Successive PAR Reduction in (MIMO) OFDM

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***Abstract*— A successive scheme for PAR reduction in (MIMO/ SISO) OFDM is presented, where K (parallel/consecutive) OFDM frames are treated jointly. Employing Reed–Solomon codes further candidate OFDM frames are generated and assessed successively; the currently best K are selected for possible transmission. The procedure stops if (i) all K best frames stay below a given tolerable PAR limit, or (ii) the maximally allowed number of candidates is exhausted. Thereby complexity compared to other PAR reduction schemes can be reduced significantly. Analytical derivations show that for PAR limits in the region of the "critical PAR" value $\xi_{\rm crit} = \log(D)$, with D being the number of carriers, the average number candidates is close to Euler's number $\mathrm{e} = 2.71828\ldots$, which is particularly low.**

I. Introduction

Algorithms for reducing the *peak-to-average power ratio (PAR)* in *orthogonal frequency-division multiplexing (OFDM)* schemes are widely discussed in literature, e.g., [8], [10]. Using PAR reduction schemes, the demand on the linearity of the power amplifiers can be relaxed.

Recently, a combination of the multiple representation/ selection principle—as is present in *selected mapping (SLM)*—and coding via general purpose channel codes and has been proposed in [7]. Thereby a number of, say K, OFDM frames—most prominently the parallel signals at an antenna array, but consecutive OFDM frames over time are also possible—are processed jointly. Reed–Solomon codes are used to generate further candidate OFDM frames, from which the K best are selected and transmitted. This scheme is very flexible, shows very good performance but has only moderate complexity.

In this paper, we extend the coding and selection scheme given in [7] by performing the encoding and candidate assessment successively. The procedure stops if a predefined PAR limit is met or if the maximum allowed number of candidates is reached. The main advantage is that complexity, which is mainly given by the calculation of the *inverse discrete Fourier transform ((I)DFT)* and the PAR, is reduced significantly. The average number of tested candidates and the achieved *complementary cumulative distribution function (ccdf)*, the main performance criterion, are derived anylytically.

This organization of the paper is as follows: In Section II the OFDM system model and the performance measure are defined. Moreover, the Reed–Solomon coding scheme for PAR reduction is briefly reviewed. Successive candidate generation and selection is presented in Section III. A motivation based on the derivation of a "critical PAR" and an in-depth analytical performance analysis are given. Finally, Section IV presents numerical simulation results supporting the analytical derivations and some conclusions are given.

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II. System Model, Performance Measure, and PAR Reduction Based on Reed–Solomon Codes

Throughout the paper a standard discrete-time OFDM system model based on an (I)DFT of length D [3], [13] is assumed; all carriers are expected to be active. I.i.d. binary data (vector $\boldsymbol{Q} = [Q_0, \ldots, Q_{\mu D-1}]$) is demultiplexed and mapped (mapping $\mathcal{M}$) to complex-valued frequency-domain zero-mean ($M = 2^\mu$)-ary QAM or PSK data symbols A_d, $d = 0, \ldots, D-1$, with variance $\sigma_a^2 = \mathrm{E}\{|A_d|^2\}$. These symbols are combined into the frequency-domain OFDM frame, denoted by $\boldsymbol{A} = [A_0, \ldots, A_{D-1}]$ and with $\boldsymbol{A} = \mathcal{M}\{\boldsymbol{Q}\}$. The frequency-domain frame is transformed into the time-domain OFDM frame $\boldsymbol{a} = [a_0, \ldots, a_{DI-1}] = \mathrm{IDFT}_I\{\boldsymbol{A}\}$ via I-times oversampled IDFT, i.e.,

$$a_k = \frac{1}{\sqrt{D}} \sum_{d=0}^{D-1} A_d \cdot \mathrm{e}^{\mathrm{j}2\pi kd/(DI)} \;, \quad k = 0, \ldots, DI-1 \;. \tag{1}$$

Already for moderate values of I ($I \approx 4$) the samples a_k very closely reflect the continuous-time transmit signal [15].

Due to the superposition of D statistically independent terms within the IDFT, the time-domain samples a_k tend to be Gaussian and hence exhibit a high *peak-to-average power ratio (PAR)*

$$\xi \overset{\text{def}}{=} \max_{k=0,\ldots,DI-1} |a_k|^2/\sigma_a^2 \;. \tag{2}$$

As common in literature, we study the probability that the PAR of an OFDM frame exceeds a certain threshold $\xi_{\rm th}$, hence we consider the complementary cumulative distribution function (ccdf)

$$\mathrm{C}(\xi_{\rm th}) \overset{\text{def}}{=} \Pr\{\xi > \xi_{\rm th}\} \;. \tag{3}$$

In case of multi-antenna transmission where K signals are transmitted in parallel, the *worst-case PAR* is a reasonable performance criterion. Having K OFDM frames $\boldsymbol{A}_\kappa$ with their respective PAR ξ_κ, $\kappa = 1, \ldots, K$, we assess

$$\mathrm{C}_K(\xi_{\rm th}) \overset{\text{def}}{=} \Pr\{\max_{\kappa=1,\ldots,K} \xi_\kappa > \xi_{\rm th}\} \;. \tag{4}$$

Although not exactly true in practice, cf. [14], [9], for the derivation of analytical results it is very convenient to consider the time-domain signal to be complex Gaussian distributed. For Gaussian samples at Nyquist rate ($I = 1$), the ccdf of the original OFDM signal is simply given by [2]

$$\mathrm{C}_{\rm o}^{(\rm G)}(\xi_{\rm th}) \overset{\text{Gauss}}{=} 1 - (1 - \mathrm{e}^{-\xi_{\rm th}})^D \;. \tag{5}$$

The ccdf of PAR of multi-antenna transmission is related to that of single-antenna transmission by $\mathrm{C}_{\mathrm{o},K}(\xi_{\mathrm{th}}) = 1 - (1 - \mathrm{C}_{\mathrm{o}}(\xi_{\mathrm{th}}))^K$, which for Gaussian samples simplifies to [1]

$$\mathrm{C}^{(\mathrm{G})}_{\mathrm{o},K}(\xi_{\mathrm{th}}) \overset{\text{Gauss}}{=} 1 - (1 - \mathrm{e}^{-\xi_{\mathrm{th}}})^{KD} \ . \tag{6}$$

In the PAR reduction scheme proposed in [7] K OFDM frames[1] are treated jointly. As in channel coding via (linear) block codes, given the K initial OFDM frames a collection of n_{cand} frames is calculated. Out of these n_{cand} candidates, a number of (at least) K exhibiting the lowest PAR—any other optimization criterion is possible at this step—are selected for actual transmission.

In particular, a Reed–Solomon (RS) code over the Galois field $\mathbb{F}_{2^m}$ with (maximum) code length $n_{\mathrm{RS}} = 2^m - 1$ ($n_{\mathrm{cand}} \leq n_{\mathrm{RS}}$) and dimension equal to K is employed [4]. The binary vectors $\boldsymbol{Q}_\kappa$, $\kappa = 1, \ldots, K$, are partitioned into blocks of m bits which constitute one RS code symbol. Given each K information symbols, n_{cand} coded symbols are calculated; systematic encoding is assumed. Since $D\mu$ binary symbols are contained in each OFDM frame, $D\mu/m$ RS codes are present in parallel.

After mapping the bits of the RS symbols to data symbols $A_{\nu,d}$, n_{cand} OFDM frames $\boldsymbol{A}_\nu = [A_{\nu,0}, \ldots, A_{\nu,D-1}]$, $\nu = 1, \ldots, n_{\mathrm{cand}}$, are obtained, which are finally transformed into time domain and their respective ξ_ν is calculated. From the n_{cand} OFDM frames, the K best frames are selected for transmission. Figure 1 sketches the arrangement of the code words over the OFDM frames to generate the candidates.[2]

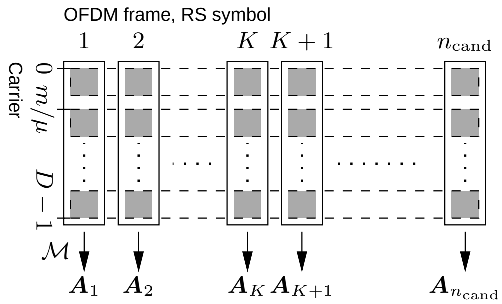

Fig. 1. Illustration of arrangement of the RS code words (dashed) over the K initial OFDM frames. m/μ carriers are combined into one RS symbol (depicted in gray).

Having received the K selected and transmitted frames, the initial data can be recovered by RS erasure decoding. Because the RS codes are *maximum distance separable (MDS)* [4], any selection of K code symbols out of N suffices; the transmitter may chose any set of K candidates. However, the RS decoders have to be informed which K symbols (position within the code word) are non-punctured. Hence, some small amount of side information has to be transmitted as well; we will ignore the required side information in the following.

Given the ccdf of the original single-antenna OFDM system, $\mathrm{C}_{\mathrm{o}}(\xi_{\mathrm{th}})$, (without any PAR reduction) with D carriers and using I-times oversampling, the ccdf of the K^{th} best OFDM frame of the RS coding scheme which limits performance according to (4) then calculates to [7]

$$\mathrm{C}_{\mathrm{RS}}(\xi_{\mathrm{th}}) = \sum_{l=0}^{K-1} \binom{n_{\mathrm{cand}}}{l} (1 - \mathrm{C}_{\mathrm{o}}(\xi_{\mathrm{th}}))^l \, \mathrm{C}_{\mathrm{o}}^{n_{\mathrm{cand}}-l}(\xi_{\mathrm{th}}) \ . \tag{7}$$

III. Successive Candidate Generation and Stopping Criterion

A. *"Critical PAR"*

We start with single-antenna OFDM and its $\mathrm{C}_{\mathrm{o}}(\xi_{\mathrm{th}})$. Having, as in SLM, n candidates representing the same data from which the best is selected, an ccdf according to

$$\mathrm{C}_{\mathrm{SLM}}(\xi_{\mathrm{th}}) = (\mathrm{C}_{\mathrm{o}}(\xi_{\mathrm{th}}))^n \tag{8}$$

is achieved. When assuming Gaussian samples at Nyquist rate, we hence have

$$\mathrm{C}^{(\mathrm{G})}_{\mathrm{SLM}}(\xi_{\mathrm{th}}) \overset{\text{Gauss}}{=} \left(1 - (1 - \mathrm{e}^{-\xi_{\mathrm{th}}})^D\right)^n \ . \tag{9}$$

Using the relation $1 - (1 - \mathrm{e}^{-x})^y < y\mathrm{e}^{-x}$, which is an upper bound for $y \geq 2$ and $10\log_{10}(x) \leq 12.7$ dB, an upper bound on the ccdf of SLM can be given:

$$\mathrm{C}^{(\mathrm{G})}_{\mathrm{SLM}}(\xi_{\mathrm{th}}) < \mathrm{C}^{(\mathrm{G})}_{\mathrm{SLM,up}}(\xi_{\mathrm{th}}) \overset{\text{def}}{=} D^n \mathrm{e}^{-\xi_{\mathrm{th}} n} \ , \tag{10}$$

which, in logarithmic[3] scale, corresponds to a linear function

$$\log(\mathrm{C}^{(\mathrm{G})}_{\mathrm{SLM,up}}(\xi_{\mathrm{th}})) = n \cdot (\log(D) - \xi_{\mathrm{th}}) \ . \tag{11}$$

Independent of n, this function becomes zero (hence the ccdf becomes 1) at

$$\xi_{\mathrm{th}} \overset{!}{=} \xi_{\mathrm{crit}} \overset{\text{def}}{=} \log(D) \ , \tag{12}$$

and the slope of this function is given by n, the number of assessed candidates. Hence, the more candidates are taken into account, the steeper gets the slope and a PAR in the range of the "critical PAR" ξ_{crit} can be guaranteed. For $n \to \infty$ the ccdf of SLM with Gaussian signaling is then upper bounded by a stair-step function with jump discontinuity at ξ_{crit}. This "critical" value has already been observed in [11], [14] in the sense that there exist schemes which (with high probability) can guarantee that this PAR will not be exceeded.

B. *Successive Encoding*

Having in mind that a PAR of ξ_{crit} can be guaranteed and OFDM frames with PAR exceeding ξ_{crit} can asymptotically be completely by eliminated, we can argue contrariwise that OFDM frames with PAR below this limit should readily be accepted for transmission. Hence the candidate search should be stopped if ξ_{crit} has been achieved. Instead of generating and testing a fixed number of candidate OFDM frames, it may be advantageous to do the candidate generation and selection successively and to stop if some desired performance—in

[1]We here study multi-antenna OFDM transmission. The PAR reduction scheme can also be applied to single-antenna schemes when consecutive frames over time are considered—the worst-case PAR has then to be replaced by the average PAR [6].

[2]The scheme has to be slightly modified for $K = 1$ (single-antenna case), since the RS code then degrades to a repetition code; all candidates would be equal. Random binary vectors, known to transmitter and receiver, may be added (XOR-ed) to the encoded binary OFDM frames, basically leading to an SLM scheme.

[3]$\log(\cdot)$ denotes the natural logarithm.

particular a PAR below a given limit ξ_0—is achieved. Thereby, IDFTs and PAR calculations (hence complexity) can be saved.

Using the RS code scheme, successive candidate generation and evaluation is straightforward—e.g., using the well-known feedback shift register systematic encoder for cyclic codes [4] the successive calculation of coded symbols is immediate. Whenever the new candidate OFDM frame has a PAR lower than the worst PAR of the K current OFDM frames this new frame is accepted and replaces the worst.

C. Performance Analysis

We now derive the performance of successive candidate generation and stopping. First, assume that potentially an infinite number of candidates can be tested and a PAR below the given limit ξ_0 should be guaranteed. Then the ccdf reads

$$\begin{aligned} \mathrm{C}_{\mathrm{succ},\infty}(\xi_{\mathrm{th}}) &= \Pr\{\max_\kappa \xi_\kappa > \xi_{\mathrm{th}} \mid \max_\kappa \xi_\kappa \le \xi_0\} \\ &= \frac{\Pr\{\xi_{\mathrm{th}} < \max_\kappa \xi_\kappa \le \xi_0\}}{\Pr\{\max_\kappa \xi_\kappa \le \xi_0\}} \\ &= \begin{cases} \frac{\mathrm{C}_{\mathrm{o},K}(\xi_{\mathrm{th}})-\mathrm{C}_{\mathrm{o},K}(\xi_0)}{1-\mathrm{C}_{\mathrm{o},K}(\xi_0)}, & \xi_{\mathrm{th}} < \xi_0 \\ 0, & \xi_{\mathrm{th}} \ge \xi_0 \end{cases}, \end{aligned} \quad (13)$$

where $\mathrm{C}_{\mathrm{o},K}(\xi_{\mathrm{th}})$ is again the ccdf of the original K-antenna OFDM scheme.

In practice, the maximum number of trials has to be restricted to some given value $n_{\max}$. If after testing $n_{\max}$ candidates the PAR is still above the desired value, for $\xi_{\mathrm{th}} \ge \xi_0$ the ccdf is identical to that of the RS code scheme with $n_{\mathrm{cand}} = n_{\max}$ and for $\xi_{\mathrm{th}} < \xi_0$ it is one. This event occurs with probability $\mathrm{C}_{\mathrm{RS}}(\xi_0)$. With the complementary probability a PAR below the limit is obtained and the ccdf is given by (13). The ccdf of the successive scheme with limitation to $n_{\max}$ candidate is the average of these two contributions and reads

$$\mathrm{C}_{\mathrm{succ},n_{\max}}(\xi_{\mathrm{th}}) = \begin{cases} \frac{\mathrm{C}_{\mathrm{o},K}(\xi_{\mathrm{th}})-\mathrm{C}_{\mathrm{o},K}(\xi_0)}{1-\mathrm{C}_{\mathrm{o},K}(\xi_0)}(1-\mathrm{C}_{\mathrm{RS}}(\xi_0)) \\ \qquad + \mathrm{C}_{\mathrm{RS}}(\xi_0), & \xi_{\mathrm{th}} < \xi_0 \\ \mathrm{C}_{\mathrm{RS}}(\xi_{\mathrm{th}}), & \xi_{\mathrm{th}} \ge \xi_0 \end{cases} . \quad (14)$$

Next, we assess the average number of candidates to be tested. The probability that after testing exactly n candidates the PAR threshold is met and the search stops is given by a negative binomial distribution[4] [12] (the argument "(ξ_0)" is omitted for brevity)

$$\Pr\{n\} = \binom{n-1}{K-1} \mathrm{C}_{\mathrm{o}}^{n-K}(1-\mathrm{C}_{\mathrm{o}})^K , \quad n = K, K+1, \ldots \quad (15)$$

Hence, stopping the candidate generation after $n_{\max}$ trials, average complexity (per antenna) reads

$$\bar{n}(\xi_0) \overset{\mathrm{def}}{=} \mathrm{E}\{n\} ; \quad (16)$$

unfortunately, no analytic expression for this quantity can be given. However, two bounds can be stated: (i) $\mathrm{E}\{n\} \le n_{\max}$, and (ii) average complexity for finite $n_{\max}$ is strictly lower than average complexity in case of $n_{\max} \to \infty$. The later reads (mean of a negative binomial distribution [12])

$$\bar{n}_\infty(\xi_0) = \frac{1}{K} \sum_{n=K}^{\infty} n \cdot \Pr\{n\} = \frac{1}{1-\mathrm{C}_{\mathrm{o}}(\xi_0)} = \frac{1}{\mathrm{cdf}_{\mathrm{o}}(\xi_0)} . \quad (17)$$

Noteworthy, asymptotically average complexity per antenna is independent of the number of antennas K and is simply given by the reciprocal of the cdf of PAR of the original single-antenna OFDM scheme; this result holds for all oversampling factors I and all modulation alphabets.

Moreover, using (5), for Gaussian symbols at Nyquist rate the expected number of candidates reads $\bar{n}_\infty(\xi_0) = (1 - \mathrm{e}^{-\xi_0})^{-D}$, which at the "critical" value $\xi_{\mathrm{crit}} = \log(D)$ and large number D of carriers converges to

$$\bar{n}_\infty(\log(D)) = \left(1 - \tfrac{1}{D}\right)^{-D} \overset{D\to\infty}{\longrightarrow} \mathrm{e} , \quad (18)$$

where $\mathrm{e} = 2.71828\ldots$ is Euler's number. For finite D and non-Gaussian symbols only slight deviations from this number have been observed.

Interestingly, using the successive RS coding scheme, at desired PAR values around ξ_{crit} average complexity (in number of assessed candidates) is only 2.72. This is extremely low compared to the usual application of SLM or other PAR reduction scheme which typically evaluate more than ten candidates per antenna in order to achieve similar PAR performance.

D. Design Criterion

Basically two parameters for the design of the PAR reduction scheme can be chosen: the limit ξ_0 and the maximum number of candidates $n_{\max}$. The ccdf of PAR then concentrates below ξ_0; at the limit ξ_0 the ccdf has the value $\mathrm{C}_{\mathrm{RS}}(\xi_0)$ and for larger threshold PARs the ccdf has a slope,[5] which from (7) is asymptotically given as [6]

$$s = n_{\max} - K + 1 . \quad (19)$$

From a practical point of view, it is more desirable to specify some *ccdf mask*, similar to spectral masks in transmission systems. A suited definition of such a mask can be

- for PAR values below the given limit ξ_0 the ccdf may be arbitrary,
- at the limit ξ_0 the ccdf may not exceed a given probability Pr_0,
- for PAR values larger ξ_0 the ccdf should decay with a slope of at least s_0, i.e., $\mathrm{C}(\xi_{\mathrm{th}}) \le \left(D \cdot \mathrm{e}^{-\xi_{\mathrm{th}}}\right)^{s_0}$.

Since the last two demands are linked to each other, the more restrictive one is active. From (14) and (19) we have

$$n_{\max} \ge \max\{n_{\min},\ s_0 + K - 1\} , \quad (20)$$

where $n_{\min}$ is the minimum number of candidates n_{cand} in (7) such that $\mathrm{C}_{\mathrm{RS}}(\xi_0) \le \mathrm{Pr}_0$. In Figure 2 a ccdf mask is exemplarily plotted together with a ccdf fulfilling the criteria.

[4] K out of the n candidates have lower and $n-K$ larger PAR than ξ_0. The probabilities of these events are $1-\mathrm{C}_{\mathrm{o}}(\xi_0)$ and $\mathrm{C}_{\mathrm{o}}(\xi_0)$, respectively. Since the last candidate has to have a PAR lower than ξ_0—otherwise the search would have stopped earlier—$\binom{n-1}{K-1}$ combinations exist, leading to the given distribution.

[5] As in (11), $\log(\mathrm{C}(\xi_{\mathrm{th}}))$ decreases almost linearly over ξ_{th} in linear scale. By "slope" we denote the decay of this straight line.

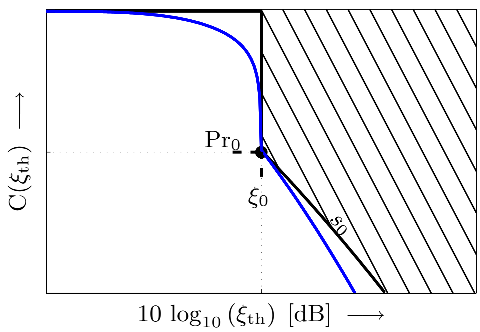

Fig. 2. Visualization of ccdf mask (forbidden region is dashed) with design parameters PAR limit ξ_0, maximum probability Pr_0, and slope s_0. A ccdf fulfilling the criteria is also shown.

IV. Numerical Results and Conclusions

We now assess the proposed successive scheme by means of numerical simulations. An OFDM system with $D = 512$ carriers (all used), 4-PSK modulation per carrier, and $K = 4$ transmit antennas is assumed. Noteworthy, the results are representative for all oversampling factors; since all derivations start from the ccdf of the original, I times oversampled signal and this ccdf basically shifts to somewhat higher PAR values when increasing I, we concentrate on Nyquist-rate sampling ($I = 1$).

In Figure 3 the ccdfs of PAR are plotted over the threshold $\xi_{\rm th}$. In addition to the successive scheme (solid), the results for the RS coding scheme with a fixed number of candidates equal to $n_{\max}$ (dashed), SLM with the same complexity as the RS scheme (dash-dotted), and the original OFDM system (dotted) are shown for comparison. The theoretical curves are shown in solid gray. The slight differences are due to the used Gaussian approximation.

From top to bottom figure, the PAR limit ξ_0 is fixed to 8 dB, 9 dB, and 10 dB. Noteworthy, in the present situation the "critical PAR" is $10\log_{10}(\log(512)) = 7.95$ dB. As expected, the ccdf concentrates below the given limit; in this region the ccdfs are almost identical for all maximum numbers of candidates. If $n_{\max}$ is chosen large enough, a vertical fall of the ccdf is achieved. For smaller $n_{\max}$, a shoulder (similar to the side lobes in a power spectral density) occurs which is given by the ccdf of the coding scheme with the same number of $n_{\max}$ candidates.

Figure 4 shows the situation for a varying number of antennas, starting from $K = 1$ (single-antenna scheme) over 2 and 4 to 6. In each case the maximum number of candidates is $n_{\max} = 3K$, i.e., the number of candidates per antenna is the same, hence the average complexity per antenna is also the same (cf. (17), here 1.20 candidates per antenna). However, performance gets much better when increasing K, i.e., joint PAR reduction even gains when treating more symbols jointly. With respect to the ccdf, some kind of *diversity gain* (increased slope of the curve) is achieved; besides the RS coding scheme such a behavior has just been observed in directed SLM [5].

In Figure 5 the average number of tested candidates is shown. The theoretical curves for Gaussian signaling are shown in solid gray, and the value of the "critical PAR" is shown via the dash-dotted line. In the top plot, the number

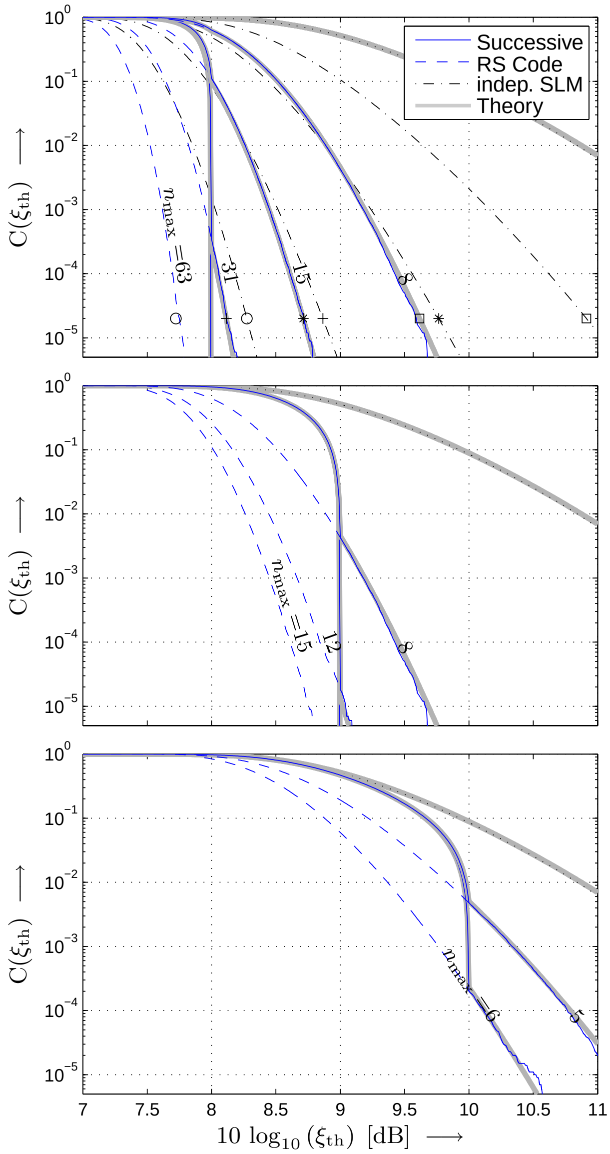

Fig. 3. Ccdf of PAR over $\xi_{\rm th}$. OFDM with $D = 512$ carrier, 4-PSK modulation, $K = 4$ antennas, $I = 1$. Solid: proposed successive scheme; dashed: RS coding scheme with a fixed number of candidates equal to $n_{\max}$; dash-dotted: SLM per antenna with the same (same marker) total number of tested candidates as the RS scheme; dotted: original OFDM; gray solid: theoretical curves (here based on the Gaussian assumption). Top: PAR limit $\xi_0 \mathrel{\hat{=}} 8$ dB; $n_{\max} = 8, 15, 31, 63$. Middle: PAR limit $\xi_0 \mathrel{\hat{=}} 9$ dB; $n_{\max} = 8, 12, 15$. Bottom: PAR limit $\xi_0 \mathrel{\hat{=}} 10$ dB; $n_{\max} = 5, 6$.

of antennas is fixed to $K = 4$ and the maximum number of assessed candidates in the RS coding scheme is chosen to be $n_{\max} = 8, 12, 16, 20$. The two asymptotes (low PAR: $\bar{n} \to n_{\max}/K$; high PAR: $\bar{n} \to \bar{n}_\infty$) are clearly visible. Moreover, for large $n_{\max}$, $\bar{n} \approx \mathrm{e}$ can also be recognized at $\xi_0 = \log(D)$.

The situation for fixed ratio between K and $n_{\max}$ is shown in the bottom plot. Here, the curves converge at low and high PAR limits. In the middle region, for $K = 1$ (single-antenna scheme) the lowest average complexity is observed (but also the worst performance, see Figure 3); for increasing K the curves tend to both asymptotes.

Finally in Figure 6 the ccdf of PAR is plotted over the

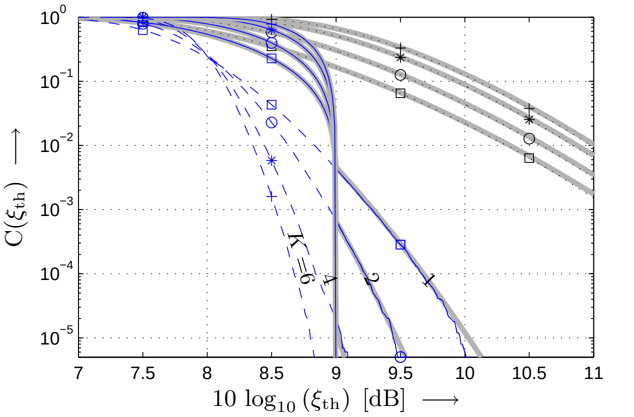

Fig. 4. Ccdf of PAR over $\xi_{\rm th}$. OFDM with $D = 512$ carrier, 4-PSK modulation, $K = 4$ antennas, $I = 1$. Solid: proposed successive scheme; dashed: RS coding scheme with a fixed number of candidates equal to $n_{\max}$; dotted: original OFDM; gray solid: theoretical curves (here based on the Gaussian assumption). PAR limit $\xi_0 \hat{=} 9$ dB. $K = 1, 2, 4, 6$ antennas with $n_{\max} = 3K$.

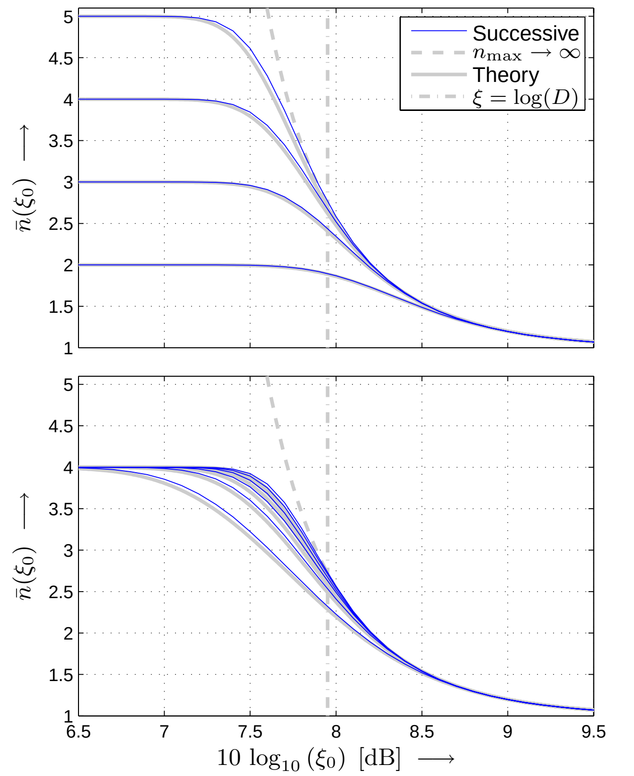

Fig. 5. Average number $\bar{n}$ of candidates over the PAR limit ξ_0. OFDM with $D = 512$ carrier, 4-PSK modulation, $I = 1$. Solid: proposed successive RS coding scheme; gray solid: theoretical curves; gray dashed: theoretical curve for $n_{\max} \to \infty$; gray dash-dotted: "critical PAR" value $\xi = \log(D)$. Top: $K = 4$, $n_{\max} = 8, 12, 16, 20$ (bottom to top, RS coding). $n_{\max} = 8, 12, 15$ in case of Simplex coding (curves are marked with circles). Bottom: $n_{\max} = 4, 8, 12, 16, 20, 24$, $K = n_{\max}/4$ (bottom to top).

threshold $\xi_{\rm th}$ and the maximum number of tested candidates for $\xi_0 \hat{=} 9$ dB. It can be seen that performance is governed by $n_{\max}$. However, for almost all $n_{\max}$ average number of tested candidates is equal to only $\bar{n} = 1.20$; for $n_{\max} = 4$ it is (of course) 1; for $n_{\max} = 4$, 5 we have $\bar{n} = 1.13$, and 1.18.

In conclusion it can be stated that using coding for candidate generation and selection is an very interesting approach for PAR reduction in multi-antenna OFDM schemes. When stopping the candidate generation after a given PAR limit is met, significant reduction in the average number of assessed candidates is achieved. Thereby the scheme is very flexible: it can be used for all DFT sizes and all modulation alphabets. Similar to channel coding, the PAR reduction capability improves if the number of antennas increases; original OFDM and most other PAR reduction schemes deteriorate in this case.

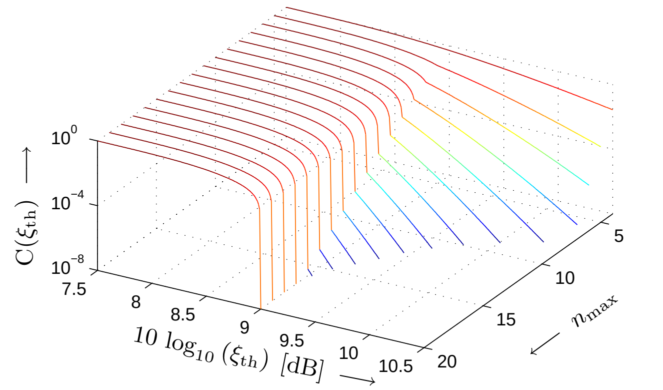

Fig. 6. Ccdf of PAR over $\xi_{\rm th}$ and $n_{\max}$. OFDM with $D = 512$ carrier, 4-PSK modulation, $K = 4$ antennas, $I = 1$. PAR limit: $\xi_0 \hat{=} 9$ dB.

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