Distortionless Reduction of Peak Power without Explicit Side Information

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Abstract— **Selected mapping (SLM) peak–power reduction [1] is distortionless as it selects the actual transmit signal from a set of alternative signals, which all represent the same information. Usually, the specific signal generation information needs to be transmitted and carefully protected against bit errors. Here, we propose an extension of SLM, which employs scrambling and refrains from transmitting explicit side information. Some additional complexity and nearly vanishing redundancy is introduced to achieve markedly improved transmit signal statistics. Even though SLM is applicable with any modulation, we concentrate on Orthogonal Frequency–Division Multiplexing (OFDM) in this paper.**

Keywords— **Peak–Power Reduction, PAR, SLM, Out–of–Band Power, OFDM**

I. Introduction

ONE major drawback of OFDM is the high peak–to–average power ratio (PAR). If nonlinearities are overloaded by large signal peaks, intermodulation among subcarriers and undesired out–of–band radiation is caused. Amplifiers operate with large power back–offs, to keep out–of–band power below specified limits, leading to inefficient amplification and expensive transmitters so that it is highly desirable to reduce PAR. In this paper, we extend the distortionless and flexible SLM scheme [1], [2], which reduces PAR by introducing little redundancy [3], [4]. In SLM, the transmitter generates a set of sufficiently different candidate signals which all represent the same information and selects the most favorable for transmission. Generation of alternative candidates can be realized in many ways and SLM does neither impose restrictions on signal alphabets used in the subcarriers, their number nor on the type of channel coding. The modified SLM approach is also applicable with all types of modulation [5], but here, we focus on OFDM.

The paper is organized as follows: Section II recapitulates OFDM signaling. In Section III, the SLM approach is reviewed and our extension is presented. Section IV provides simulation results for bit error performance and transmit signal spectra for one specific implementation.

II. OFDM Transmission

OFDM utilizes D_u (used) "orthogonal" subcarriers with uniform frequency spacing. Frequency multiplexing is implemented by D–point inverse discrete Fourier transform (IDFT) ($D \geq D_\mathrm{u}$) in the transmitter. Data is mapped onto D_u subcarrier amplitudes A_ν with $0 \leq \nu < D$ being the discrete subcarrier index. Here, all D_u active subcarriers use the same signal set $\mathcal{A}$, but the proposed scheme also works for mixed signal constellations. Inactive (so–called virtual) subcarriers [6] are set to zero to shape the transmit signal's power spectral density (PSD).

The *subcarrier vector* $\boldsymbol{A} = [A_0, \ldots, A_{D-1}]$ with all subcarrier amplitudes of the current OFDM symbol interval is transformed by IDFT to obtain T–spaced samples of the signal in the current block. This so–called *transmit sequence* $\boldsymbol{a} = [a_0, \ldots, a_{D-1}]$ is given by $a_\kappa = \frac{1}{\sqrt{D}} \sum_{\nu=0}^{D-1} A_\nu \, \mathrm{e}^{+\mathrm{j}\frac{2\pi}{D}\kappa\nu}$, $0 \leq \kappa < D$, where κ is the discrete sample index within an OFDM symbol. The block processing can be described by $\boldsymbol{a} = \mathrm{IDFT}\{\boldsymbol{A}\}$. The samples a_κ are transmitted by ordinary T–spaced pulse amplitude modulation. A guard interval [6] is not introduced here, as it does not affect the PAR.

The receiver performs the DFT to obtain received subcarrier amplitudes $Y_\nu = \frac{1}{\sqrt{D}} \sum_{\kappa=0}^{D-1} y_\kappa \, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{D}\nu\kappa}$, $0 \leq \nu < D$, for demapping and decoding from the received samples y_κ. The received sample sequence is $\boldsymbol{y} = [y_0, \ldots, y_{D-1}]$ and the received subcarrier amplitude vector is $\boldsymbol{Y} = [Y_0, \ldots, Y_{D-1}]$, and, hence, this block processing can be described by $\boldsymbol{Y} = \mathrm{DFT}\{\boldsymbol{y}\}$.

III. Selected Mapping (SLM) with Scrambling

A. Review of Selected Mapping (SLM)

In SLM [1], [2] (and related [7], [8]), it is assumed that U statistically independent alternative transmit sequences $\boldsymbol{a}^{(u)}$, $0 \leq u < U$, which represent the same information are generated by some suitable algorithm. Finally, the sequence $\boldsymbol{a}^{(\tilde{u})}$ with lowest peak power is selected for transmission. Hence,

$$\tilde{u} = \underset{0 \leq u < U}{\operatorname{argmin}} \left(\max_\kappa |a_\kappa^{(u)}| \right) = \underset{0 \leq u < U}{\operatorname{argmin}} \, \| \boldsymbol{a}^{(u)} \|_\infty$$

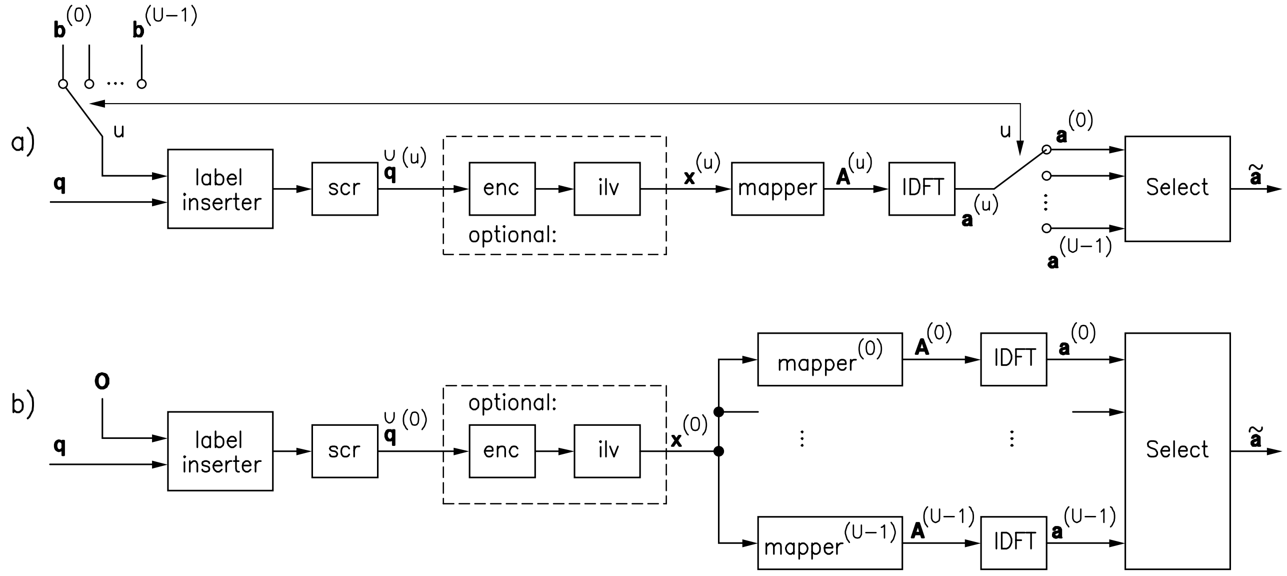

Fig. 1. SLM–OFDM transmitter with label insertion and scrambling approach.

needs to be determined, usually by exhaustive search. Clearly, $\|\cdot\|_\infty$ denotes the ∞–norm (Chebyshev norm) of the vector in the argument. The peak–power criterion is used instead of PAR, because the average power may be slightly modified by SLM, if signal sets with nonequal power of the signal points are used.

To perform the appropriate inverse operation, the SLM receiver requires knowledge about transmit sequence selection in the current symbol period. Thus, the number $\tilde{u}$ needs to be transmitted to the receiver unambiguously. Obviously, $\lceil \log_2 U \rceil$ bits are required to represent this side information, which is of highest importance to recover the data.

B. SLM with Scrambling (and Coding)

We propose a scrambling scheme, which abstains from explicit transmission and careful protection of side information. Fig. 1a displays a block diagram of the transmitter. To generate U different transmit sequences $\boldsymbol{a}^{(u)}$, $0 \le u < U$, representing the same binary information word $\boldsymbol{q}$, labels $\boldsymbol{b}^{(u)}$ are inserted as a prefix to $\boldsymbol{q}$. The labels are U different binary vectors of length $\lceil \log_2 U \rceil$, and we assume in this paper without loss of generality that $\boldsymbol{b}^{(0)} = \boldsymbol{0}$. The concatenated vector $\bar{\boldsymbol{q}}^{(u)}$ of the label prefix and the information word is then fed into a scrambler consisting of a shift–register with a feedback branch only as shown in Fig. 2, which is reset to the zero–state before the scrambling takes place. The labels are hence used to drive the scrambler into one of U different states before scrambling the information word $\boldsymbol{q}$ itself. The scrambled output vector $\breve{\boldsymbol{q}}^{(u)}$ is then processed as usual, i.e., in our example, it is channel encoded, interleaved, and mapped to a signal constellation. After the IDFT, we obtain the transmit sequence $\boldsymbol{a}^{(u)}$ associated with the inserted label $\boldsymbol{b}^{(u)}$. This proceeding is executed for $u = 0, \ldots, U-1$, and finally the specific transmit sequence number $\tilde{u}$, which possesses the lowest peak power, is selected and transmitted $\tilde{\boldsymbol{a}} = \boldsymbol{a}^{(\tilde{u})}$.

The transmitter scheme shown in Fig. 1b is equivalent to the scheme in Fig. 1a. The linearity (in GF(2)) of the label inserter, the scrambler, the channel encoder, and the interleaver can be exploited by processing the label vectors $\boldsymbol{b}^{(u)}$ and the information word $\boldsymbol{q}$ separately in these stages. Only a single interleaved codeword $\boldsymbol{x}^{(0)}$ needs to be generated, which is obtained from concatenating only the zero–label $\boldsymbol{b}^{(0)} = \boldsymbol{0}$ with $\boldsymbol{q}$ in the label inserter. Owing to the aforementioned linearity in GF(2), the U different subcarrier vectors $\boldsymbol{A}^{(u)}$, $u = 0, \ldots, U-1$ can then be generated by applying U different vector mappings to $\boldsymbol{x}^{(0)}$. Observe that the vector mapping number u needs to be calculated only once from the associated label $\boldsymbol{b}^{(u)}$ and can be stored in a read–only memory, and that the mapping may be different for each element of the input vector $\boldsymbol{x}^{(0)}$. These U parallel mappings are followed by IDFTs and the selection of the most favorable transmit sequence $\tilde{\boldsymbol{a}}$ as exhibited above. The second scheme in Fig. 1b clearly reveals the relationship to the SLM algorithm in [1].

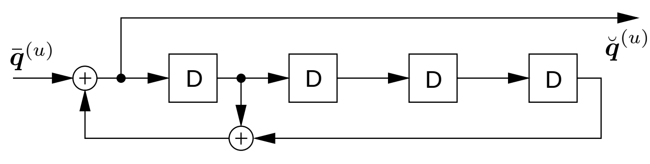

Fig. 2. Scrambler of memory 4 with feedback polynomial $1+x+x^4$.

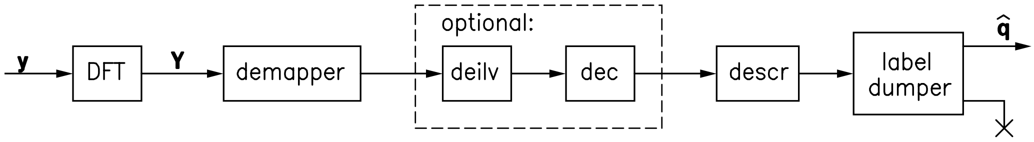

Fig. 3. SLM–OFDM receiver for scrambling approach. No explicit side information is required.

If the period length[1] of the scrambler is $\geq U-1$, then the U subcarrier vectors $\boldsymbol{A}^{(u)}$ are usually satisfyingly different from each other, such that they appear to be generated statistically independent from each other. Starting a scrambler in one of $U-1$ different nonzero states before scrambling $\boldsymbol{q}$ results in a $\breve{\boldsymbol{q}}^{(u)}$, which is the sum of $\breve{\boldsymbol{q}}^{(0)}$ and one out of $U-1$ pseudo–noise sequences. Additionally, the interleaver and the nonlinear (in GF(2)) mapper boost this pseudo–randomness effect of the mappings from $\boldsymbol{q}$ to $\boldsymbol{A}^{(u)}$, which is the essential ingredient of SLM schemes.

The corresponding receiver is depicted in Fig. 3. The received sample vector $\boldsymbol{y}$ is processed as in ordinary OFDM systems, i.e. DFT, demapped (or detected), deinterleaved, and decoded. The only additional devices are a descrambler and a label dumper. The descrambler performs the inverse operation to the scrambler in the transmitter and is hence a shift–register with a feedforward branch, only, as shown in Fig. 4. The descrambler is reset to the zero–state before descrambling starts for an OFDM symbol. In the Figure, $\hat{\breve{\boldsymbol{q}}}$ represents the estimated concatenated vector of the label prefix and the information word, and $\hat{\mathring{\boldsymbol{q}}}$ is this vector before descrambling (or equivalently: after scrambling). If no transmission errors have occured, the output of the descrambler is the concatenation of the transmitted $\boldsymbol{b}^{(\tilde{u})}$ and $\boldsymbol{q}$. The label dumper only needs to strip off the label prefix and output the estimated information word $\hat{\boldsymbol{q}}$. We see that the receiver can explicitly determine the number $\tilde{u}$ of the sequence selected for transmission, but it does not need this information for data recovery. If errors occur during transmission, the descrambler causes a moderate multiplication of these errors. As an example, consider the case that the transmitted binary vector is the all zero–word $\breve{\boldsymbol{q}}^{(\tilde{u})} = \boldsymbol{0}$. Then a transmission error manifests itself in the presence of a binary one symbol in $\hat{\mathring{\boldsymbol{q}}}$ before descrambling in the receiver. When shifted through the descrambler, for every weight–one tap this error can produce an error (i.e. a binary one) at the descrambler output $\hat{\breve{\boldsymbol{q}}}$. If two or more errors are shifted simultanously through the descrambler, then they can extinguish each other partially in the descrambler output. Therefore the total number of errors at the descrambler output is at most equal to the number of errors before descrambling multiplied by the number of weight–one taps of the descrambler, i.e. the weight of the (de–)scrambler polynomial. Therefore, a scrambler polynomial of low weight should be chosen. Since the channel decoder usually emits errors in bursts, the error multiplication is in practice even less severe.

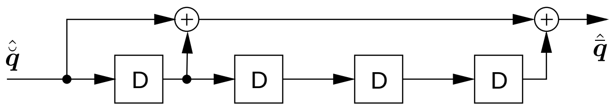

Fig. 4. The corresponding descrambler for the scrambler polynomial $1+x+x^4$.

[1]A maximum scrambler period is obtained, if the feedback polynomial of the scrambler is primitive in GF(2) [9].

In the case that the channel code is a convolutional code, the scrambler and the channel encoder — both are shift–registers — as well as the channel decoder and the descrambler can be integrated into a single device without further cost, respectively. This is possible for the case of Maximum Likelihood Sequence Decoders, such as a Viterbi decoder, and also for Symbol-By-Symbol Decoders, e.g. the BCJR algorithm [10]. Thus, the additional requirements in the receiver are indeed kept at a minimum. As regards to the transmitter, the main cost is constituted by the U vector mappings, the parallel IDFTs (cf. Fig. 1b) and determining the optimal sequence. Assuming that the most expensive part of the transmitter are the IDFTs, the cost in the transmitter is roughly multiplied by U.

Observe that the bits comprised in the label are transmitted in the same way as the information bits of $\boldsymbol{q}$; thus, the label bits are also encoded and thereby protected like the information bits. In fact, they represent information that has to be transmitted; if U is a power of 2, then the information content of every label bit is exactly 1 bit, since the transmission of all U candidate sequences is equally probable for the receiver. The information in the bits of the information word $\boldsymbol{q}$ and of the selected label is however not transmitted *directly*. Due to the presence of the scrambler this information is rather contained in the *combination* of *several* successive bits after the scrambler; thus, the information of any bit is "smeared" over a larger part of the scrambled sequence and eventually the transmit sequence.

IV. Simulation Results

A. System Description

To demonstrate the proposed SLM extension, we consider a system which could be used for wireless Asynchronous Transfer Mode (ATM) devices. We employ $D = 256$ and $D_u = 219$ used carriers. The SLM processing is performed with an oversampling factor of two, i.e., IDFTs of size $2D$ with zero–padding are used to generate the candidate sample sequences. Those are analyzed with respect to peak power to obtain its classification for the selection process. The oversampled signal representation yields sufficiently accurate peak–power information on the final continuous–time signal $s(t)$ after impulse shaping. The root–raised cosine Nyquist transmit filter to generate $s(t)$ in our simulations has a rolloff factor of $\alpha = 0.12$. The continuous–time signal simulation is performed with an oversampling with factor eight, i.e., $s(kT/8)$ is used to quantify the continuous–time characteristic of $s(t)$ and to simulate the power spectral density of $s'(t)$, which is the distorted transmit signal obtained by passing $s(t)$ through a possibly nonlinear device.

An additive white Gaussian noise (AWGN) channel with one–sided power spectral density N_0 is used. Hence, no guard interval needs to be implemented. Further, perfect synchronization is assumed.

We use a scrambler with a feedback polynomial $1+x+x^4$ and a rate–1/2 industry–standard convolutional code with 64 states. The bits are interleaved and mapped onto the 16QAM symbols of one OFDM symbol. Gray labeling is used, such that a bit–interleaved coded modulation (BICM) [11] is realized. Hence, an OFDM–symbolwise blocked convolutional–coded system results. Note that our proposal does not actually require this blocking.

B. Improved Statistics of Signal Magnitude

Fig. 5 illustrates simulation results for the probability that the instantaneous power $|s(kT/8)|^2$ corresponding to samples from the complex–valued continuous–time signal $s(t)$ exceeds the power threshold s_0^2. The abscissa is normalized to the average transmit signal power σ_s^2 of $s(t)$ and given in decibels.

We want to coin the term *probabilistic PAR* for the normalized power threshold s_0^2/σ_s^2 which is connected with some fixed power–excess probability on a sample–by–sample basis. Hence, we can say that the probabilistic PAR of conventional OFDM at excess probability 10^{-5} is 10.6 dB. This expresses that one in 100000 samples exceeds a power threshold s_0^2, which is by 10.6 dB larger than σ_s^2. Obviously, we have a reduction of probabilistic PAR by 1.8 and 2.5 dB (at 10^{-5}), if we consider SLM with $U = 4$ and $U = 16$, respectively.

16QAM modulation (PAR of the signal constellation $\mathcal{A}$: 2.55 dB) is used, but the statistics do not differ greatly, when 4PSK or 8PSK modulation is used (PAR of $\mathcal{A}$: 0 dB).

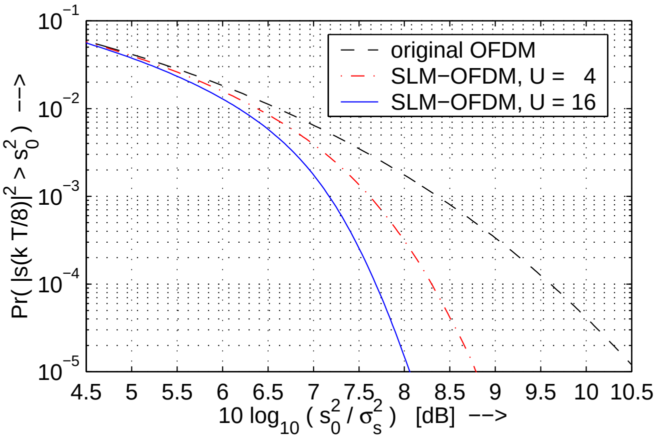

Fig. 5. Power–excess probability for transmit signal $s(t)$.

C. Bit–Error Performance

The demapper computes bitwise probabilities from the received subcarrier amplitudes. Perfect knowledge of noise variance and channel state (AWGN) is assumed.

In Fig. 6, we show the bit residual error rate (BRER) vs. SNR per bit information for conventional transmission (no scrambler) and SLM transmission (with scrambler). The rate loss for the label insertion is negligible. Due to error propagation in the receiver, we observe a small degradation in BRER which is typically around 0.2 dB caused by the increase in BRER by a factor lower than the weight of the scrambler polynomial. The BRERs for the case without scrambler correspond well with literature [11, Fig. 11].

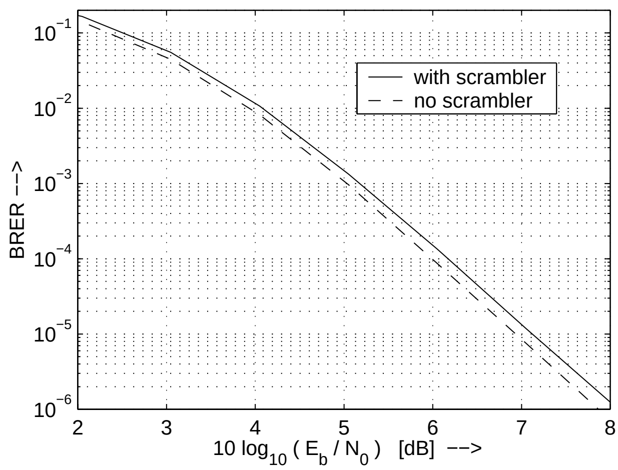

Fig. 6. Bit residual error rate in AWGN, i.e., after convolutional decoding for original OFDM and scrambled SLM–OFDM.

The beneficial effect of the reduced nonlinear distortion on the interesting range of bit error rates is negligible at back–offs larger than 6 dB, especially if coded transmission is considered.

D. Spectral Benefit

The continuous–time transmit signal $s(t)$ is generated from the OFDM sample sequence by a square–root Nyquist transmit filter with rolloff factor of $\alpha = 0.12$. This signal is passed through a possibly nonlinear device to obtain $s'(t)$. The out–of–band power after a simple soft–limiting nonlinearity[2] is evaluated by measuring the PSD of the distorted transmit signal $s'(t)$.

Fig. 7 shows the PSDs $\Phi_{s's'}(f)$ for the transmitted signal after an ideally linear as well as after a nonlinear characteristic. The transmission under ideal conditions leads to no spectral spread, while the transmission of original OFDM via a nonlinear device produces considerable out–of–band power for 6, 7, 8, and 9 dB back–off from saturation point. For comparison, we plotted the PSDs for SLM–OFDM with $U = 4$ and 16 candidates for 6 and 7 dB back–off. It can be concluded that — depending on the tolerated level of out–of–band density — between 1 and 2 dB can be saved in back–off with 4 bits redundancy per OFDM symbol and additional complexity.

As is the case with sample magnitude statistics, the results do not differ greatly, when 4PSK or 8PSK instead of 16QAM modulation is used in the subcarriers.

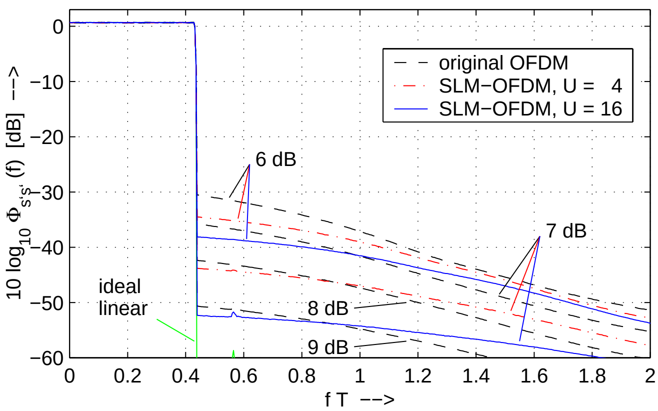

Fig. 7. Power spectral density of $s'(t)$ after ideally linear and soft–limiting nonlinear characteristic for original OFDM and SLM–OFDM for various back–offs.

V. Summary and Conclusions

We proposed a powerful extension for SLM PAR reduction and demonstrated its operability for the special case of convolutionally–coded OFDM transmission. The scheme refrains from explicit transmission of side information by a label insertion and scrambling approach. Only little redundancy is introduced into the signal and the BRER performance is degraded by 0.2 dB due to error propagation in the descrambler. Often, a scrambler is anyhow present in the transmitter to destroy long zero–bit sequences, so that this is no additional loss. On the other hand, the transmit signal statistics and the spectral properties in presence of transmitter nonlinearities are decisively improved such that a saving of 1 to 2 dB in back–off can easily be achieved.

With this proposal, the SLM scheme for PAR reduction gains additional attraction for practical implementation. We emphasize again that SLM is also suitable for other modulation schemes, e.g. single–carrier modulation or code–division multiple access.

[2]The soft–limiter amplitude characteristic is ideally linear up to the perfectly horizontal saturation region. The signal phase is not modified by this model.

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References

[1] Robert Bäuml, Robert Fischer, and Johannes Huber, “Reducing the Peak–to–Average Power Ratio of Multicarrier Modulation by Selected Mapping,” *Electronics Letters*, vol. 32, no. 22, pp. 2056–2057, October 1996.

[2] Stefan Müller, Robert Bäuml, Robert Fischer, and Johannes Huber, “OFDM with Reduced Peak–to–Average Power Ratio by Multiple Signal Representation,” *Annals of Telecommunications*, vol. 52, no. 1-2, pp. 58–67, February 1997.

[3] “German patent no. 199 58 210.6,” 1999.

[4] Stefan Müller–Weinfurtner, *OFDM for Wireless Communications: Nyquist Windowing, Peak–Power Reduction, and Synchronization*, Reihe Kommunikations- und Informationstechnik, Band 16, ISBN 3-8265-7658-6. Shaker Verlag, Aachen, 2000, Dissertation Universität Erlangen–Nürnberg.

[5] Michael Beer, *Reduktion des Spitzenwertfaktors bei steilflankiger Einträgermodulation (in German)*, Diplomarbeit am Lehrstuhl für Nachrichtentechnik II, Universität Erlangen–Nürnberg, Oktober 1998.

[6] Hikmet Sari, Georges Karam, and Isabelle Jeanclaude, “Transmission Techniques for Digital Terrestrial TV Broadcasting,” *IEEE Communications Magazine*, vol. 33, no. 2, pp. 100–109, 1995.

[7] P. Van Eetvelt, G. Wade, and M. Tomlinson, “Peak to Average Power Reduction for OFDM Schemes by Selective Scrambling,” *Electronics Letters*, vol. 32, no. 21, pp. 1963–1964, October 1996.

[8] Denis J.G. Mestdagh and Paul M.P. Spruyt, “A Method to Reduce the Probability of Clipping in DMT–Based Transceivers,” *IEEE Trans. on Commun.*, vol. 44, no. 10, pp. 1234–1238, October 1996.

[9] Solomon Golomb, *Shift Register Sequences*, Aegean Park Press, revised edition, 1982.

[10] L.R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal Decoding of Linear Codes for Minimizing Symbol Error Rate,” *IEEE Transactions on Information Theory*, vol. IT-24, no. 3, pp. 284–287, 1974.

[11] Giuseppe Caire, Giorgio Taricco, and Ezio Biglieri, “Bit–Interleaved Coded Modulation,” *IEEE Transactions on Information Theory*, vol. 44, no. 3, pp. 927–946, 1998.