Hidden Markov Modeling of Error Patterns and Soft Outputs for Simulation of Wideband CDMA Transmission Systems

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Abstract - **The purpose of this paper is the presentation of the Hidden Markov Modeling (HMM) technique for a fast and accurate simulation of bit errors and soft outputs in wireless communication systems. HMMs with continuous probability distributions are considered. We focus on binary phase–shift keying (BPSK) modulation for direct sequence spread spectrum transmission as proposed e.g. for the uplink in the frequency division duplex mode (FDD) of the third generation wireless communication system UMTS. Comparisons of pdfs of error patterns, autocorrelation functions and bit error rates with convolutional coding for Rake receivers and HMM models are shown for AWGN, flat fading and vehicular channel conditions.**

I. INTRODUCTION

Hidden Markov Modeling is a mathematical technique that has turned out to be useful in a number of applications including speech recognition and signal processing [6]. More recently, Hidden Markov Models (HMMs) have been applied in the field of digital communications with convincing results [1, 14]. Due to emerging new applications in the field of transmission of multimedia content over the physical layer in third generation (3G) wireless communication systems, fast simulation techniques for assessment of the quality of transmission are demanded. The benefit of HMM based techniques is a high potential gain in simulation time compared to standard wireless channel simulation and the ability to adapt the statistical behaviour of HMMs to reference models (e.g. to a real hardware performance). Many papers are devoted to the modeling of fading channels with memory. However, most of them only discuss error models for the case of hard decision at the receiver side [2]. The HMM proposed in this paper is designed for modeling the soft outputs of a Rake receiver for direct sequence spread spectrum wireless communications [4] using BPSK modulation. One possible application, on which we focus in this contribution, is the simulation of the uplink transmission (data channel) in the FDD mode of UMTS. Fig. 1 shows the principle for the substitution of modulation, channel, and Rake receiver by a HMM generating bit errors and soft outputs combined to error patterns.

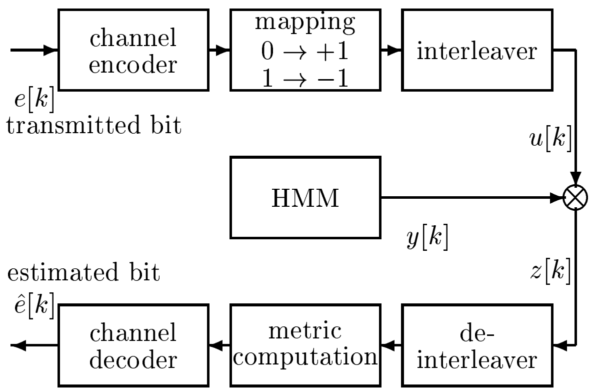

Figure 1: Transmission model with HMM.

II. UMTS CHANNEL MODEL

Fig. 2 shows the frame structure of the Dedicated Physical Control Channel (DPCCH) and the Dedicated Physical Data Channel (DPDCH) used for the uplink transmission in accordance with the UMTS evaluation document [3]. All presented simulation results have been obtained using the highest spreading factor which corresponds to 10 symbols per 625 μs slot (or equivalently to $m = 0$ in Fig. 2) for the DPDCH. It is assumed that the UMTS transmitter, channel, and receiver (including ideal matched filtering and maximum ratio combining but excluding interleaving and channel decoding) can be modeled with

$$r[k] = \mu[k] \cdot u[k] + n[k], \quad (1)$$
$$u[k] \in \{+1, -1\},\ \mu[k] \in \{\mu_1, \ldots, \mu_N\},$$

where $r[k]$ denotes the output signal of the Rake receiver at discrete time k, $u[k]$ the transmitted symbol, and $n[k]$ AWGN with variance σ_n^2. The fading gain $\mu[k]$ is a colored random process, whose statistics depend on those of the different channel paths, which are coherently combined in the receiver. For Markov modeling, it is assumed that $\mu[k]$ is an amplitude–discrete process, which can take on N different values. This is justified by the numerical results presented later. The log likelihood ratio is de-

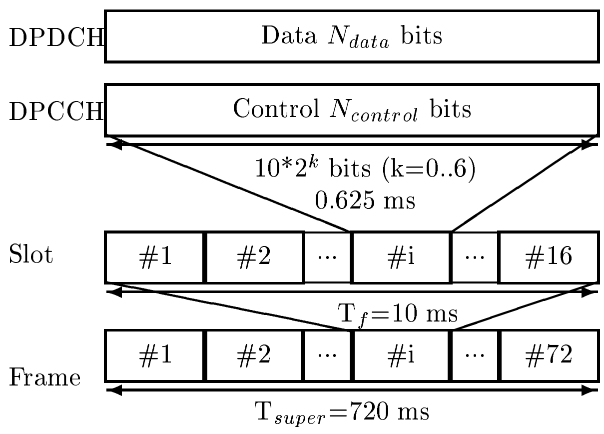

Figure 2: Structure of the uplink transmission.

fined as

$$z[k] = \ln\left(\frac{p(r[k] \mid u[k] = +1)}{p(r[k] \mid u[k] = -1)}\right), \qquad (2)$$

with the conditional probability density of $r[k]$ given $u[k]$

$$p(r[k] \mid u[k]) = \frac{1}{\sigma_n\sqrt{2\pi}} \exp\left[-\frac{(r[k]-\mu[k]u[k])^2}{2\sigma_n^2}\right]. \qquad (3)$$

It is straightforward to show that the log likelihood ratio $z[k]$ is directly proportional to the output sample $r[k]$:

$$z[k] = 2\,\frac{\mu[k]}{\sigma_n^2}\,r[k]. \qquad (4)$$

Due to the symmetry in the BPSK modulation we can introduce a model independent of the actual binary input sequence $u[k]$. An *error pattern* (EP) sample $y[k]$ can be viewed as the soft output for a transmitted symbol $u[k]$ equal to $+1$, or equivalently for a transmitted bit $e[k]$ equal to zero (see Fig. 1).

$$y[k] = \begin{cases} +z[k] & \text{if} \quad u[k] = +1,\ e[k] = 0 \\ -z[k] & \text{if} \quad u[k] = -1,\ e[k] = 1 \end{cases} \qquad (5)$$

III. HIDDEN MARKOV MODELS

Let us define q_k to be the state of a HMM at a given time $k \in [1, L]$ with L defining the duration of the training sequence as an integer number of samples. The state sequence of length L of the Markov chain is a stochastic process assuming values in the finite *state space* $\mathcal{Q}$ composed of N states. We can encode the states by assigning the label j to the state s_j and use for simplicity of notation the set

$$\mathcal{Q} = \{1, 2, \ldots, N\}. \qquad (6)$$

The initial state distribution $\mathbf{\Pi}$ is defined as:

$$\mathbf{\Pi} = \{\pi_i\}_{i=1}^{N} \in [0;1]^{1\times N} \qquad (7)$$

with

$$\pi_i = \Pr(q_1 = i) \in [0;1], \qquad (8)$$

where q_1 denotes the initial state at time $k = 1$. Furthermore, the HMM is characterized by the state transition probability distribution $\mathbf{A}$,

$$\mathbf{A} = \{a_{ij}\}_{i=1,j=1}^{N,N} \in [0;1]^{N\times N} \qquad (9)$$

with the conditional probabilities

$$a_{ij} = \Pr(q_{k+1} = j \mid q_k = i). \qquad (10)$$

The stationary state distribution contained in row vector $\mathbf{P}$

$$\mathbf{P} = [P_1\ P_2\ \ldots\ P_N] \in [0;1]^{1\times N} \qquad (11)$$

with

$$P_i = \Pr(q_k = i),\ k \in \{1, 2, \ldots, L\} \qquad (12)$$

can be calculated using

$$\mathbf{PA} = \mathbf{P}, \quad \sum_{i=1}^{N} P_i = 1. \qquad (13)$$

In accordance with the proposed channel modelisation (Eq. (4)) the state conditional observation pdf $p_Y(y[k] \mid i)$ in state i can be directly modeled by a Gaussian distribution. For a complete description of the applied HMM model we introduce the row vectors of fading gains μ_i and noise variances σ_i^2 assigned to the states i

$$\boldsymbol{\mu} = [\mu_1\ \mu_2\ \ldots\ \mu_N],\ \boldsymbol{\sigma} = \left[\sigma_1^2\ \sigma_2^2\ \ldots\ \sigma_N^2\right], \qquad (14)$$

resulting in a compact notation λ of the model:

$$\lambda = (\mathbf{\Pi}, \mathbf{A}, \boldsymbol{\mu}, \boldsymbol{\sigma}). \qquad (15)$$

The Baum Welch algorithm (BWA) [8, 7] has been used to adapt the fading gains μ_i and the noise variances σ_i^2. Fig. 1 shows the principle of error pattern modeling for UMTS using a HMM. The state distribution $\mathbf{\Pi}$ is initialized in that way that all N states composing the HMM are initially equiprobable. The transition matrix $\mathbf{A}$ is initialized with random coefficients fulfilling the stochastic constraint. One crucial task is to determine an efficient measure to compare in a statistical sense the original UMTS error pattern generated by the UMTS simulator and the EP sequence delivered by the HMM model. There are many different approximation quality measures which can be taken for assessment of the ability of the model to produce sequences statistically close to the training sequence. One of them is the *relative entropy* also called *Kullback-Leibler divergence* (KLD) [5, 9]. The continuous pdf of the process generated by the HMM and the pdf of the original process (Rake receiver error patterns) are used to calculate the KLD. The advantage of the KLD measure is the possibility to determine an upper bound (*differential entropy*) from the pdf of the process to be modeled. Another frequently used method

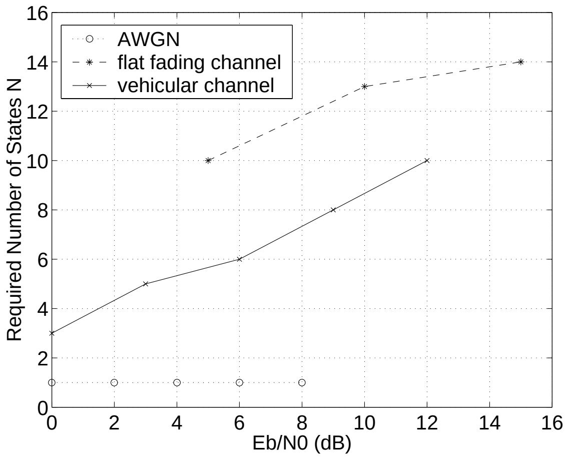

Figure 3: Required number of HMM states.

is the *HMM likelihood function* introduced by Liporace [10]. We used a related probabilistic distance measure, the *scoring indicator* introduced in [11, 7]. For all channels and each E_b/N_0 value considered, we first evaluated the simplest one state model, investigating then more elaborate models with more states by inspection of the KLD and the scoring indicator. The main advantage of the scoring indicator is its guaranteed continuous improvement (hence it has been used as scoring measure for the Baum Welch algorithm) whereas its drawback is that an upper bound can not be calculated. On the other hand, the upper bound for the KLD can be calculated quite simple. Combining these two measures makes it possible to find optimal models efficiently. In all cases, the training sequence had a length of $L = 10^5$.

IV. SIMULATION RESULTS

Fig. 3 shows the number of states required for modeling the EP sequences for the different channels considered. Obviously a one state model is sufficient to model the AWGN channel for all SNRs considered. In case of flat fading, in contrast to the AWGN channel, a high number of states is required, due to the memory of the channel. For $E_b/N_0 > 10$ dB, the considered range for N ($N \leq 14$) seemed to be not sufficient since the scoring indicator curve was still increasing for the chosen HMM state number indicating that a model with higher number of states might achieve better performance. However, simulations for $N > 15$ could not be performed because of memory limitations and we decided to choose these models. In case of the vehicular channel, which has been simulated using the vehicularA definition in [3], it was much simpler to find the optimal models than in the flat fading case since the maximal achievable scoring values seem to be closely approached for each E_b/N_0 value. Fig. 4 shows examples for EP pdfs measured from the training sequence and calculated with the optimal HMM for the AWGN, vehicular and flat fading channel. The notation HMM(v) in Fig. 4 means that a model with v states has been used.

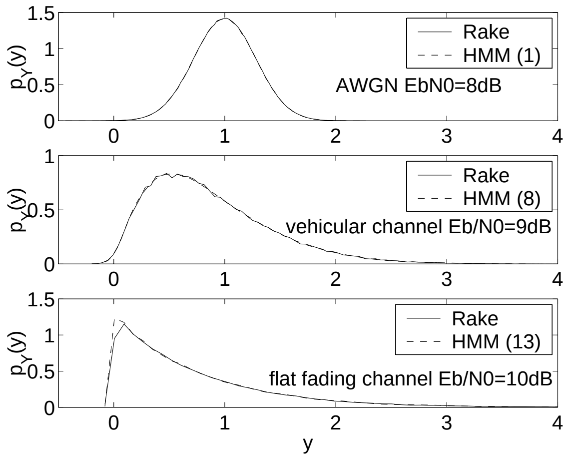

Figure 4: Pdfs obtained for different channels.

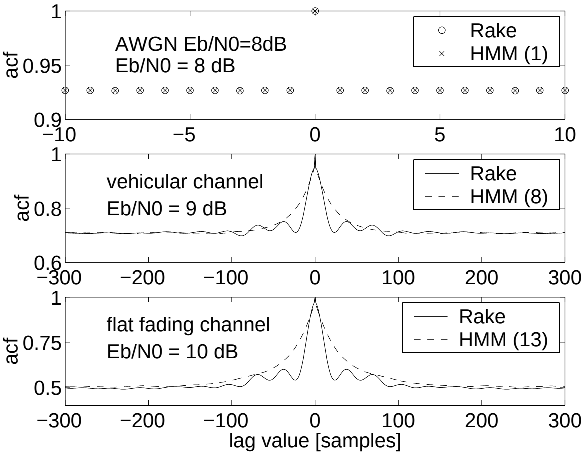

Figure 5: Autocorrelation functions.

Comparing the flat fading case with the AWGN case, it is obvious that the EP value range is much larger for flat fading and the pdf of the original process is far away from a Gaussian density. Hence a greater number of states is required to model the flat fading channel. Each HMM model for flat fading was obtained after 214 iterations of the BWA. For the vehicular as well as for the AWGN channel, the EP pdf corresponding to each model chosen as optimal fits the original process pdf with a remarkable accuracy, which was also proven by KLD measurements. For the flat fading case small differences between the pdfs can be noticed. In addition to a statistical assessment via one-dimensional pdfs, it is also of interest to study the dynamical behaviour of the generated sequences and to compare it with the UMTS EP training sequence in terms of autocorrelations. As obvious from Fig. 5 (top), autocorrelations (acfs) of EP signals generated by HMMs are in perfect accordance with autocorrelations of the training sequence for the case of an AWGN channel. Both resemble a Dirac pulse. Fig. 5 (bottom) shows results for the flat fading channel. The Bessel autocorrelation sequence of the fading gain [12] is reflected in the oscillations of the UMTS autocorrelation sequence. For the vehicular channel (Fig. 5 (mid)), the autocorrelations

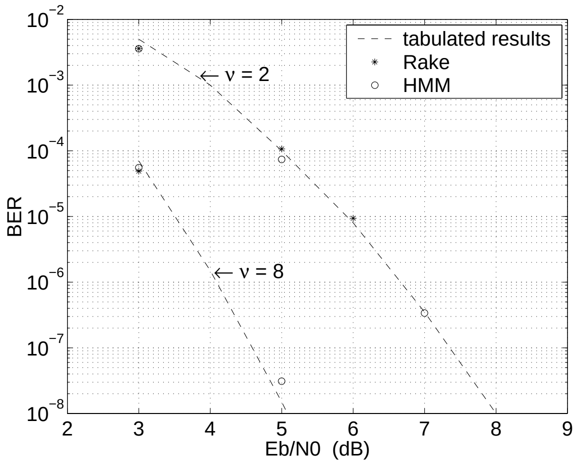

Figure 6: BER for AWGN channel.

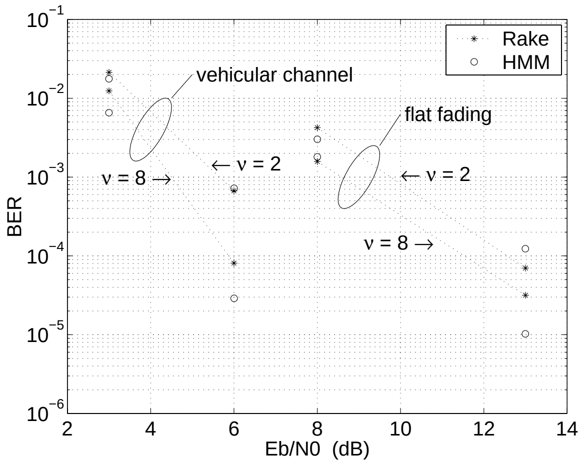

Figure 7: BER for flat fading and vehicular channel.

are similar as for the case of flat fading. The HMMs are unable to model the oscillations of the autocorrelation functions but fit them as good as possible. In order to assess the ability of the HMM to model soft-decision information including its dynamics, decoding results for the error patterns generated by HMMs are compared to results obtained using the Rake receiver simulation software. As the error patterns are generated independently of the transmitted bits, the error pattern sequence corresponds to the log likelihood ratios delivered by the Rake receiver for a bit sequence composed of zeros only. Since the convolutional code is linear, the HMM may be used directly for determination of the bit error rate after deinterleaving and decoding. A rate 1/2 convolutional encoding scheme according to the UMTS evaluation document [3] (constraint length $\nu = 8$ with generator polynomials $g_0 = 561, g_1 = 753$ in octal form) and a simpler encoding scheme (constraint length $\nu = 2$ with generator polynomials $g_0 = 5, g_1 = 7$) have been investigated. Figs. 6 and 7 show BER results. For the flat fading channel the simulations yield a higher BER than expected. This is a consequence of the quite short interleaver size of 16×10, for a frame with $N_{frame} = 160$ bits. Hence, interleaving does not randomize the error pattern samples sufficiently and the channel does not appear approximately memoryless. The BER results obtained with HMMs for the three channels considered are in a good agreement with the BERs obtained with the UMTS channel simulator. This confirms the efficiency of the proposed modeling technique.

V. CONCLUSIONS

It can be seen from the discussion of the AWGN and vehicular channel that the error patterns of an UMTS Rake receiver can be sufficiently accurately modeled by an HMM. In the case of flat fading, for high signal-to-noise ratios only HMMs with large number of states seem to describe the channel memory sufficiently. This, however, is in contradiction to limitations in computation time and memory for the BWA iterations. Monitoring of the KLD in addition to the scoring indicator provides a useful method for HMM performance assessment. The application of HMMs allows very fast error pattern generation, which can be directly used as soft input for simulations with linear codes.

VI. ACKNOWLEDGEMENTS

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