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Optimum Nyquist Windowing for Improved OFDM Receivers

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Abstract— **Conventional Orthogonal Frequency–Division Multiplexing (OFDM) receivers disregard oversized guard intervals. An (adaptive) Nyquist–shaped receive window could exploit signal samples in its unconsumed portion to improve demodulation. The OFDM transmitter is not modified, subcarrier orthogonality is preserved, and the DFT size may be retained. In this paper, we optimize the window shape by considering additive noise and intercarrier interference due to carrier frequency offsets, jointly.**

Keywords— **OFDM, Optimum Demodulation, Carrier Frequency Offset, Nyquist Window**

I. Introduction

CONVENTIONAL OFDM reception [1] ignores signal samples in the guard interval (cyclic prefix/postfix). Only the so–called useful samples are input to a discrete Fourier transform (DFT) which performs OFDM demodulation. This corresponds to rectangular windowing of the received signal in time domain.

Real–world scenarios suffer from residual frequency offsets, oscillator phase noise and Doppler spread, causing intercarrier interference (ICI) among subcarriers. OFDM is standardized as modulation scheme in single frequency networks for broadcasting (cf. DAB or DVB) and as soon as receiver mobility is involved, we may encounter worst–case Doppler shifts by antipodal frequencies. Concludingly, it is highly desirable to lower the sensitivity of OFDM to carrier frequency offsets of any kind.

Spectral properties improve by allowing excess time for demodulation windows [2]. In many channels, the guard interval is oversized, i.e., the actual channel–impulse–response duration is shorter than the implemented guard interval. We refer to that part of the guard interval which is not affected by echoes from previous OFDM symbols as *unconsumed guard interval*. Samples in that region can be exploited to improve performance.

Nonrectangular windows emerged in [3], where the root raised–cosine shape is used in transmitter and receiver to lower synchronization requirements. In [2], a raised–cosine shape is used in the receiver, only. In [4], a Nyquist shape minimizes noise power in the subcarriers. Other approaches to OFDM transmission with modified window shapes [5], [6] include transmitter modifications to reduce out–of–band power of the transmitter by waveform shaping which differs from our intention.

The paper is organized as follows: After description of the transmission model and the concept of a Nyquist–shaped receive signal windowing in Section II, the derivation of optimum window shapes for joint noise and ICI is presented in Section III together with results and extensions. Section IV compares different window shapes.

II. Transmission Model

A. Conventional OFDM Transmitter

We consider the transmission of one single OFDM symbol generated by a D–point inverse DFT. Subcarrier number ν is modulated by the complex–valued signal point A_ν. The signal points for all D_u $(\leq D)$ active subcarriers are taken from the same zero–mean signal set $\mathcal{A}$ with variance $\sigma_\mathcal{A}^2$. The $D - D_\mathrm{u}$ virtual [1] subcarriers are set to zero. The modulation interval (sample spacing) for the system is denoted by T so that the subcarrier spacing is $\Delta f_\mathrm{sub} = \frac{1}{DT}$. The OFDM symbol is equipped with D_g samples guard interval [1] in total. Here, they are split up into a cyclic prefix and a cyclic postfix of D_pr and D_po samples, respectively. For efficiency, we can definitely assume $D_\mathrm{g} = D_\mathrm{pr} + D_\mathrm{po} < D$. The discrete–time complex baseband transmit samples are $s_k = \frac{1}{\sqrt{D}}\sum_{\nu=0}^{D-1} A_\nu \, \mathrm{e}^{+\mathrm{j}\frac{2\pi}{D}\nu k}$, $-D_\mathrm{pr} \leq k < D+D_\mathrm{po}$, where k is the discrete time. The average signal power is $\sigma_s^2 \triangleq \mathcal{E}\{|s_k|^2\} = (D_\mathrm{u}/D)\sigma_\mathcal{A}^2$ via Parseval's theorem.

B. OFDM Receiver with Nyquist Window

Transmission over a nondispersive channel with residual carrier frequency offset Δf_co is assumed so that the noiseless receive sample is $\tilde{r}_k = \mathrm{e}^{+\mathrm{j}2\pi\Delta f_\mathrm{co}Tk}\, s_k$, $-D_\mathrm{pr} \leq k < D + D_\mathrm{po}$. Clearly, we have $\mathcal{E}\{|\tilde{r}_k|^2\} = \sigma_s^2$. If we further assume (zero–mean) additive white Gaussian noise with $\sigma_n^2 \triangleq \mathcal{E}\{|n_k|^2\}$, the received sample in the complex baseband is $r_k = \tilde{r}_k + n_k$. The signal–to–noise power ratio (SNR) at the receiver input (channel SNR) is $\varsigma_\mathrm{c} \triangleq \sigma_s^2/\sigma_n^2$.

General OFDM receivers obtain the noisy frequency–domain detection variables by [7]

$$Y_\nu = \frac{1}{\sqrt{D}} \sum_{k=-\infty}^{+\infty} w_k\, r_k\, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{D}\nu k} = \frac{1}{\sqrt{D}} \sum_{\kappa=0}^{D-1} y_\kappa\, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{D}\nu\kappa} \quad (1)$$

$$y_\kappa \triangleq \sum_{n=-1}^{+1} w_{nD+\kappa}\, r_{nD+\kappa}, \quad (2)$$

where we substituted $k = \kappa + nD$. The modification consists in a windowing of the received samples with w_k

and for validity of the summation limits in (2), we have to assume $w_k = 0$ at least for $k < -D$ and $k \geq 2D$. In practical implementations, the time window will be even much shorter. Now, y_κ is input to the DFT of identical size D like in conventional OFDM. In a conventional receiver, we have $y_\kappa = r_\kappa\,,\, 0 \leq \kappa < D$ as direct copy of the received signal. In (2), the simple preprocessing prior to the DFT of the modified OFDM receiver is clearly visible. The structure is tightly related with the polyphase implementation of a decimated DFT filter bank [8, p.127]. This is in contrast to [2], where (1) is directly calculated with a DFT of size $2D$, finally evaluating every second frequency–domain result, only. The latter solution is valid but not efficient for implementation.

Windowing is also applicable in OFDM systems which have *no postfix*. Then, an appropriate time shift of the receive window and/or an appropriate frequency–domain phase correction needs to be performed.

Incorporating $\tilde{r}_k$ into (1) and introducing the normalized frequency offset (NFO) $\xi_f \triangleq \Delta f_{\mathrm{co}}/\Delta f_{\mathrm{sub}} = \Delta f_{\mathrm{co}} T D$ (we restrict our discussion to $|\xi_f| < 0.5$), we arrive at the noiseless frequency–domain transmission characteristic

$$Y_\nu = \sum_{\nu_{\mathrm{t}}=0}^{D-1} A_{\nu_{\mathrm{t}}}\, H_{\xi_f}^{\mathrm{STF}}\left[\nu_{\mathrm{t}}, \nu\right], \tag{3}$$

with the *subcarrier transfer factor* (STF)

$$H_{\xi_f}^{\mathrm{STF}}\left[\nu_{\mathrm{t}}, \nu\right] \triangleq \frac{1}{D} \sum_{k=-\infty}^{+\infty} w_k\; \mathrm{e}^{+\mathrm{j}\frac{2\pi}{D}(\nu_{\mathrm{t}}-\nu\; +\xi_f)k}, \tag{4}$$

which describes the interference characteristic between transmitted subcarrier ν_{t} and received subcarrier ν for a given NFO ξ_f. We clearly recognize its dependency on the receive window shape w_k. For $\nu = \nu_{\mathrm{t}}$, we obtain the useful STF. Clearly, the time window must comply with the time–domain *Nyquist condition*

$$\sum_{i=-\infty}^{+\infty} w_{k+iD} = \mathrm{const}\,, \quad 0 \leq k < D \tag{5}$$

to ensure subcarrier *orthogonality* for $\xi_f = 0$. This is perfectly dual to the Nyquist criterion in frequency–domain [9, p.543]. A second (practical) demand is the finite duration of the window shape.

III. MMSE–Optimum Nyquist Window Shape

A. Decomposition of Nyquist Window

Now, we assume the receiver time window to be composed of a rectangular component

$$c_k = \begin{cases} 1\,, & 0 \leq k < D \\ 0\,, & k < 0 \text{ or } k \geq D \end{cases}, \tag{6}$$

being equivalent to the time–limited receive window in conventional OFDM, plus a second additive component g_k which appropriately modifies both rolloff regions. This component is assumed to be time–limited such that $g_k = 0,\, \forall\, |k| > D_\theta$. The time–excess parameter $0 \leq D_\theta < D/2$ is variable. Hence,

$$w_k = c_k + g_k - g_{k-D} \tag{7}$$

is an adequate starting point for the optimization, as the composition in (7) automatically enforces the Nyquist criterion (5) but does not impose further unnecessary restrictions on the rolloff region's shape. For a good impression of this decomposition into a rectangular and two rolloff components we refer to Fig. 1. The window is extended by D_θ samples on the left and $D_\theta + 1$ samples on the right, when compared to original OFDM, with c_k as time window. The prefix size $D_{\mathrm{pr}} \geq D_\theta$ and postfix size $D_{\mathrm{po}} \geq D_\theta + 1$ is required. In perfect duality to the bandwidth–excess factor (rolloff) of impulse shaping transmit filters, the time–excess factor $\theta \triangleq (2D_\theta + 1)/D$, indicates the fraction of time, which the duration of the receive window is larger than the minimum of D samples, necessary for subcarrier orthogonality.

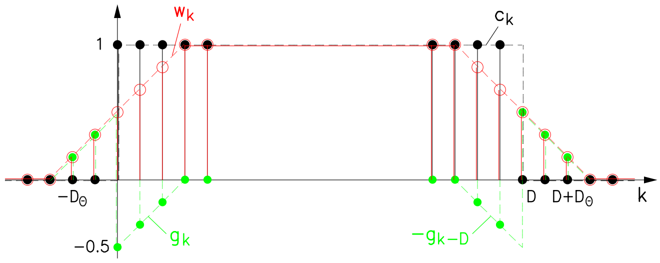

Fig. 1. Exemplary decomposition [7] of a trapezoidal Nyquist receiver time window w_k into the sum of a rectangular (i.e., c_k) and two rolloff modifications (i.e., g_k and $-g_{k-D}$) with $D_\theta = 2$.

B. Subcarrier Transfer Factor

We introduce the subcarrier number $\nu_{\mathrm{t}} = \nu + \Delta\nu$, where the discrete (cyclic) subcarrier index spacing $\Delta\nu$ is integer. With (7) and the time–limited nature of the respective components, the STF from (4) is rewritten as

$$\begin{aligned} H_{\xi_f}^{\mathrm{STF}}\left[\nu+\Delta\nu, \nu\right] \;&=\; \frac{1}{D}\,\frac{\mathrm{e}^{+\mathrm{j}2\pi(\Delta\nu+\xi_f)}-1}{\mathrm{e}^{+\mathrm{j}\frac{2\pi}{D}(\Delta\nu+\xi_f)}-1} + \\ &+ \frac{1}{D}\sum_{k=-D_\theta}^{+D_\theta} g_k \left(1-\mathrm{e}^{+\mathrm{j}2\pi(\Delta\nu+\xi_f)}\right)\mathrm{e}^{+\mathrm{j}\frac{2\pi}{D}(\Delta\nu+\xi_f)k} \end{aligned} \tag{8}$$

$$= \frac{q-1}{D}\left(\frac{1}{\mathrm{e}^{+\mathrm{j}\frac{2\pi}{D}(\Delta\nu+\xi_f)}-1} - \sum_{k=-D_\theta}^{+D_\theta} \mathrm{e}^{+\mathrm{j}\frac{2\pi}{D}(\Delta\nu+\xi_f)k} g_k\right), \tag{9}$$

where we introduced $q \;\triangleq\; \mathrm{e}^{+\mathrm{j}2\pi(\Delta\nu+\xi_f)} \;\overset{\Delta\nu\in\mathbb{Z}}{=}\; \mathrm{e}^{+\mathrm{j}2\pi\xi_f}$ and used the closed form for the finite geometrical series component emerging from c_k, cf. [1, Eq. (11)]

The STF is independent of the absolute subcarrier index ν so that we can set it to zero. Clearly, $H_{\xi_f}^{\mathrm{STF}}\left[\Delta\nu, 0\right]$

expresses the crosstalk between any two subcarriers which have a spacing $\Delta\nu$. Given all subcarriers are operated with identical average power, the average interference power from all others into any one subcarrier is identical for each subcarrier in the multiplex.

C. Definitions

At this point we introduce the following vector and matrix definitions:

$$\boldsymbol{H} = \left[H_{\xi_f}^{\mathrm{STF}}[1,0]\,,\,\ldots\,,\,H_{\xi_f}^{\mathrm{STF}}[D-1,0]\right]^{\mathsf{T}} \quad (10)$$

$$\boldsymbol{a} = \left[\frac{1}{\mathrm{e}^{+\mathrm{j}\frac{2\pi}{D}(1+\xi_f)}-1}\,,\,\ldots\,,\,\frac{1}{\mathrm{e}^{+\mathrm{j}\frac{2\pi}{D}((D-1)+\xi_f)}-1}\right]^{\mathsf{T}} \quad (11)$$

$$\boldsymbol{B} = \left[\mathrm{e}^{+\mathrm{j}\frac{2\pi}{D}(\Delta\nu+\xi_f)k}\right]_{\substack{\Delta\nu=1,\ldots,D-1\\ k=-D_\theta,\ldots,D_\theta}} \quad (12)$$

$$\boldsymbol{g} = \left[g_{-D_\theta}\,,\,\ldots\,,\,g_{D_\theta}\right]^{\mathsf{T}}, \quad (13)$$

where $\boldsymbol{X}^{\mathsf{T}}$ is the transposition of matrix/vector $\boldsymbol{X}$. Vector $\boldsymbol{H}$ contains the interfering STFs, only, as the useful STF for $\Delta\nu = 0$ is not included. Vector $\boldsymbol{g}$ represents the rolloff modification of length $2D_\theta + 1$ to be optimized. Matrix $\boldsymbol{B}$ has $D-1$ rows and $2D_\theta + 1$ columns.

We rewrite (9) simultaneously for $\Delta\nu = 1, \ldots, D-1$ in a compact notation and obtain

$$\boldsymbol{H} = ((q-1)/D)\,(\boldsymbol{a} - \boldsymbol{B}\boldsymbol{g})\,. \quad (14)$$

Further, the window composition from (7) is written as

$$\boldsymbol{w} = \boldsymbol{d} + \boldsymbol{C}\boldsymbol{g}, \quad (15)$$

where we introduced

$$\boldsymbol{w} = \left[w_{-D_\theta}\,,\,\ldots\,,\,w_{D+D_\theta}\right]^{\mathsf{T}} \quad (16)$$

$$\boldsymbol{d} = [\underbrace{0,\,\ldots,\,0}_{D_\theta},\,\underbrace{1,\,\ldots,\,1}_{D},\,\underbrace{0,\,\ldots,\,0}_{D_\theta+1}]^{\mathsf{T}} \quad (17)$$

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{I}_{(2D_\theta+1)\times(2D_\theta+1)} \\ \boldsymbol{O}_{(D-2D_\theta-1)\times(2D_\theta+1)} \\ -\boldsymbol{I}_{(2D_\theta+1)\times(2D_\theta+1)} \end{bmatrix}. \quad (18)$$

The matrices $\boldsymbol{I}_{a\times a}$ and $\boldsymbol{O}_{a\times b}$ represent the identity and zero matrix of given sizes, respectively. The vector $\boldsymbol{w}$ expresses the overall receiver window of length $D + 2D_\theta + 1$, while $\boldsymbol{d}$ is the rectangular component of conventional OFDM being modified by the rolloff contribution $\boldsymbol{C}\boldsymbol{g}$.

D. MMSE–Optimization of Window Shape

According to (3), the noiseless and interference–free useful received subcarrier amplitude component in subcarrier ν is $A_\nu H_{\xi_f}^{\mathrm{STF}}[\nu,\nu] = A_\nu H_{\xi_f}^{\mathrm{STF}}[0,0]$. We introduce the average power of ICI induced into one specific subcarrier from all other active subcarriers in the frequency multiplex as σ_I^2. The average noise power per subcarrier is denoted by σ_N^2. Signal interference and noise are uncorrelated, so that the effective mean–squared error (MSE) in one subcarrier is

$$\sigma_I^2 + \sigma_N^2, \quad (19)$$

and this cost function is desired to be minimized by the optimum $\boldsymbol{g}$. The Nyquist criterion is already enforced by the construction principle, so that $H_{\xi_f}^{\mathrm{STF}}[0,0] \approx 1$ under the constraint that ξ_f is not too large. Consequently, the rolloff shape which minimizes $\sigma_I^2 + \sigma_N^2$ will in good approximation maximize the overall SNR for each subcarrier

$$\varsigma_{\mathrm{s}} \triangleq \sigma_{\mathcal{A}}^2 \left|H_{\xi_f}^{\mathrm{STF}}[0,0]\right|^2 / \,(\sigma_I^2 + \sigma_N^2). \quad (20)$$

Two factors cause the subcarrier SNR ς_{s} to become larger than channel SNR ς_{c} for $\xi_f = 0$. A first gain with factor D/D_{u} is obtained for $D_{\mathrm{u}} < D$ (i.e., signal subspace smaller than noise subspace) and the second factor is the signal combining gain caused by the Nyquist receive window. The maximum achievable gain factor for a window with time–excess factor θ is $2/(2-\theta)$ [4] such that we have the subcarrier SNR upper bounded by

$$\varsigma_{\mathrm{s}} \leq \frac{D}{D_{\mathrm{u}}}\,\frac{2}{2-\theta}\,\varsigma_{\mathrm{c}}. \quad (21)$$

To keep the derivation simple, we now assume that all subcarriers are used (i.e., $D_{\mathrm{u}} = D$), so that the improvement of ς_{s} over ς_{c} will be solely caused by the signal combining gain by Nyquist windowing. Hence, the average power received in each subcarrier is $\sigma_{\mathcal{A}}^2 = \sigma_s^2$ and we have the average ICI power $\sigma_I^2 = \sigma_s^2\,\boldsymbol{H}^{\mathsf{H}}\boldsymbol{H}$, which is identical for each subcarrier. $\boldsymbol{X}^{\mathsf{H}}$ is the conjugate (Hermitian) transposition of matrix/vector $\boldsymbol{X}$. The additive noise in the subcarriers is acquired by windowing of time–domain noise samples with the receive time window $\boldsymbol{w}$. For the average noise power we have $\sigma_N^2 = \sigma_n^2\,\boldsymbol{w}^{\mathsf{H}}\boldsymbol{w}/D$. This expression requires white Gaussian noise for validity.

For optimization the minimum MSE (MMSE) approach is chosen. According to the Appendix, we form the derivative $\partial(\sigma_I^2 + \sigma_N^2)/\partial\boldsymbol{g}$ and set it to zero, yielding the optimum SNR–adaptive rolloff modification and with $\varsigma_{\mathrm{c}} = \sigma_s^2/\sigma_n^2$ it is given by

$$\begin{aligned}\boldsymbol{g} &= \left(\varsigma_{\mathrm{c}}\left(|q-1|^2/D\right)\Re\{\boldsymbol{B}^{\mathsf{H}}\boldsymbol{B}\} + 2\boldsymbol{I}\right)^{-1}\cdot \\ &\quad\cdot\left(\varsigma_{\mathrm{c}}\left(|q-1|^2/D\right)\Re\{\boldsymbol{B}^{\mathsf{H}}\boldsymbol{a}\} - \boldsymbol{C}^{\mathsf{T}}\boldsymbol{d}\right),\end{aligned} \quad (22)$$

because $\boldsymbol{C}^{\mathsf{T}}\boldsymbol{C} = 2\boldsymbol{I}_{(2D_\theta+1)\times(2D_\theta+1)}$, Here, the derivation aimed at obtaining a real–valued vector $\boldsymbol{g}$. In contrast, a complex–valued optimization of $\boldsymbol{g}$ would obtain the best solution for one specific NFO ξ_f. The real–valued $\boldsymbol{g}$ is the best solution for NFOs with unknown sign. Similarly, it is a good choice for some specified maximum Doppler spread centered at frequency zero.

E. Asymptotic Optimum Results

1) For channel SNRs $\varsigma_c \to \infty$ (∞ dB), i.e., dominating ICI, we obtain from (22) the asymptotic result

$$\boldsymbol{g} = \left(\Re\{\boldsymbol{B}^{\mathsf{H}}\boldsymbol{B}\}\right)^{-1} \Re\{\boldsymbol{B}^{\mathsf{H}}\boldsymbol{a}\}. \tag{23}$$

2) Another interesting asymptotic case is observed for $\xi_f = 0$ or at extremely low SNRs, where we have $q = 1$ or $\varsigma_c \to 0$ ($-\infty$ dB), respectively. In this case the additive noise dominates and we arrive at the constant solution

$$\boldsymbol{g} = -(2\boldsymbol{I})^{-1}\boldsymbol{C}^{\mathsf{T}}\boldsymbol{d} = \Big[\underbrace{\tfrac{1}{2}, \ldots, \tfrac{1}{2}}_{D_\theta}, \underbrace{-\tfrac{1}{2}, \ldots, -\tfrac{1}{2}}_{D_\theta+1}\Big]^{\mathsf{T}}, \tag{24}$$

which is identical with the SNR–maximizing optimum window shape given in [4]. This special window is constantly 0.5 in both rolloff regions which is a maximum ratio combining rule for all those signal samples which are received twice due to the unconsumed guard interval. We refer to this window as *constant* Nyquist shape. It allows cheap implementation, as a multiplication with 0.5 is a simple bit–shift operation in binary format.

F. Numerical Results for MMSE–Optimum Window

In Fig. 2, we find an illustration of the results for window shape $\boldsymbol{w}$ for $D = 64$ with $\theta = 0.266$ obtained from (22) for $\xi_f = 0.15$ and $\varsigma_c = 10$ (10 dB) ($\to$ MMSE, $*$), and (24) for $\varsigma_c \to 0$ ($-\infty$ dB) or $\xi_f = 0$ ($\to$ constant, $\triangledown$), leading to an optimum window, which is constantly 0.5 in the rolloff region. To the left and to the right we find discontinuities of the optimum window shapes, as both jump to zero and one, respectively.

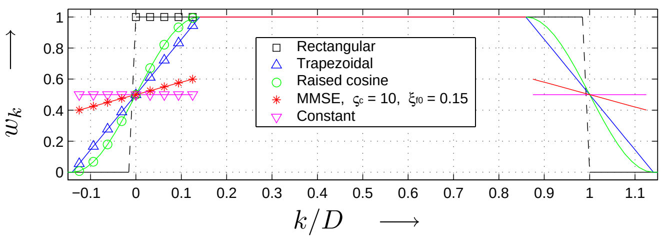

Fig. 2. Various Nyquist rolloff shapes $\boldsymbol{w}$ with $D_\theta = 8$ and $D = 64$ so that the window has a time–excess factor of $\theta = 0.266$. The conventional rectangular window is dashed with □ marks.

The optimum solutions run approximately linear inside the rolloff region, allowing a simplified derivation with a–priori linear rolloff segment [10]. There, a near–optimum solution is obtained which requires no matrix inversion.

G. Extensions

Thus far, we fixed the time–excess factor θ. In various receiver applications with possibility of channel length estimation, we can adapt θ according to the actual channel length [2] and the receiver automatically makes the best use of the received OFDM signal.

IV. Comparison with other Windows

In Fig. 2, we find three more window shapes which will be compared to the constant and the MMSE–optimum shape in the following. We investigate the rectangular shape of conventional OFDM, a Nyquist shape with trapezoidal and raised–cosine shape [3], [2]. The Nyquist window shapes all have a time–excess factor of $\theta = 0.266$.

The possible gains in joint noise and ICI power are demonstrated in Fig. 3, where the subcarrier SNR ς_s is plotted over the NFO ξ_f for all five window shapes from Fig. 2. The channel SNR is constantly $10\log_{10}\varsigma_c = 10$ dB. We evaluate Fig. 3 at $\xi_f = 0$ and learn that with $\theta = 0.266$ a maximum signal combining gain of (cf. Eq. (21)) $10\log_{10}\frac{2}{2-0.266} = 0.618$ dB results for the optimum time window with respect to conventional OFDM which can not exceed the channel SNR of 10 dB. The constant shape is optimum in achieving the maximum gain at $\xi_f = 0.0$ but becomes suboptimum in presence of nonzero NFOs. The MMSE–optimum shape shows superior behaviour with nonzero NFOs, while the degradation at $\xi_f = 0$ is hardly visible. The diagram can also be read like this: Conventional OFDM achieves a subcarrier SNR of 9.5 dB for $\xi_f = 0.06$, while we can allow twice that offset for the same effective subcarrier SNR with Nyquist windowing.

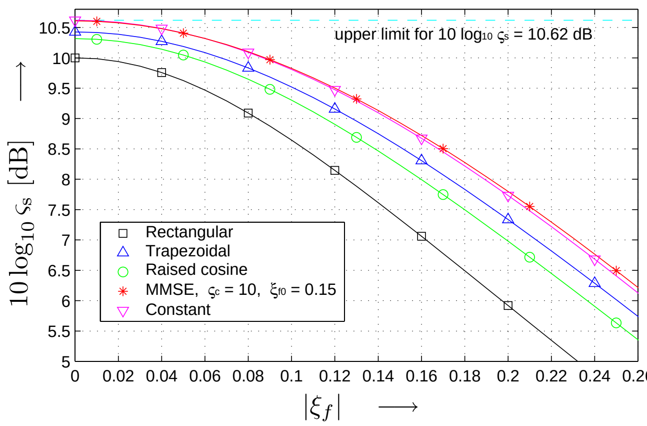

Fig. 3. Subcarrier SNR vs. NFO for various Nyquist rolloff shapes with $D_\theta = 8$ at $D = 64$, i.e., $\theta = 0.266$. The channel SNR is $10\log_{10}\varsigma_c = 10$ dB.

For small θ, the biggest portion of the gain is realized by the simple constant Nyquist window. The performance advantage of the MMSE–optimum window over the constant solution is increasing for larger θ, so that there the presented general solution is required for optimality. For $\theta \to 1$, the MMSE solution approaches a triangular window shape. The ad–hoc choice of a raised cosine rolloff region [2] is globally suboptimum. It offers only little more than half of the possible combining gain and half of the increase in robustness against nonzero NFOs over conventional OFDM.

To provide a frequency–domain explanation for optimality, we investigate the normalized frequency response

$W(\mathrm{e}^{+\mathrm{j}2\pi fT}) \triangleq \frac{1}{D}\sum_{k=-\infty}^{+\infty} w_k\,\mathrm{e}^{-\mathrm{j}2\pi fTk}$ of four receive window shapes $\boldsymbol{w}$ with $\theta = 0.266$ in Fig. 4. All Nyquist–type time windows offer frequency responses with reduced sidelobes when compared to original OFDM with rectangular window. Further, MMSE–optimized solutions have significantly reduced side–lobes directly adjacent to the main lobe (see between subcarrier index 1 and 3), when compared to the trapezoidal Nyquist window. It is this feature, which causes the decisively decreased interference power in case of residual carrier frequency offsets.

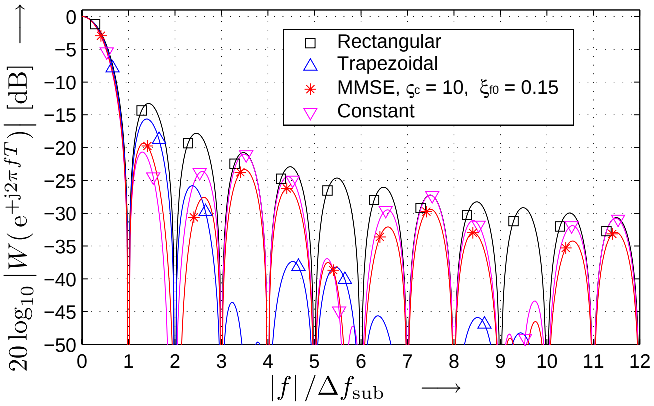

Fig. 4. Effective frequency responses for subcarrier demodulation with four different receiver window shapes with $D_\theta = 8$ and $D = 64$, i.e., $\theta = 0.266$.

The trapezoidal shape has no discontinuities in time domain so that the frequency response is decreasing very fast. The MMSE–optimum solutions exhibit discontinuities with step size sum up to one in the window; thus, they decrease with (little less than) the asymptotic slope of the rectangular window.

In some scenarios we have to cope with narrowband interference which does not comply with the subcarrier frequencies [11]. Hence, it influences a wide range of subcarriers. Nyquist windowing also has a positive effect in this case, and here, the linear or raised–cosine Nyquist shape would have a performance advantage due to the higher stopband attenuation.

V. Summary and Conclusions

In this paper, we proposed to exploit a possibly oversized guard interval in the (adaptive) receiver to enable a more robust demodulation in joint additive noise and ICI due to carrier frequency offsets. We derived the MMSE–optimum Nyquist window shape and compared it to known windows. The intuitive choice of a raised–cosine shape is far from optimum and the special solution given in [4] is an asymptotic case of our solution.

Our presentation assumed an OFDM system with pre- *and* postfix which is not compulsory for windowing. In existing OFDM systems *without postfix*, an appropriate time shift of the receive window and/or a frequency–domain phase correction needs to be performed.

With optimum time windowing, a significant and cheap improvement in demodulation performance is obtained over conventional OFDM. The upper limit for signal combining gain is achieved by the MMSE–optimum window and the robustness towards NFOs can be adapted.

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Appendix

The derivative of the real–valued function $\boldsymbol{g}^\mathsf{T}\boldsymbol{a} + \boldsymbol{a}^\mathsf{H}\boldsymbol{g}$ with respect to the real–valued column vector $\boldsymbol{g}$ is $\partial(\boldsymbol{g}^\mathsf{T}\boldsymbol{a} + \boldsymbol{a}^\mathsf{H}\boldsymbol{g})/\partial\boldsymbol{g} = \boldsymbol{a} + \boldsymbol{a}^* = 2\,\Re\{\boldsymbol{a}\}$, where $\boldsymbol{a}$ is an arbitrary complex–valued row vector. For the quadratic form $\boldsymbol{g}^\mathsf{T}\boldsymbol{A}\boldsymbol{g}$ with an arbitrary complex–valued matrix with Hermitian symmetry $\boldsymbol{A}^\mathsf{H} = \boldsymbol{A}$, it can be shown that $\partial(\boldsymbol{g}^\mathsf{T}\boldsymbol{A}\boldsymbol{g})/\partial\boldsymbol{g} = (\boldsymbol{A} + \boldsymbol{A}^\mathsf{T})\boldsymbol{g} = 2\,\Re\{\boldsymbol{A}\}\boldsymbol{g}$.

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