SIGNAL SHAPING FOR REDUCTION OF PEAK-POWER AND DYNAMIC RANGE IN PRECODING SCHEMES

Robert F.H. Fischer, Roman Tzschoppe, Johannes B. Huber
Lehrstuhl für Nachrichtentechnik II, Universität Erlangen-Nürnberg, Cauerstraße 7/NT, D-91058 Erlangen, Germany
Phone: +49-9131-85-28902, Fax: +49-9131-85-28849, email: fischer@LNT.de, WWW: http://www.LNT.de/LNT2/

***Abstract* — Precoding at the transmitter side has been proved to be a very efficient strategy for channel equalization in single-carrier digital transmission schemes. It enables the application of coded modulation in a seamless fashion. A drawback of precoding is that the signal at the input of the decision device exhibits a huge dynamic range. Based on dynamics shaping, a combined precoding/shaping technique introduced in [3], a new shaping strategy is developed in this paper. This technique enables a flexible trade-off between (i) reduction of the average transmit power, (ii) avoidance of peaks in the transmit signal in order to facilitate line driver implementation, and (iii) restriction of the maximum amplitude at the receiver side to a prescribed value. Over a wide range, all three demands can be met simultaneously. As the scheme is fully compatible with Tomlinson-Harashima precoding, it can replace the precoder even in existing and standardized schemes. Simulation results for a typical DSL scenario show the achievable gains.**

I. INTRODUCTION

When transmitting over channels which produce severe intersymbol interference (ISI), equalization is of major concern. Precoding at the transmitter side, e.g., *Tomlinson-Harashima precoding (THP)* [6, 7, 13], has been proved to be a very efficient strategy for single-carrier systems. It enables the application of coded modulation in a seamless fashion, and is able to come close to the channel capacity of the underlying channel [10, 6].

A drawback of THP is that the signal at the input of the decision device exhibits a huge dynamic range for the present application. These dynamics cause very high sensitivity to equalization inaccuracies and symbol clock jitter. In order to overcome these problems a combined precoding/shaping technique, called *dynamics shaping*, was proposed in [3]. The main aim of this approach is to limit the maximum amplitude of the signal at the output of the receive filter. In doing so, dynamics shaping is fully compatible with THP.

In this paper, we extend this shaping method in order to additionally reduce the dynamic range of the transmit signal. Thereby, implementation of power amplification (line driver) is facilitated as the range of linear operation can be narrowed. This moreover reduces the power loss of the line driver, as it is directly related to the peak-to-average power ratio of the transmitted signal. As an example, we study the application to *single-pair digital subscriber lines (SDSL)*, by which the "last mile" from the central office to the customer can be bridged cost-effectively on ordinary telephone lines.

After introducing the system model of transmitter precoding schemes in Section 2, we briefly review THP. Starting from dynamics shaping as proposed in [3], the new approach for additional reduction of the peak power at the transmitter is presented in Section 3. Simulation results for a typical DSL scenario are given in Section 4.

II. BASICS OF TRANSMITTER PRECODING

II.1. Discrete-Time Channel Model

The system model of the transmission scheme is shown in Figure 1. Here we mainly deal with baseband transmission, hence all signals are real. The extension to passband transmission is straightforward by using complex-valued signals and systems in the equivalent low-pass domain [11].

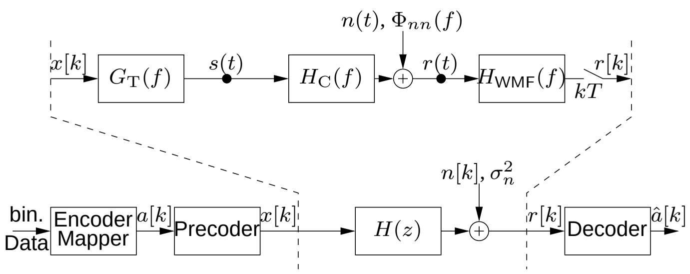

Figure 1: Block diagram of transmission schemes employing precoding.

First, the binary data stream is channel encoded and mapped to the PAM symbols $a[k]$ ($k \in \mathbb{Z}$: discrete-time index), taken from an M-ary PAM signal set $\mathcal{A} = \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$, M even. The sequence $\langle a[k] \rangle$ is fed into the Tomlinson-Harashima precoder which generates channel symbols $x[k]$.

In order to generate the analog, continuous-time transmit signal $s(t)$, the channel symbols are passed through the transmit filter with transfer function $G_{\mathrm{T}}(f)$. The symbol interval is denoted by T. The transmit signal is fed to the subscriber line which is as usual modeled as a linear, dispersive system with transfer function $H_{\mathrm{C}}(f)$. Crosstalk of other systems installed in the same binder group is combined into the additive Gaussian noise signal $n(t)$. Because of phase randomizing, the noise is expected to be stationary with power spectral density $\Phi_{nn}(f)$. The receiver consists of a whitened-matched filter [4] and subsequent T-spaced sampling, which is optimal for various equalization methods [4].

Regarding the end-to-end transmission including transmit filter, channel, and whitened-matched filter, a T-spaced discrete-time model can be established. Assuming proper scaling, the signal transfer function is given by the monic polynomial $H(z) = 1 + \sum_{k=1}^{p} h[k] z^{-k}$ of some order p, and the additive white Gaussian noise $n[k]$ has variance σ_n^2. For details see e.g. [3]. Note, without noticeable loss, the overall impulse response $\langle h[k] \rangle$ can be shortened to a great extent, by using an approximate whitened-matched filter [3]. Finally, the channel decoder produces estimates $\hat{a}[k]$ of the data symbols, from which the binary data can be extracted.

II.2. Tomlinson-Harashima Precoding

The intersymbol interference present in the end-to-end cascade with transfer function $H(z)$ may be removed by precoding at the transmitter side, which avoids the disadvantages of decision-feedback equalization (DFE). Therefore, during start-up, the impulse response $\langle h[k] \rangle$ is communicated back to the transmitter. Having knowledge on the channel transfer function, the transmitter can pre-equalize the transmit signal. Since linear pre-equalization boosts transmit power, or does not exist if $H(z)$ has spectral zeros, a nonlinear technique employing modulo arithmetics, usually referred to as *Tomlinson-Harashima precoding (THP)* was introduced in [7, 13].

For an M-point PAM signal set $\mathcal{A}$ the operation of THP can be interpreted as follows [2, 6]: A unique sequence $\langle d[k] \rangle$, $d[k] \in 2M\mathbb{Z}$, called the precoding sequence, is added to the data sequence $\langle a[k] \rangle$ in order to create the "*effective data sequence*" *(EDS)* $\langle v[k] \rangle$, with $v[k] = a[k] + d[k]$. By this, the initial signal set $\mathcal{A}$ is extended periodically. All points spaced by $2M$ are equivalent and represent the same information. Then $\langle v[k] \rangle$ is filtered with $1/H(z)$. The samples of the precoding sequence are chosen symbol-by-symbol, so that the channel symbols $x[k] = v[k] - \sum_{\kappa=1}^{p} h[\kappa] \cdot x[k-\kappa]$ fall into the interval $[-M, +M)$. This selection can be described by a simple memoryless modulo operation.

Since the precoder is matched to the channel impulse response its postcursors are canceled. Hence, the effective data symbols $v[k]$ corrupted by additive white Gaussian noise are present at the input of the decision device / channel decoder. Applying the same mod operation into the interval $[-M, +M)$ as at the transmitter, the precoding sequence may be eliminated, but equalization (e.g. adaption of the whitened-matched filter) still has to be done with respect to $v[k]$ prior to the nonlinear device.

II.3. Problems in Precoded Systems

Dynamic Range at the Receiver: From the above discussion it follows that the receiver has to work on the dynamic range of the periodically extended signal constellation. For high orders p, the pdf of $v[k]$ is nearly discrete Gaussian, and since $v[k]$ is an odd integer, its range can be as large as

$$|v[k]| \le V_{\text{THP}} \stackrel{\text{def}}{=} 2 \cdot \left\lfloor \left(M \cdot \sum_{k=0}^{p} |h[k]| + 1 \right) \Big/ 2 \right\rfloor - 1 \,, \quad (1)$$

cf. [3], where $\lfloor \cdot \rfloor$ denotes rounding to the next smaller integer. Only for $H(z) = 1$ (no ISI) the minimum $V_{\text{THP}} = M - 1$ occurs. For severe ISI producing channels as in xDSL the dynamic range of $v[k]$ becomes very large ($V_{\text{THP}} \gg M$). This in turn rather complicates implementation. Especially, the demands on the accuracy of equalization and symbol timing become extremely high. Even very small residual pre- and post-cursors of pulses weighted by very large samples $v[k]$ of the effective data sequence cause intolerable ISI. Jitter of the symbol timing, which may be still tolerable in DFE has to be avoided because of the much smaller horizontal opening of the eye pattern in precoded transmission schemes.

Hence, a first requirement for DSL transmission is to do precoding with a smaller, and well prescribed dynamic range for the effective data symbols.

Peak-to-Average Power Ratio at the Transmitter: In order to obtain the actual transmit signal, the channel symbols $x[k]$ are passed through the transmit filter $G_{\text{T}}(f)$. Due to the dispersive character of this filter, the uniform distribution of $x[k]$ is smeared and the average[1] pdf of $s(t)$ is broadened. As the time duration of the transmit pulse gets longer—which is the case as the band edges get steeper—the pdf becomes Gaussian. Now peaks may occur in the transmit signal, and hence its peak-to-average power ratio (PAR) is increased compared to that of $x[k]$.

The line driver, which amplifies the analog transmit signal, has to cope with the full dynamic range of $s(t)$, and work linearly over the entire region. Hence, implementation becomes more difficult and the power loss at the driver tends to be significant. Especially if the transceiver at the customer or repeaters should be supplied remotely with power (using the same twisted pair line as the data signal) power consumption is a major concern. In the central office were a large number of modems are collocated, waste heat is undesired, too. The loss at the drivers—which is directly related to PAR—has to be as low as possible.

Hence, a second desire for DSL transmission is to create an analog transmit signal with peak-to-average power ratio as low as possible.

Average Transmit Power: Finally, in DSL applications, near-end crosstalk of other high-rate systems is the main source of disturbance. Within a binder group many systems may be installed, and thus operate on lines in close proximity. Here, transmitted signal power of one user directly translates to noise in the other systems. A reduction of average transmit power without sacrificing performance in the regarded system is thus highly appreciated.

Hence, a third desire for DSL transmission is to transmit at lowest average power as possible.

There exists a trade-off between the three above demands. E.g., decreasing the dynamic range at the receiver increases the PAR at the transmitter, as do shaping techniques for reducing average transmit power. System design has to look for the best exchange of dynamic range at the receiver, PAR, and average power at the transmitter.

III. SHAPING FOR PAR REDUCTION

The objective of *signal shaping* is to create a signal which exhibits some desired properties. From literature, a number of techniques for reducing the average transmit power (power shaping)

[1] Note that the transmit signal in PAM schemes is cyclo-stationary with period equal to the symbol duration T.

are known. We now propose a transmitter, which is fully compatible with THP, but enables a trade-off over a wide range between the above mentioned three demands.

III.1. Dynamics Shaping

On ISI channels, combined precoding and shaping can be done by selecting the effective data sequence $\langle v[k]\rangle$, or equivalently the precoding sequence $\langle d[k]\rangle$ suitably. Based on the desired shaping aim, an algorithm has to determine ("decode") the best sequence $\langle d[k]\rangle$ on the long run. Since we aim to reduce the dynamic range at the *receiver* side, only those values $d[k]$ will be considered, for which $|v[k]| = |a[k] + d[k]| \le V_{\max}$ holds. Here, $V_{\max}$ is the dynamic limit stipulated by the designer.

For power shaping it is shown in [2, 5] that it is sufficient to double the support of the signal constellation—the decoder has to decide only between two candidate signal points. Following this idea, we divide the expanded signal set $\mathcal{V}$ into 2 subsets ($\mathcal{V} = \mathcal{V}_0 \uplus \mathcal{V}_1$) with

$$\begin{aligned} \mathcal{V}_0 &= (\mathcal{A} + 4M\mathbb{Z}) \cap [-V_{\max},\, +V_{\max}] \ , \\ \mathcal{V}_1 &= (\mathcal{A} + 4M\mathbb{Z} + 2M) \cap [-V_{\max},\, +V_{\max}] \ . \end{aligned} \qquad (2)$$

The decoder makes a binary decision $b[k] \in \{0,\, 1\}$ on the subset. Then the point from $\mathcal{V}_{b[k]}$ is chosen which results in the channel symbol $x[k]$ with the least magnitude.

This two-stage selection can be characterized as follows: Given the binary decision of the decoder, the signal $p[k] = a[k] + 2M \cdot b[k]$ is generated. Then, $p[k]$ is fed to the precoder, where compared to THP the modulo device is replaced by a memoryless, nonlinear function $f_p(\cdot)$ (cf. Figure 2). Mathematically this set of functions is defined as

$$f_p(q) = q + 4M \cdot \underset{\delta\in\mathbb{Z},\ |p+4M\delta|\le V_{\max}}{\operatorname{argmin}} |q + 4M\delta| \ . \qquad (3)$$

Finally, in order to replace sequential decoding, we *imagine* that the binary sequence $\langle b[k]\rangle$ is the output of a scrambler (rate-1 linear, dispersive system over the Galois field $\mathbb{F}_2$). This scrambler defines a trellis, whose branches are labeled with scrambler output bits $b[k]$. This trellis is searched for the "best" sequence $\langle b[k]\rangle$ using a Viterbi algorithm. Thereby, for calculating the channel symbols, the influence of the predistortion filter $1/H(z)$ has to be taken into account, e.g., by means of a parallel decision-feedback (PDF) decoder [2]. For power shaping the branch metric

$$\lambda[k] = |x[k]|^2 \ , \qquad (4)$$

i.e., the instantaneous power of the channel symbols $x[k]$, is used. Noteworthy, in order to guarantee the desired restriction of the dynamic range continuous-path integrity is essential.

One of the main advantages of this shaping technique is the compatibility with THP. Since it is also based on the concept of modulo-equivalence, a simple modulo device at the receiver is sufficient to eliminate the precoding sequence.

III.2. Reducing the Peak-to-Average Power Ratio

In addition to the signals shaped by dynamics shaping, we now want to reduce the peaks in the continuous-time *transmit* signal $s(t)$, too. Therefore, for each path at the shaping decoder its associated signal $s(t)$ is calculated. In practice, a sampled version with U samples per modulation interval T is sufficient, which can be efficiently calculated by a poly-phase filter. From these samples $s[lT_s] = s(t = lT_s)$, $T_s = T/U$, an appropriate metric increment $\lambda[k] = f(s[kT], s[kT + T_s], \ldots, s[kT + (U-1)T_s])$ has to be calculated. Figure 2 sketches the block diagram of dynamics shaping and its modification for peak-power control. Noteworthy, the decoding delay which is unavoidable in implementation is not shown.

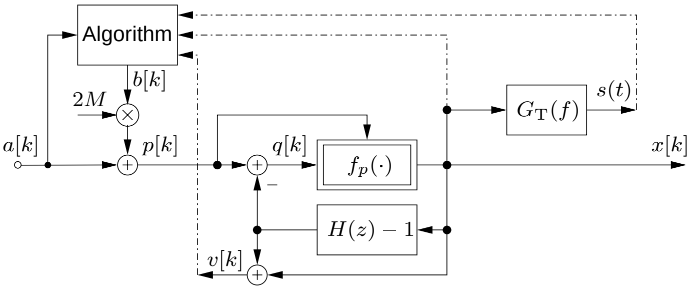

Figure 2: Structure of dynamics shaping (decoding delay not shown).

The remaining task is to select a suited branch metric $\lambda[k]$ for peak-reduction.

Maximum Amplitude: An obvious strategy is to look only at the maximum values of the transmit signal. Unfortunately, using this approach, the average power will increase significantly, since transmit power is disregarded. Consequently, the metrics has to incorporate some power term.

Clipping: In [12] it was proposed to use power metrics combined with clipping of paths which exceeds a given threshold. A disadvantage is that the clipping level has to be selected very carefully. For high clipping levels, signal peaks above the chosen level will be eliminated. But if the threshold is chosen too low, the algorithm may run into dead ends and produce signals inconsistent with the shaping aim.

Soft Clipping: Instead of setting the metric to infinity if the threshold is exceeded, a course which punishes large signal values harder than the quadratic function is suitable. Thereby, a proper operation of the shaping algorithm can always be guaranteed. The disadvantage is that still the threshold has to be optimized for the present situation.

m^{th}*-Power Metric:* A very promising approach is to simply replace the instantaneous power metric $|s|^2$ by the m^{th} power, $m > 2$,

$$\lambda[k] = \sum\nolimits_{u=0}^{U-1} |s[kT + uT_s]|^m \ . \qquad (5)$$

Now, only the exponent m has to be optimized. For increasing m, large signal peaks are punished even harder, whereas the contribution of signal with small amplitudes to the accumulated metric tends to zero. Setting $m = 2$, pure power shaping is active. Because of its simplicity, subsequently we focus on the m^{th}-power metric for PAR reduction.

IV. NUMERICAL RESULTS

IV.1. Simulation Parameters

For the examples a typical scenario of single-pair digital subscriber lines within European networks is regarded [8]. We assume transmission of 2.312 Mbit/s in the up-stream direction over a wire pair with field length 3.0 km. The transfer characteristics of the loop model with 0.4 mm diameter is chosen from [8, Appendix II; PE 04] (ETSI ETR152 loop2). As the additive disturbance, alien noise model B of [8, Appendix B.3] is assumed, which reflects a medium penetration scenario.

For SDSL, a $(M = 16)$-ary PAM signal set is used, which carriers 3 bits information; trellis coded modulation adds one bit redundancy. Pulse shaping is done according to [9], i.e., a rectangular pulse with width equal to the symbol duration $T = 3/(2.31\ \text{Mbit/s}) = 1.3\ \mu$s is filtered with a 6^{th}-order Butterworth filter with 3 dB cut-off frequency at half the symbol rate of $1/T = 771$ kHz. The nominal transmit power is fixed to be 14.5 dBm. For metric generation, $U = 4$ samples per symbol interval of the transmit signal are calculated.

From the above parameters the discrete-time channel model $H(z)$ can be calculated. As $H(z)$ does not vary significantly as the parameters are changed slightly, the results are valid for a broad spectrum of scenarios.

In each case, for shaping a Viterbi algorithm with 16 states based on the trellis defined by the scrambler with generator polynomial $1+D^3+D^4$ is employed. The path register length equals 64 symbols. Noteworthy, it is essential that the pulse shaping filter is minimum-phase. Otherwise the impact of the current selection is needlessly delayed, which can only be combatted by increasing the number of states.

IV.2. Results

The performance of the shaping technique is assessed by regarding the clipping probability. First, we neglect the restriction of the dynamic range at the receiver side, i.e., $V_{\max} \to \infty$ is chosen.

In Figure 3, the complementary cumulative density function of $|s(t)|^2$ when using the m^{th}-power metric is plotted for different values of m. From the diagram, the probability of exceeding a certain threshold can be extracted. For convenience, the instantaneous signal power is normalized to the average transmit power S_{THP} in a THP system (dashed line), which constitutes the base line performance. Thus, $S_{\text{norm}} = |s(t)|^2/S_{\text{THP}}$ holds.

For $m = 2$, pure power shaping is active. Since the distribution of $s(t)$ tends to be Gaussian, the average power is reduced by about 0.54 dB (shaping gain), but the peak value is increased significantly. By going to larger m the peaks are eliminated at the cost of average power, and for given clipping probability, e.g. 10^{-6}, a gain up to 1.5 dB in clipping level can be achieved. In practice, clipping probabilities lower than $\approx 10^{-6}$ should be tolerable; due to channel coding no errors will occur. Subsequently we focus on $m = 16$, which seems to be a good trade-off between complexity (4 sucessive multiplications are required, since $s^{16} = (((s^2)^2)^2)^2$ holds) and achievable performance.

Next, we consider an additional restriction of the dynamic range at the receiver side. Note, for THP the maximum possible amplitude calculates to about $V_{\text{THP}} = 22 \cdot M$. Figure 4 plots the complementary cumulative density function of the transmit signal $s(t)$ for $m = 16$ and various amplitude restrictions $V_{\max}$ (solid lines). Additionally, the performance of THP is shown, and for dynamics shaping, based solely on the energy of the discrete-time channel symbols $x[k]$ (as originally proposed in [3]).

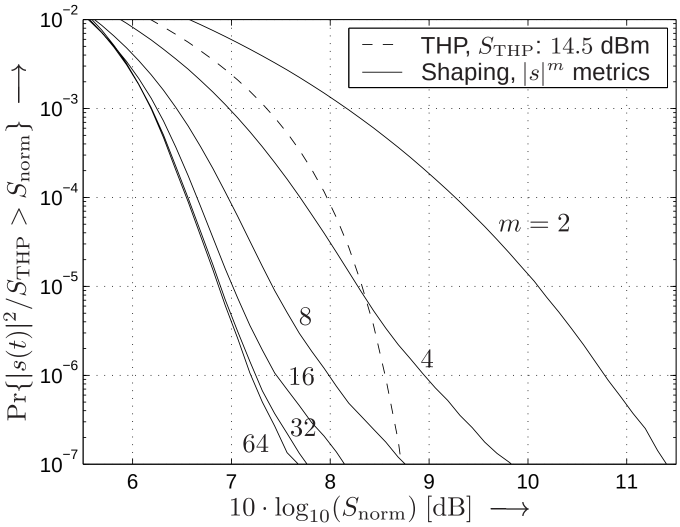

Figure 3: Complementary cumulative density function of $|s(t)|^2$. m^{th}–power metrics; $m = 2, 4, 8, 16, 32, 64$. Dashed line: THP.

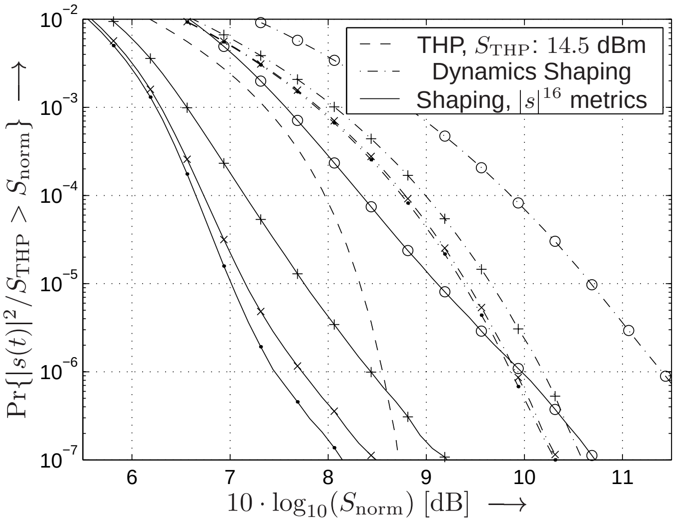

Figure 4: Complementary cumulative density function of $|s(t)|^2$. Dashed line: THP. Dash-dotted lines: Dynamics shaping with respect to discrete-time channel input signal. Solid lines: Shaping on the continuous-time transmit signal $s(t)$ using 16^{th}-power metrics. Restriction of the dynamic range at the receiver: $V_{\max} = 3M$ ($\circ$), $5M$ ($+$), $7M$ ($\times$), and ∞ ($\cdot$).

Restricting the dynamic range at the receiver side clearly leads to an increase of the peak power at the transmitter. This effect is increased as the restriction $V_{\max}$ on the dynamic range is tightened. If shaping is done based on the signal $x[k]$ prior to pulse shaping, the performance is worse than that for THP. But if the metric is calculated from $s(t)$, even for a restriction to $V_{\max} = 5M$ at the receiver, a gain in peak-power at the transmitter is possible. Moreover, in this case, the average transmit power is decreased by 0.29 dB compared to THP.

A summary of the performance is given in Figure 5. The curves are parameterized by the restriction $V_{\max}$ on the dynamic range, and are normalized to the performance of THP ($V_{\mathrm{THP}} = 23M$), which corresponds to the origin (square). The x-axis shows the gain in clipping level, when the clipping probability at the transmitter is fixed to be 10^{-6}, and the y-axis gives the respective reduction in average transmit power. Restricting $V_{\max}$

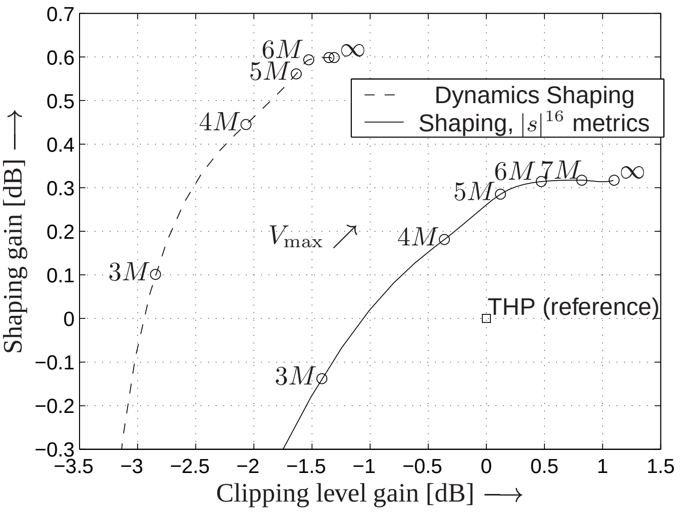

Figure 5: Shaping gain versus gain in clipping level (clipping probability 10^{-6}). Upper left curve: Dynamics shaping with respect to the discrete-time channel input signal $x[k]$. Lower right curve: Shaping on the continuous-time transmit signal $s(t)$ using 16^{th}-power metrics. Variation of restriction $V_{\max}$ on the dynamic range.

as low as $5M$ causes almost no loss. Shaping gain as well as gain in reduction of the dynamic range is possible by the newly proposed algorithm. Specifically, for $V_{\max} = 5M$ still 0.29 dB shaping gain, and 0.15 dB clipping level gain are achievable. Only if the maximum amplitude at the receiver is limited below $4M$, signal power as well as clipping level are increased. For dynamics shaping, as described in [3], higher shaping gains are possible but at the price of increased dynamic range at the transmitter. In summary, employing the new algorithm, over a wide range the three demands discussed above can be fulfilled simultaneously.

Finally, a remark concerning the influence of shaping on the power spectral density. When comparing the spectra with that of THP, around Nyquist frequency (≈ 380 kHz) some accentuation occur. Since the spectral mask is not exhausted in this region, regimentation will not be violated. Only if the dynamic range at the receiver is restricted to less than $3M$, the power spectral density will exceed the spectral mask.

The same investigations have also been performed for the OPTIS power spectrum, defined for HDSL2 [1], and also used in [8, Appendix A]. Compared to the PSD used above, it has a much steeper roll-off region. In turn, even for THP precoding an almost Gaussian distributed transmit signal $s(t)$ is present. In this case, even pure power shaping is rewarding, since reducing average power of a Gaussian distribution also reduces the clipping probability. It can be observed that the proposed shaping algorithm can achieve even better results for transmit spectra with sharper band edges.

V. Conclusions

This paper proposes a combined precoding/shaping technique which is especially suited for fast digital transmission over twisted pair lines. The shaping algorithm aims (i) to reduce average transmit power, (ii) to mitigate peaks in the transmit signal, and (iii) to restrict the maximum amplitude at the receiver side to a well prescribed value. Over a wide range, all three demands can be met simultaneously. Simulation results for a typical SDSL scenario show the achievable gains of shaping. In particular, the opinion that reducing the average transmit power and the maximum amplitude at the receiver always increases the dynamic range at the transmitter is rebutted.

Implementation can be done efficiently by the Viterbi algorithm, which needs only a moderate number of states. Although only explained for baseband transmission, the extension to complex signals and systems is immediately possible. The proposed transmitter is fully compatible with THP, which permits a seamless application to all systems where precoding is used or standardized.

References

[1] ANSI draft for HDSL2 Standard. 1999.

[2] M.V. Eyuboğlu and G.D. Forney. Trellis Precoding: Combined Coding, Precoding and Shaping for Intersymbol Interference Channels. *IEEE Tr. IT*, 301–314, 1992.

[3] R. Fischer, W. Gerstacker, and J. Huber. Dynamics Limited Precoding, Shaping, and Blind Equalization for Fast Digital Transmission over Twisted Pair Lines. *IEEE JSAC*, 1622–1633, 1995.

[4] G.D. Forney. Maximum Likelihood Sequence Estimation of Digital Sequences in the Presence of Intersymbol Interference. *IEEE Tr. IT*, 363–378, 1972.

[5] G.D. Forney. Trellis Shaping. *IEEE Tr. IT*, 281–300, 1992.

[6] G.D. Forney and M.V. Eyuboğlu. Combined Equalization and Coding using Precoding. *IEEE Comm. Mag.*, 25–34, 1991.

[7] H. Harashima and H. Miyakawa. Matched-Transmission Technique for Channels with Intersymbol Interference. *IEEE Tr. COM*, 774–780, 1972.

[8] ITU. Draft Recommendation G.991.2: Single-Pair High–Speed Digital Subscriber Line (SHDSL) Transceivers, 2000.

[9] M. Kimpe. Optional Asymmetric PSDs at 2.048 and 2.304 Mbps Data Rates. TD40, ETSI TM6, Amsterdam, 1999.

[10] R. Price. Nonlinear Feedback Equalized PAM versus Capacity for Noisy Filter Channels. In *ICC'72*, 22.12–22.17, 1972.

[11] J.G. Proakis. *Digital Communications*. McGraw-Hill, New York, 3. edition edition, 1995.

[12] I. Reuven. Combined Constellation Shaping and Reduction of Peak-to-Average Ratio. TD24, ETSI TM6, Edinburgh, 1999.

[13] M. Tomlinson. New Automatic Equaliser Employing Modulo Arithmetic. *Electr. Letters*, 138–139, 1971.