A Novel Decision–Feedback Differential Detection Scheme for 16 DAPSK

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Abstract

In this letter a novel decision–feedback differential detection (DF–DD) scheme for 16–level differentially encoded amplitude phase shift keying (16 DAPSK) signals is proposed. It is shown that a significant performance gain of the new technique based on multiple–symbol detection (MSD) [1] may be obtained compared to a previously proposed DF–DD scheme [2]. This gain increases with decreasing number of feedback symbols, which makes the novel scheme attractive for implementation.

1 Introduction

16–level differentially encoded amplitude phase shift keying (16 DAPSK) with differential detection is very attractive for mobile communications because of its simplicity of implementation and its robustness against carrier phase variations [3]. In order to improve the power efficiency of differential detection for transmission over an additive white Gaussian noise (AWGN) channel a decision feedback scheme has been proposed by Adachi et al. in [3]. Recently, it has been shown that an improved version of this scheme approaches the error performance of coherent detection (CD) if the number of feedback symbols tends to infinity [2]. In contrast to the DF–DD schemes of [2, 3], our scheme is not based on the maximum–likelihood estimation of a reference symbol but on multiple–symbol detection (MSD) [1] and yields superior performance if only a finite number of feedback symbols is applied.

2 DF–DD Scheme Based on MSD

In the following, we assume perfect synchronisation and T-spaced sampling of the received signal. For simplicity all signals are represented by their complex–valued baseband equivalents. The received signal sample $r[k]$ at time k may be written as

$$r[k] = R[k]b[k]\mathrm{e}^{j\Theta} + n[k], \tag{1}$$

where Θ is an unknown, constant, uniformly distributed phase and $n[\cdot]$ is a zero mean white Gaussian noise process. Due to an appropriate normalization $n[k]$ has the variance $\sigma^2 = N_0/E_S$, where N_0 and E_S are the single–sided power spectral density of the underlying passband noise process and the mean received energy per symbol, respectively. 16 DAPSK can be viewed as a combination of independent 2 DASK (differential amplitude shift keying) and 8 DPSK (differential phase shift keying) [3]. The transmitted 2 DASK symbols are denoted by $R[k] \in \mathcal{A}_R = \{R_H, R_L\}$ ($0.5(R_H^2 + R_L^2) = 1$, $a_0 = R_H/R_L > 1$), whereas the information bearing amplitude ratios $\Delta R[k] \in \mathcal{A}_{\Delta R} = \{a_0,\, 1,\, 1/a_0\}$ are given by $\Delta R[k] = R[k]/R[k-1]$. $\Delta R[k] = 1$ corresponds to message bit '0', whereas $\Delta R[k] = a_0$ and $\Delta R[k] = 1/a_0$ correspond to message bit '1'. The 8 DPSK symbols $b[k]$ depend exclusively on the information bearing phase difference symbols $a[k] = b[k]b^*[k-1] \in \mathcal{A}_a = \{\mathrm{e}^{j\pi\nu/4} | \nu \in \{0,\, 1,\, \ldots, 7\}\}$ ($(\cdot)^*$ denotes complex conjugation).

The derivation of our DF–DD scheme starts from the MSD metric for quadrature amplitude modulation (QAM) as presented in [1] (eqn. 20). The metric to decide upon a block of $(N-1)$ 16 DAPSK symbols, while $N \geq 2$ received symbols are observed, is given by

$$\begin{aligned}\eta(\boldsymbol{\Delta R}[k],\, \boldsymbol{a}[k]) = &-\frac{1}{2\sigma^2}\hat{R}^2[k-N+1]\sum_{\nu=0}^{N-1}\prod_{\mu=\nu}^{N-2}\Delta R^2[k-\mu] + \\ &\ln \mathrm{I}_0\left(\frac{1}{\sigma^2}\left|\sum_{\nu=0}^{N-1} r[k-\nu]\hat{R}[k-N+1]\prod_{\mu=\nu}^{N-2}\Delta R[k-\mu]\prod_{\xi=0}^{N-\nu-2}a^*[k-\nu-\xi]\right|\right),\end{aligned} \tag{2}$$

where $\mathrm{I}_0(\cdot)$ is the zeroth order modified Bessel function of the first kind and the definitions $\boldsymbol{\Delta R}[k] \triangleq [\Delta R[k],\, \Delta R[k-1],\, \ldots, \Delta R[k-N+2]]$, $\boldsymbol{a}[k] \triangleq [a[k],\, a[k-1],\, \ldots, a[k-N+2]]$ are used. Note, that in contrast to eqn. 20 of [1], eqn. 2 shows explicitly the dependence of $\eta(\boldsymbol{\Delta R}[k],\, \boldsymbol{a}[k])$ on the amplitude *ratios* contained in $\boldsymbol{\Delta R}[k]$. $\hat{R}[k-N+1] \in \mathcal{A}_R$ is the estimated absolute amplitude of the last symbol of the current block of N symbols and can be determined from the previously decided block of symbols since adjacent blocks overlap in one symbol [1]. The MSD decision rule demands to maximize $\eta(\boldsymbol{\Delta R}[k],\, \boldsymbol{a}[k])$ over $\{\boldsymbol{\Delta R}[k],\, \boldsymbol{a}[k]\}$, which requires 16^{N-1} metric calculations to decide upon $(N-1)$ 16 DAPSK symbols, i.e., complexity grows exponentially with N. To overcome this problem, we propose to introduce desicion feedback, i.e., in eqn. 2 the previously decided symbols $\Delta\hat{R}[k-\nu]$ and $\hat{a}[k-\nu]$, $1 \le \nu \le N-2$, are used instead of $\Delta R[k-\nu]$ and $a[k-\nu]$, $1 \le \nu \le N-2$, while a decision is made only upon $\Delta R[k]$ and $a[k]$. In order to simplify eqn. 2, the approximation $\ln(\mathrm{I}_0(x)) \approx x$, which is valid for high E_S/N_0 ratios [1], is used and additive terms which do not depend on $\Delta R[k]$ or $a[k]$ are omitted. Furthermore eqn. 2 is multiplied by $2\sigma^2/(\hat{R}[k-N+1]\prod_{\mu=1}^{N-2}\Delta\hat{R}[k-\mu])$ and the relation $\hat{R}[k-1] = \hat{R}[k-N+1]\prod_{\mu=1}^{N-2}\Delta\hat{R}[k-\mu]$ is applied. Finally, the term within the magnitude operator $|\cdot|$ is divided by $\prod_{\xi=1}^{N-2}\hat{a}^*[k-\xi]$ which does not change its magnitude. The resulting metric $\eta'(\Delta R[k],\, a[k])$ is given by

$$\eta'(\Delta R[k],\, a[k]) = -\hat{R}[k-1]\Delta R^2[k] + 2|r[k]\Delta R[k]a^*[k] + \tilde{r}[k-1]|, \tag{3}$$

where the symbol $\tilde{r}[k-1]$ is defined as

$$\tilde{r}[k-1] \triangleq \sum_{\nu=1}^{N-1} r[k-\nu] \prod_{\mu=1}^{\nu-1} \frac{\hat{a}[k-\mu]}{\Delta\hat{R}[k-\mu]}. \tag{4}$$

From eqn. 3 it can be seen, that the value of $a[k]$ which maximizes $\eta'(\Delta R[k],\, a[k])$ does not depend on $\Delta R[k]$. In order to maximize $\eta'(\Delta R[k],\, a[k])$ with respect to $a[k]$ it suffices to maximize $|r[k]\Delta R[k]a^*[k] + \tilde{r}[k-1]|$. Equivalently we might maximize $|r[k]\Delta R[k]a^*[k] + \tilde{r}[k-1]|^2$ which yields the decision rule

$$\hat{a}[k] = \underset{a[k]}{\mathrm{argmax}}\ \left\{\mathrm{Re}\left\{r[k]a^*[k]\tilde{r}^*[k-1]\right\}\right\}, \tag{5}$$

where $\mathrm{Re}\{\cdot\}$ is the real part of $\{\cdot\}$. If the complex plain is divided into 8 sectors corresponding to the 8 possible values of $a[k]$, $\hat{a}[k]$ is determined by the sector into which $r[k]\tilde{r}^*[k-1]$ falls.

Now, $\hat{a}[k]$ may be inserted into eqn. 3 instead of $a[k]$ and $\Delta R[k]$ can be obtained from

$$\Delta\hat{R}[k] = \underset{\Delta R[k]}{\mathrm{argmax}}\ \left\{-\hat{R}[k-1]\Delta R^2[k] + 2|r[k]\Delta R[k]\hat{a}^*[k] + \tilde{r}[k-1]|\right\}, \tag{6}$$

where $\Delta R[k] \in \{1,\, a_0\}$ if $\hat{R}[k-1] = R_L$ and $\Delta R[k] \in \{1,\, 1/a_0\}$ if $\hat{R}[k-1] = R_H$. For the decision in the next time step $k+1$ the symbol $\hat{R}[k]$ is calculated according to $\hat{R}[k] = \Delta\hat{R}[k]\hat{R}[k-1]$.

Eqns. 5 and 6 are the decision rules of our DF–DD scheme. $\tilde{r}[k-1]$ of eqn. 4 is calculated from the last $(N-1)$ received symbols $r[k-\nu]$, $1 \le \nu \le N-1$, in a nonrecursive way. It is also possible to derive a recursive formula for $\tilde{r}[k-1]$ if we assume $N \to \infty$. In this case, from eqn. 4

$\tilde{r}[k-1] = r[k-1] + (\hat{a}[k-1]/\Delta\hat{R}[k-1])\tilde{r}[k-2]$ is obtained. As will be shown by computer simulations, it is convenient to introduce an additional forgetting factor α $(0 \leq \alpha \leq 1)$:

$$\tilde{r}[k-1] = r[k-1] + \alpha \frac{\hat{a}[k-1]}{\Delta\hat{R}[k-1]} \tilde{r}[k-2]. \tag{7}$$

3 Simulation Results

In Fig. 1 our DF–DD scheme with nonrecursively generated symbol $\tilde{r}[k-1]$ according to eqn. 4 is compared with the DF–DD scheme of [2] and CD for ring ratio $a_0 = 2$. $E_b = E_S/4$ denotes the mean received energy per bit. If the number N of observed symbols is small, our technique performs significantly better than that of [2]. For example, at BER $= 10^{-3}$ the gain of the new scheme is 1 dB and 0.4 dB for $N = 2$ and $N = 3$, respectively. Note, that small values of N might be preferred in a practical application since the robustness of DF–DD schemes against carrier phase variations increases with decreasing N. At high E_b/N_0 ratios the gap between both DF–DD schemes becomes smaller and for large N both schemes perform almost equally well and approach CD. (Note, that in Fig. 1 of [2] the number of feedback symbols $L = N - 1$ is used as parameter instead of the number of observed symbols N.)

Fig. 2 shows the error performance of the proposed scheme when $\tilde{r}[k-1]$ is calculated recursively according to eqn. 7. As can be observed, CD is approached when α approaches 1. For smaller values of α, however, the DF–DD scheme is more robust against carrier phase variations, i.e., like for the nonrecursively generated symbol $\tilde{r}[k-1]$ there is a trade–off between robustness against phase variations and power efficiency under AWGN conditions. This has to be taken into account in order to choose the optimum value for N or α in a practical application.

Finally we would like to mention, that $a_0 = 2$ was chosen for a direct comparison of our results with those of [2]. For the optimum ratio $a_0 = 1.8$ [2, 4], however, the performance gain of our DF–DD scheme over that of [2] becomes even larger.

4 Acknowledgement

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Figure Captions

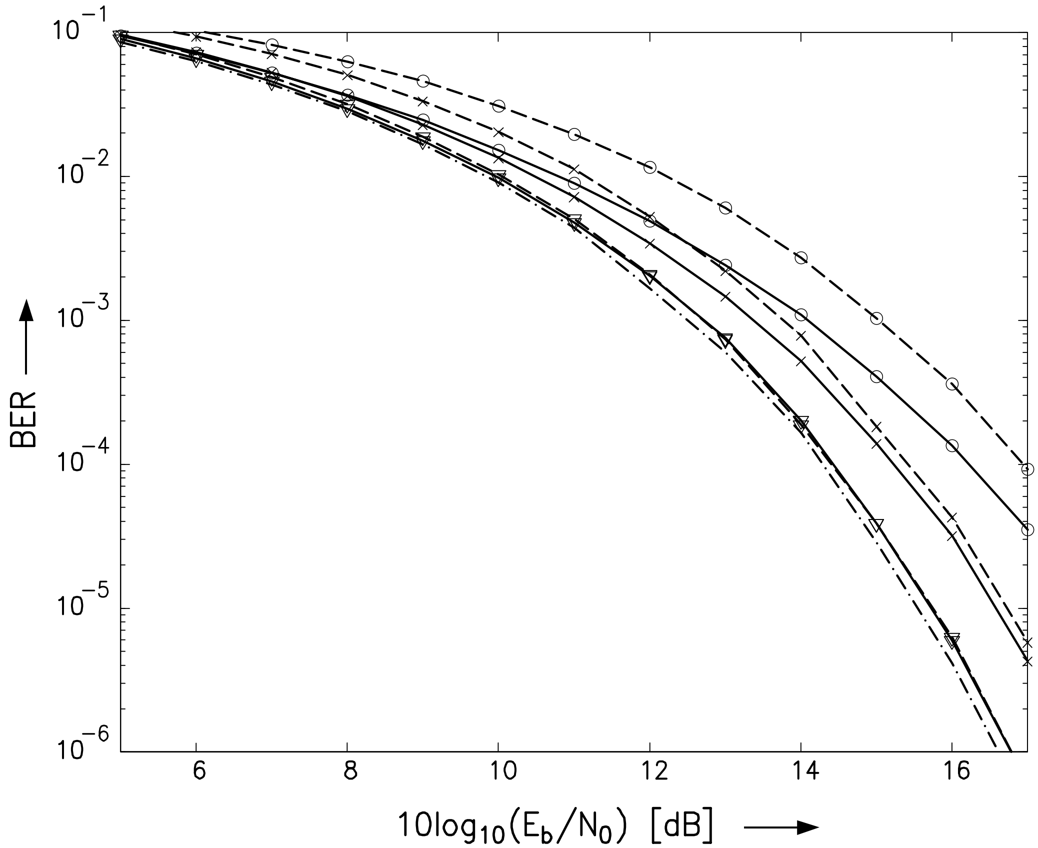

Figure 1: Comparison of the proposed DF–DD scheme with nonrecursively generated symbol $\tilde{r}[k-1]$ according to eqn. 4 and the scheme of [2].

——— proposed DF–DD scheme

– – – DF–DD scheme of [2]

○ ○ $N = 2$

× × $N = 3$

▽ ▽ $N = 10$

– · –· CD

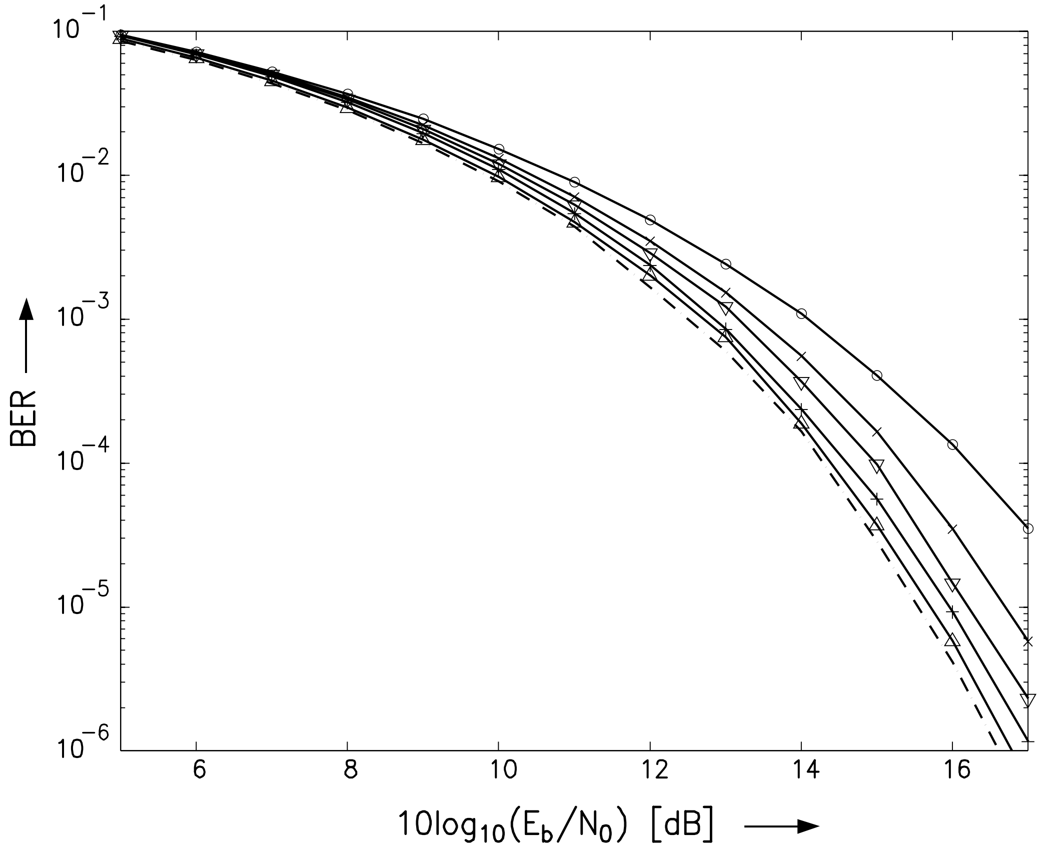

Figure 2: Error performance of the proposed DF–DD scheme with recursively generated symbol $\tilde{r}[k-1]$ according to eqn. 7.

○ ○ $\alpha = 0$

× × $\alpha = 0.2$

▽ ▽ $\alpha = 0.4$

+ + $\alpha = 0.6$

△ △ $\alpha = 0.8$

$- \cdot - \cdot$ CD