N_u$, this leads to an overdetermined system of equations which may be preferably solved in a least square sense. Finally, the data part $\widehat{\tilde{\mathbf{d}}} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{P}^{-1}\widehat{\tilde{\mathbf{c}}}$ can be processed further as usual. One can show that the error $\tilde{\mathbf{e}} = \tilde{\mathbf{c}} - \widehat{\tilde{\mathbf{c}}}$ has zero mean, and its covariance matrix is given by $\mathbf{C}_{\tilde{e}\tilde{e}} = \sigma_d^2\left(\mathbf{C}_{\tilde{c}\tilde{c}} - \mathbf{G}^{'}\mathbf{P}_1^{'}\mathbf{C}_{\tilde{c}\tilde{c}} - \mathbf{C}_{\tilde{c}\tilde{c}}(\mathbf{G}^{'}\mathbf{P}_1^{'})^H + \mathbf{G}^{'}\mathbf{P}_1^{'}\mathbf{C}_{\tilde{c}\tilde{c}}(\mathbf{G}^{'}\mathbf{P}_1^{'})^H\right) + \mathbf{G}^{'}\mathbf{P}_1^{'}\mathbf{C}_{\tilde{v}\tilde{v}}(\mathbf{P}_1^{'})^H$. $\mathbf{C}_{\tilde{e}\tilde{e}}$ can further be used in the case when additional channel coding is applied. Especially, varying noise variances along the subcarriers within the data vector $\tilde{\mathbf{d}}$ may be exploited as well known from coded transmission over time variant channels, cf. e.g. [8].

B. LMMSE UW-OFDM Receiver

In the Bayesian approach, we exploit the fact that the redundant subcarrier symbols have been calculated out of the data symbols by (4), and thus are correlated with the data symbols and among each other. Because of that we propose to apply an LMMSE Wiener smoother [9] on $\tilde{\mathbf{y}}^{'}$, which results in the noise reduced estimate

$$\widehat{\tilde{\mathbf{c}}} = \tilde{\mathbf{W}}\tilde{\mathbf{H}}^{-1}\tilde{\mathbf{y}} \tag{13}$$

with the Wiener smooting matrix defined as

$$\tilde{\mathbf{W}} = \mathbf{C}_{\tilde{c}\tilde{c}}\left(\mathbf{C}_{\tilde{c}\tilde{c}} + \mathbf{C}_{\tilde{v}\tilde{v}}\right)^{-1}, \tag{14}$$

where $\mathbf{C}_{\tilde{c}\tilde{c}}, \mathbf{C}_{\tilde{v}\tilde{v}} \in \mathbb{C}^{(N_d+N_u)\times(N_d+N_u)}$ denote the covariance matrices of $\tilde{\mathbf{c}}$ and $\tilde{\mathbf{v}}$, respectively. If we assume uncorrelated and zero-mean data QAM symbols with variance σ_d^2, and uncorrelated and zero-mean noise with variance σ_n^2, these covariance matrices are given as follows:

$$\mathbf{C}_{\tilde{c}\tilde{c}} = E\left[\tilde{\mathbf{c}}\tilde{\mathbf{c}}^H\right] = \mathbf{G}E\left[\tilde{\mathbf{d}}\tilde{\mathbf{d}}^H\right]\mathbf{G}^H = \sigma_d^2\mathbf{G}\mathbf{G}^H, \tag{15}$$

$$\mathbf{C}_{\tilde{v}\tilde{v}} = E\left[\tilde{\mathbf{v}}\tilde{\mathbf{v}}^H\right] = N\sigma_n^2\tilde{\mathbf{H}}^{-1}(\tilde{\mathbf{H}}^{-1})^H. \tag{16}$$

Similar to the algebraic RS decoding approach, the data part can be extracted by $\widehat{\tilde{\mathbf{d}}} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{P}^{-1}\widehat{\tilde{\mathbf{c}}}$. The error $\tilde{\mathbf{e}} = \tilde{\mathbf{c}} - \widehat{\tilde{\mathbf{c}}}$ has zero mean, and its covariance matrix is given by $\mathbf{C}_{\tilde{e}\tilde{e}} = \left(\mathbf{I} - \tilde{\mathbf{W}}\right)\mathbf{C}_{\tilde{c}\tilde{c}}$ [9].

IV. Simulation Results

Fig. 3 shows the block diagram of the simulated UW-OFDM system (equivalent complex baseband description is used throughout this paper). After outer channel coding, interleaving and QAM-mapping, the redundant subcarrier symbols are determined using (4). After assembling the OFDM symbol, which is composed of $\tilde{\mathbf{d}}$, $\tilde{\mathbf{r}}$, and a set of zero subcarriers, the IFFT (inverse fast Fourier transform) is performed. For the sake of simplicity, we omit any kind of specific spectral masking. At the receiver, the FFT (fast Fourier transform) operation is followed by a ZF equalization as in classical CP-OFDM. Then, either the Wiener smoother or our algebraic RS decoder is applied to the OFDM symbol, depending on the specific receiver concept. Finally, demapping, deinterleaving and decoding is performed. For the soft decision Viterbi decoder, the main diagonal of matrix $\mathbf{C}_{\tilde{e}\tilde{e}}$ is used to specify the varying noise variances along the subcarriers after equalization and Wiener filtering or algebraic RS decoding, respectively.

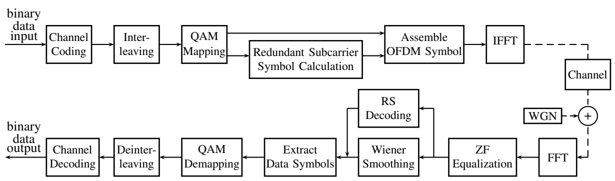

Fig. 3: Block diagram for simulation analysis.

We compare our novel UW-OFDM approach with the classical CP-OFDM concept. The IEEE 802.11a WLAN standard [10] serves as reference system. We apply the same parameters for UW-OFDM as in [10] wherever possible: $N = 64$, sampling frequency $f_s = 20$MHz, DFT period $T_{DFT} = 3.2\mu$s, guard duration $T_{GI} = 800$ns. Instead of 48 data subcarriers and 4 pilots, we use $N_d = 36$ data subcarriers and $N_u = 16$ redundant subcarriers. The zero subcarriers are chosen as in [10], the indices of the redundant subcarriers are chosen to be {2, 6, 10, 14, 17, 21, 24, 26, 38, 40, 43, 47, 50, 54, 58, 62}. This choice, which can easily also be described by (1) with an appropriately constructed matrix $\mathbf{P}$, has been found by heuristic optimization methods and minimizes the total energy of the redundant subcarriers on average (when averaging over all possible data vectors $\tilde{\mathbf{d}}$) [3].

Note that in conventional CP-OFDM like in the WLAN standard, the total length of an OFDM symbol is given by $T_{GI} + T_{DFT}$. However, the guard interval is part of the DFT period in our approach. Therefore, both systems show almost the same bandwidth efficiency.

The multipath channel has been modeled as a tapped delay line, each tap with uniformly distributed phase and Rayleigh distributed magnitude, and with power decaying exponentially. A detailed description of the model can be found in [6]. Fig. 4 shows one typical channel snapshot featuring an rms delay spread of 100ns. The frequency response shows two spectral notches within the system's bandwidth.

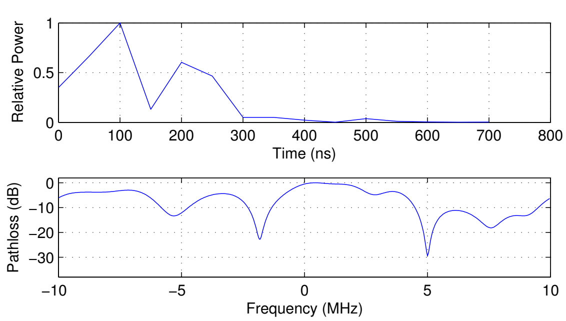

Fig. 4: Time- and frequency-domain representation of the used multipath channel snapshot.

In order to clearly demonstrate the effect of our UW-OFDM approach, the following discussions are based on results obtained for the displayed channel snapshot. Although not shown in this paper, similar results have been achieved for various other channel snapshots as well. In case of algebraic RS decoding, we assume that all subcarrier symbols have been well received except for those at the subcarriers with index 15 and 46, respectively. Hence, these two subcarrier symbols will be recalculated from the well received ones like described in III-A. Fig. 5 shows the MSE before and after algebraic RS decoding with and without slicing. In case slicing is applied, we assume perfect knowledge of the data subcarrier symbols and "slice" the received symbols to the appropriate complex constellation points, i.e. we replace the noisy symbols by the actual transmitted symbols. For the redundant subcarriers, slicing is assumed to be practically not feasible due to the huge amount of possible constellation points. We thus stick to the noisy symbols after the ZF stage. In case no slicing is applied, we use for all subcarrier symbols the values after the ZF stage. We notice that our algebraic RS decoding approach is able to slightly improve the MSE on subcarrier 15. Interestingly, with perfect slicing the MSE cannot be reduced significantly anymore. As such, the noise on the redundant subcarriers dominates the performance of our algebraic RS decoding receiver. Moreover, our algebraic RS decoding approach will even increase the MSE in case of subcarrier 46. In summary, algebraic RS decoding leads to solving a very ill-conditioned system of equations which cannot be solved adequately anymore as soon as only little noise (note that $E_b/N_0 = 20$dB for our simulations) is present. Unfortunately, applying common regularization techniques like e.g. the Tikhonov regularization also showed no success in stabilizing this system. Hence, this receiver concept is not applicable for practical systems.

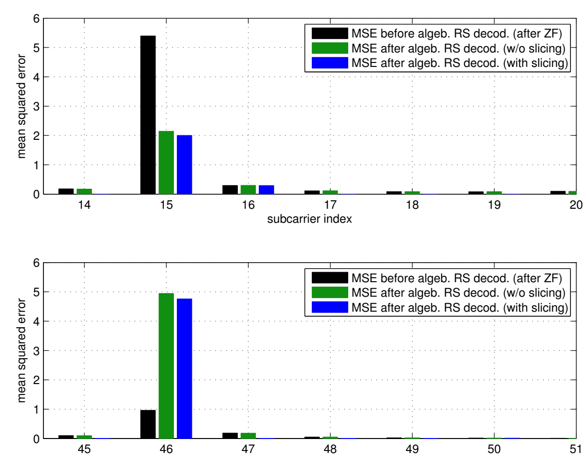

Fig. 5: Noise reduction/enhancement effect of the algebraic RS decoder in a frequency selective environment for $E_b/N_0 = 20$dB. Above: zoomed y-axis around carrier 15; below: zoomed y-axis around carrier 46.

However, we will show in the following, that UW-OFDM in combination with an LMMSE receiver performs superior. Fig. 6 compares the mean squared errors (MSE) on the $N_d + N_u$ (data + redundant) subcarriers before and after the Wiener smoothing operation. We note that all subcarriers experience a significant noise reduction by the smoother, but the effect is impressive on the subcarriers corresponding to spectral notches in the channel frequency response. The subcarriers with index 15 and 46 correspond to the spectral notches around 5MHz and -2MHz, respectively, cf. Fig. 4.

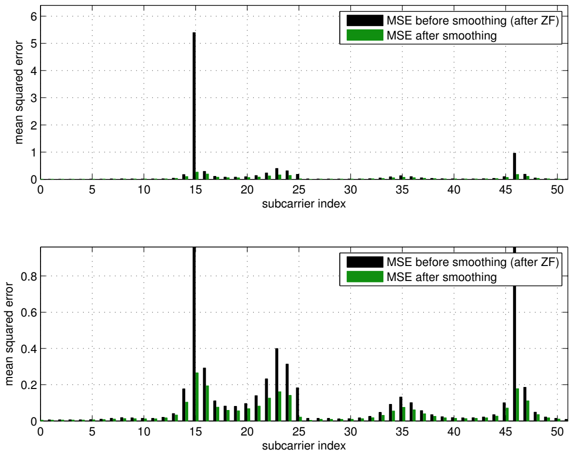

Fig. 6: Noise reduction effect of the Wiener smoother in a frequency selective environment for E_b/N_0 = 20dB. Above: full scale; below: zoomed y-axis.

In Fig. 7 the BER behavior of the IEEE 802.11a standard and the novel UW-OFDM approach is compared, both in QPSK-mode for the channel displayed in Fig. 4. The channel snapshot represents a typical indoor NLOS (non line of sight) office environment. Here, we show results of simulations with and without the usage of an additional outer code. The outer code features the coding rates $r = 3/4$ and $r = 1/2$, respectively. Both systems use the same convolutional coder with the industry standard rate 1/2, constraint length 7 code with generator polynomials (133,171). For $r = 3/4$ puncturing is used as described in [10]. Note that due to the different number of data symbols per OFDM symbol, the interleaver had to be slightly adapted compared to the WLAN standard. Perfect channel knowlegde is assumed in both approaches. We notice that UW-OFDM with algebraic RS decoding (without slicing) always performs worse than IEEE802.11a. It loses about 0.9dB and 0.5dB in case of $r = 1/2$ and $r = 3/4$, respectively. Moreover, when no further outer code is used, i.e. $r = 1$, this receiver even shows a utterly devastating BER behavior. However, UW-OFDM performs superior when a receiver based on the Bayesian estimation is applied. In the case of no further outer code, the gain achieved by the LMMSE smoother is impressive. This can be explained by the significant noise reduction on heavily attenuated subcarriers. For the coding rates $r = 3/4$ and $r = 1/2$, the novel UW-OFDM approach still achieves a gain of 1dB and 0.8dB at a bit error ratio of 10^{-6}, respectively.

V. Conclusion

In this work we introduced a novel OFDM signaling concept, where the guard intervals are built by unique words instead of cyclic prefixes. The proposed approach introduces

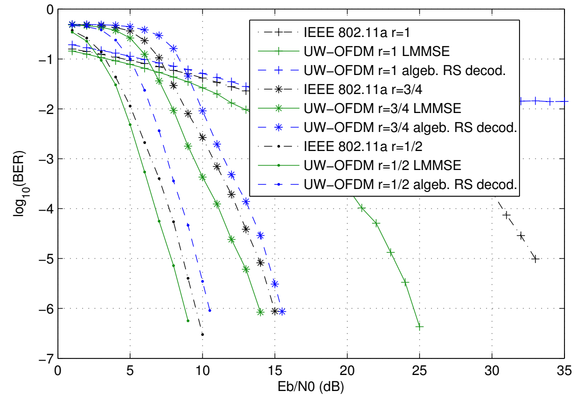

Fig. 7: BER comparison between the novel UW-OFDM approach and the IEEE 802.11a standard for the channel snapshot displayed above.

a complex number Reed-Solomon code structure within the sequence of subcarriers. As an important conclusion we can state, that besides the possibility to use the UW for synchronization and channel estimation purposes (of course for that, a UW different from the zero word needs to be chosen), the novel approach additionally allows to apply a highly efficient LMMSE Wiener smoother, which significantly reduces the noise on the subcarriers, especially on highly attenuated subcarriers. Simulation results illustrate that the novel approach outperforms classical CP-OFDM in a typical frequency selective indoor scenario. Furthermore, our novel approach of introducing UWs provides these benefits over conventional CP-OFDM while still keeping almost the same bandwidth efficiency.

References

[1] R. van Nee, R. Prasad, *OFDM for Wireless Multimedia Communications*, Artech House Publishers, Boston, 2000.

[2] H. Witschnig, T. Mayer, A. Springer, A. Koppler, L. Maurer, M. Huemer,R. Weigel, "A Different Look on Cyclic Prefix for SC/FDE" In the *Proceedings of the 13th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC 2002)*, Lisbon, Portugal, pp. 824-828, September 2002.

[3] M. Huemer, C. Hofbauer, J.B. Huber, "The Potential of Unique Words in OFDM", In the *Proceedings of the 15th International OFDM-Workshop (InOWo' 10)*, Hamburg, Germany, pp. 140 - 144, September 2010.

[4] L. Jingyi, P. Joo, J. Ro, "The effect of filling Unique Words to guard interval for OFDM", Document IEEE C802.16a-02/87, IEEE 802.16 Broadband Wireless Access Working Group, September 2002.

[5] M. Huemer, H. Witschnig, J. Hausner, "Unique Word Based Phase Tracking Algorithms for SC/FDE Systems", In the *Proceedings of the IEEE International Conference on Global Communications (GLOBECOM' 2003)*, San Francisco, USA, 5 pages, December 2003.

[6] H. Witschnig, Frequency Domain Equalization for Broadband Wireless Communication - With Special Reference to Single Carrier Transmission Based on Known Pilot Sequences, Dissertation, University of Linz, 2003.

[7] G.R. Redinbo, "Decoding Real Block Codes: Activity Detection, Wiener Estimation", In the *IEEE Transactions on Information Theory*, Vol. 46, No. 2, pp. 609-623, March 2000.

[8] J. B. Huber, "Untersuchung von Blockcodierverfahren bei gedaechtnisbehafteten Kanaelen", In *ntz-Archiv*, Bd. 6, pp. 233-241, 1984.

[9] S. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice Hall, Rhode Island 1993.

[10] IEEE Std 802.11a-1999, Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications: High-Speed Physical Layer in the 5 GHz Band, 1999.