An Analytic Solution to the Performance of Iterated Soft Decision Interference Cancellation for Coded CDMA Transmission over Frequency Selective Fading Channels

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Abstract — **In this paper coded transmission over time–variant multipath Rayleigh–fading channels employing Direct Sequence Code Division Multiple Access (DS–CDMA) is considered. Assuming ideal knowledge of the actual channel state and randomly chosen spreading sequences, the performance of iterative multiuser equalization based on adapted MMSE–filters combined with serial successive cancellation and single user decoding is solved analytically. For this, merely the statistics of the decoder must be known. Equipped with this solution, cumbersome simulations of the whole multiuser system can be avoided to evaluate the achievable bit error ratio in a wide range of interesting parameters.**

I. Introduction

The search for feasible multiuser receivers applicable to transmission with DS–CDMA has attracted considerable interest due to the deployment of the 3rd generation mobile telecommunications system and first studies with respect to the 4th generation.

While for low system loads $\beta = K/N \ll 1$ (K denotes the number of users and N the spreading factor) the use of linear receivers based on matched filter banks is sufficient, for high system loads $\beta \gtrapprox 1$ more sophisticated receiver algorithms are required instead. Among the last one it turned out that iterative schemes using decision–feedback allow to reach near single user performance while possessing manageable complexity [1, 2, 3, 4, 5, 6]. In all these studies the performance of the iterative scheme is evaluated by means of simulations.

The aim of this paper is to provide an *analytic solution to the achievable Bit Error Ratio (BER) for coded multiuser transmission over frequency selective fading channels* to a common receiver. In order to achieve near single user performance with a relatively low complexity and delay, especially small number of iterations, we restrict ourselves to the application of MMSE–filters, single user decoding with maximum a posteriori symbol by symbol estimation (MAP–SSE) and serial cancellation at the receiver.

So, a recent result on the limiting signal to noise ratio at the output of an MMSE–multiuser interference suppression filter for random spreading sequences in the case $N \gg 1$ can be used [7]. Our investigation was also motivated by the analysis of Turbo–Codes for transmission over additive white Gaussian noise (AWGN) channels given in [8] incorporating numerical evaluation of the extrinsic information's variance provided by the MAP–SSE component decoders.

As to cope with the multipath fading channel implying unequal, time–variant received powers of the users, an equivalent transmission model has to be employed. Trying to emulate the actual receiver's decision chain by means of a mathematical model, it turns out that the noise variance at the output of the interference suppression filter is one key parameter. To find a suitable formula for this variance is one of the main problems dealt with in this work. Equipped with the noise power and our equivalent channel model, the resulting BER can be obtained immediately. An additional burden is imposed by the serial cancellation carried out. As a consequence, the expected average multiuser interference differs for all users even within one iteration cycle. To devise a reasonable method for approximation of the actual soft decisions of each user is another point being addressed in this paper.

The prediction of the performance of a multiuser system for frequency selective fading channels is one goal. The other purpose of this paper is to verify that the advantages of nonorthogonal transmission compared to orthogonal multiple access schemes indicated by an information theoretical approach to this problem [9] can be realized in practical systems, too.

The paper is arranged as follows. In Section II, the transmission model is given and motivated. The receiver algorithm under study is described in Section III. The analytic solution for the BER reached by the receiver is derived in Section IV. A comparison of the actual performance of the receiver and the analytic results is given in Section V. Finally, Section VI points out conclusions.

II. Transmission Model

Coded transmission of K users over frequency selective fading channels with DS–CDMA to a single receiver is considered. The underlying discrete–time equivalent complex baseband transmission model is illustrated in Fig. 1a).

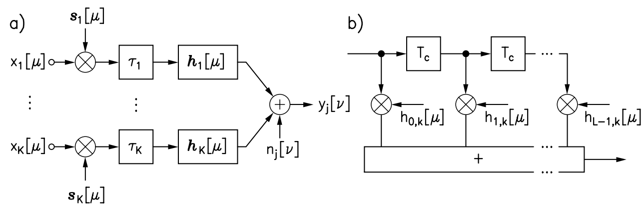

Figure 1: a) Transmission model of CDMA system with K users b) Tapped–delay–line channel model with L paths.

Here, $x_k[\mu]$ and $\boldsymbol{s}_k[\mu] = (s_{1,k}[\mu], \ldots, s_{N,k}[\mu])^T$ denote the

kth user's transmitted channel symbol and his/her spreading sequence in the μth transmission interval, respectively. The users' channel symbols are chosen from the set $x_k[\mu] \in \mathcal{X}_k$. They are obtained by convolutional encoding of the kth user's data sequence $\boldsymbol{d}_k$ and subsequent passing of the codeword through a random interleaver π_k. The elements of the unit energy spreading sequence are drawn randomly as $s_{j,k}[\mu] \in \{(\pm 1 \pm \mathrm{j})/\sqrt{2N}\}, \forall j$. The variables $\tau_1, \ldots, \tau_K$, $\tau_k \in \{0, \ldots, N-1\}$ represent the users' delays. The kth user's modulated sequence $(x_k[\mu]s_{1,k}[\mu], \ldots, x_k[\mu]s_{N,k}[\mu])$ is transmitted over a linear dispersive channel with discrete time–variant weight function $\boldsymbol{h}_k[\mu]$. In order to account for the multipath propagation the well known tapped–delay–line channel model (see Fig. 1b) assuming L resolvable paths is used. T_c is the length of one chip interval. As usual, it is assumed that the path weights $h_{0,k}[\mu], \ldots, h_{L-1,k}[\mu], \forall k$, are independent and zero mean proper complex Gaussian distributed with variance $\sigma_{l,h}^2 = \mathcal{E}\left\{|h_{l,k}[\mu]|^2\right\}, \forall k$.For clarity and comparability of the results presented, throughout this paper the study is restricted to the so–called equal (average) gain channel, i.e., $\sigma_{l,h}^2 = 1/L$, $\forall l = \{0, \ldots, L-1\}$. Supposing that the path gains are constant over one channel symbol the received signal resulting from $x_k[\mu]$ can be written as $(x_k[\mu]s'_{1,k}[\mu], \ldots, x_k[\mu]s'_{N',k}[\mu])$, where $\boldsymbol{s}'_k[\mu] = \left(s'_{1,k}[\mu], \ldots, s'_{N',k}[\mu]\right)^T$ is the kth user's *effective spreading sequence*. It is obtained by convolution of the actual spreading sequence $\boldsymbol{s}_k[\mu]$ and $\boldsymbol{h}_k[\mu]$, i.e., $s'_{j,k}[\mu] = \sum_{\eta=1}^{N} s_{\eta,k}[\mu]h_{(j-\eta),k}[\mu], j = 1, 2, \ldots, N' = N + L - 1$.

In the following the focus is on the behavior of this multiple access system when N is large compared to the number of resolvable propagation paths L, i.e., $N \gg L$. In addition, for sake of analytical tractability synchronous transmission is supposed, i.e., $\tau_k = 0, \forall k$. Hence, intersymbol interference is of minor importance in this situation and the received signal can be modeled as

$$\boldsymbol{y}[\mu] \quad = \quad \boldsymbol{S'}[\mu]\boldsymbol{x}[\mu] + \boldsymbol{n}[\mu]. \tag{1}$$

The vectors $\boldsymbol{y}[\mu] = (y_1[\mu], \ldots, y_{N'}[\mu])^T$ as well as $\boldsymbol{n}[\mu] = (n_1[\mu], \ldots, n_{N'}[\mu])^T$ represent the received signal and the additive channel noise, respectively. The i.i.d. samples $n_j[\mu], 1 \leq j \leq N'$, are zero mean complex Gaussian random variables with variance σ_n^2. Further, $\boldsymbol{x}[\mu] = (x_1[\mu], \ldots, x_K[\mu])$ and $\boldsymbol{S'}[\mu] = (\boldsymbol{s}'_1[\mu], \ldots, \boldsymbol{s}'_K[\mu])$ consist of the K users' transmitted symbols and their effective spreading sequences as well.

Before proceeding with the description of the receiver and its analytic treatment, it has to be emphasized that the restriction to the model given in Eq. (1) is merely made in order to gain theoretical insight. So, the random choice of spreading sequences leads to virtually the same performance for the general asynchronous case, too (cf.[10]). This is confirmed by our own studies.

III. Iterative Multiuser Decoding

In order to explain the serial receiver algorithm, let us consider the user with number k.[1] The main parts of the decision chain for the kth user in the mth iteration are depicted in Fig. 2.

First, the interference reduced signal $\hat{\boldsymbol{y}}_k^m[\mu] = \boldsymbol{y}[\mu] - \boldsymbol{S}'[\mu]\hat{\boldsymbol{x}}_k^{m-1}[\mu]$ is obtained using previous estimates on the

[1]A parallel version of this scheme is studied in [5]. However, for rising loads β (especially for $\beta > 1$) a performance degradation can be found for parallel cancellation compared to serial one assuming equal complexity.

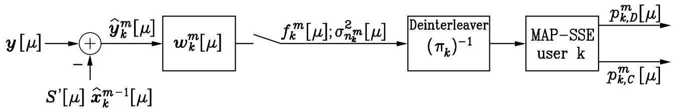

Figure 2: Receiver model for user k and mth iteration.

users' channel symbols which are arranged in $\hat{\boldsymbol{x}}_k^{m-1}[\mu] = \left(\ldots, \hat{x}_{k-1}^m[\mu], 0, \hat{x}_{k+1}^{m-1}[\mu], \ldots\right)^T$. Here, $\hat{x}_\kappa^i[\mu]$ denotes the soft/hard estimate on the κth user's channel symbol made in the ith iteration. The initialization $\hat{x}_\kappa^0[\mu] \triangleq \mathcal{E}\{x_\kappa[\mu]\}$ is applied. The soft decision employed is derived as conditional expectation $\hat{x}_\kappa^i[\mu] = \mathcal{E}\{x_\kappa[\mu]|p_{\kappa,C}^i[\mu]\}$, where $p_{\kappa,C}^i[\mu]$ denotes the probability information provided by the κth user's MAP–SSE decoder for channel symbol $x_\kappa[\mu]$ after the ith decoding loop.

The signal $\hat{\boldsymbol{y}}_k^m[\mu]$ is fed into an unbiased residual interference suppression filter designed according to the MMSE–criterion

$$(\boldsymbol{w}_k^m[\mu])^T = \frac{\sigma_{x_k}^2}{u_k^m[\mu]} \boldsymbol{s'}_k^H[\mu] \left(\boldsymbol{S'}[\mu]\boldsymbol{P}_k^{m-1}[\mu]\,(\boldsymbol{S'}[\mu])^H + \sigma_n^2 \boldsymbol{I}\right)^{-1}, \tag{2}$$

with $\sigma_{x_k}^2 = \mathcal{E}\{|x_k[\mu]|^2\}$.The exact solution to $\boldsymbol{P}_k^{m-1}[\mu] = \mathcal{E}\{(\boldsymbol{x}[\mu] - \hat{\boldsymbol{x}}_k^{m-1}[\mu])(\boldsymbol{x}[\mu] - \hat{\boldsymbol{x}}_k^{m-1}[\mu])^H\}$ requires to solve the expected cross-correlations and their complex conjugates, respectively. Although, it would be possible to solve these terms analytically, the computational effort is quite large. Instead they are treated to be 0 what is mainly backed by the fact that the decision made by the decoder for the μth interval is based on numerous symbols received in other time slots due to interleaving. In section V it will be verified that the above simplification works well in a wide range of loads β. Based on this, $\boldsymbol{P}_k^{m-1}[\mu]$ reduces to a diagonal matrix $\boldsymbol{P}_k^{m-1}[\mu] = \mathrm{diag}(\ldots, \mathcal{E}\{|x_{k-1}[\mu] - \hat{x}_{k-1}^m[\mu]|^2\}, \sigma_{x_k}^2, \mathcal{E}\{|x_{k+1}[\mu] - \hat{x}_{k+1}^{m-1}[\mu]|^2\}, \ldots)$. Employing soft decisions and conditioning the expectations on the corresponding value $p_{\kappa,C}^i[\mu]$ we have $\mathcal{E}\{|x_\kappa[\mu] - \hat{x}_\kappa^i[\mu]|^2|p_{\kappa,C}^i[\mu]\} = \mathcal{E}\{|x_\kappa[\mu]|^2|p_{\kappa,C}^i[\mu]\} - |\hat{x}_\kappa^i[\mu]|^2$. For the particular case of zero mean phase shift keying, which is considered in the sequel, follows $\mathcal{E}\{|x_\kappa[\mu] - \hat{x}_\kappa^i[\mu]|^2|p_{\kappa,C}^i[\mu]\} = \sigma_{x_\kappa}^2 - |\hat{x}_\kappa^i[\mu]|^2$. Finally, the inherent bias $u_k^m[\mu]$ of an MMSE–filter output is obtained as $u_k^m[\mu] = \sigma_{x_k}^2 \boldsymbol{s'}_k^H[\mu]\left(\boldsymbol{S'}[\mu]\boldsymbol{P}_k^{m-1}[\mu]\,(\boldsymbol{S'}[\mu])^H + \sigma_n^2\boldsymbol{I}\right)^{-1}\boldsymbol{s'}_k[\mu]$. With this, the filter output $f_k^m[\mu] = (\boldsymbol{w}_k^m[\mu])^T\,\hat{\boldsymbol{y}}_k^m$ can be written as $f_k^m[\mu] = x_k[\mu] + n_k^m[\mu]$. In the following, the interference $n_k^m[\mu]$ is modeled as zero mean complex Gaussian random variable [6]. Assuming that $\boldsymbol{w}_k^m[\mu]$ is the correct MMSE–filter the noise variance is obtained as $\sigma_{n_k^m}^2[\mu] = \mathcal{E}\{|n_k^m[\mu]|^2\} = (\sigma_{x_k}^2/u_k^m[\mu] - \sigma_{x_k}^2)$. Thus, the symbol probabilities $p_{k,X_\delta}^m[\mu] = \Pr(x_k[\mu] = X_\delta),\ \forall\, X_\delta \in \mathcal{X}_k$, at the output of the interference suppression filter $\boldsymbol{w}_k^m[\mu]$ are solved as

$$p_{k,X_\delta}^m[\mu] \quad = \quad \frac{\exp\left(-\frac{|f_k^m[\mu] - X_\delta|^2}{\sigma_{n_k^m}^2[\mu]}\right)}{\sum\limits_{X_\delta \in \mathcal{X}_k} \exp\left(-\frac{|f_k^m[\mu] - X_\delta|^2}{\sigma_{n_k^m}^2[\mu]}\right)}. \tag{3}$$

Based on these probabilities as input, the MAP–SSE algorithm delivers probabilities $p_{k,D_\delta}^m[\mu], \forall \mu$, for the kth user's data symbols as well as probabilities $p_{k,C_\delta}^m[\mu] = \Pr(x_k[\mu] = X_\delta), \forall X_\delta \in \mathcal{X}_k, \forall \mu$, for the channel symbols required for the next iteration.

Equipped with the probabilities for the channel symbols the soft estimate for the next iteration cycle is given by $\hat{x}_k^m[\mu] = \sum_{X_\delta \in \mathcal{X}_k} X_\delta \cdot p_{k,C_\delta}^m[\mu], \quad \forall \mu$, exploiting the reliability information provided by the decoder. With these new estimates the interference suppression filter for the succeeding user $k+1$ can be calculated.

This serial interference suppression and decoding procedure is carried out for iteration $m = 1, 2, \ldots$ for all users and all transmission intervals.

IV. Analytic Solution of Receiver Performance

In this part of the work we focus on the problem to obtain an analytical solution enabling the theoretical prediction of the bit error ratio (BER) resulting by application of the iterative receiver described above. This derivation is done under the supposition that the spreading factor N goes to infinity while the load $\beta = K/N$ is constant. This implies $K \to \infty$, too. The suitability of this assumption, which is made here for the theoretical approach, will be shown in the next section. There, the analytical results are compared with real systems using a spreading factor of 64.

The basis of our derivation is the asymptotic eigenvalue distribution of $N \times N$ dimensional random covariance matrices in the limit $N \to \infty$ given in [11] which was used in [7] to study the performance of a linear MMSE–filter for synchronous uncoded transmission over a single–path channel. Here, in contrast to [7], frequency selective fading, the application of serial successive cancellation based on soft outputs of the decoder as well as different iteration cycles must be taken into account. To explain the calculation, let us again consider user k and iteration m.

First, the assumed instantaneous signal to noise ratio $\mathrm{SNR}_k^m[\mu]$ at the output of the interference suppression filter $\boldsymbol{w}_k^m[\mu]$ is solved analytically. With the model $f_k^m[\mu] = x_k[\mu] + n_k^m[\mu]$, the desired value $\mathrm{SNR}_k^m[\mu] = \sigma_{x_k}^2 / \sigma_{n_k^m}^2[\mu]$, reads

$$\begin{aligned}\mathrm{SNR}_k^m[\mu] \quad = \quad & \sigma_{x_k}^2 \boldsymbol{s'}_k^H[\mu] \left(\sum_{\kappa<k} \boldsymbol{s}'_\kappa[\mu] \sigma_{x_\kappa^m}^2[\mu] \left(\boldsymbol{s}'_\kappa[\mu]\right)^H \right. \\ & \left. + \sum_{\kappa>k} \boldsymbol{s}'_\kappa[\mu] \sigma_{x_\kappa^{m-1}}^2[\mu] \left(\boldsymbol{s}'_\kappa[\mu]\right)^H + \sigma_n^2 \boldsymbol{I} \right)^{-1} \boldsymbol{s'}_k[\mu]. \qquad (4)\end{aligned}$$

Here, $\sigma_{x_\kappa^i}^2[\mu] = \mathcal{E}\left\{ |x_\kappa[\mu] - \hat{x}_\kappa^i[\mu]|^2 | p_{\kappa,C}^i[\mu] \right\}$ denotes the expected interference power of user κ remaining after soft interference cancellation in iteration i.

In order to eliminate the dependency of $\mathrm{SNR}_k^m[\mu]$ on the users' spreading sequences $\boldsymbol{s}_k[\mu], \forall k$, the actual transmission model given in Eq. (1) has to be replaced by an equivalent one. So, neglecting the first $L-1$ as well as last $L-1$ samples of each effective spreading sequence (being possible for $N \gg L$ [9]) all remaining elements of the kth user's effective spreading sequence can be modeled as $s'_{\cdot,k}[\mu] = \hat{s}_{\cdot,k}[\mu] \sqrt{\sum_{l=0}^{L-1} |h_{l,k}[\mu]|^2}, \forall k$. With the assumption that the complex variables $\hat{s}_{\cdot,k}[\mu]$ are zero mean Gaussian random variables with variance $1/N$ we found that the distribution of $\mathrm{SNR}_k^m[\mu]$ is not changed if $N \gg 1$ (for conditions see [11, 12, 9]). In fact, each user's effective spreading sequence is treated as product $\hat{\boldsymbol{s}}_k[\mu] \sqrt{\sum_{l=0}^{L-1} |h_{l,k}[\mu]|^2} \triangleq \hat{\boldsymbol{s}}_k[\mu] a_k[\mu]$. Thus, our transmission model is now

$$\boldsymbol{y}[\mu] \quad = \quad \hat{\boldsymbol{S}}[\mu] \boldsymbol{A}[\mu] \boldsymbol{x}[\mu] + \boldsymbol{n}[\mu], \qquad (5)$$

where $\hat{\boldsymbol{S}}[\mu] = (\hat{\boldsymbol{s}}_1[\mu], \ldots, \hat{\boldsymbol{s}}_K[\mu])$ and $\boldsymbol{A}[\mu] = \mathrm{diag}(a_1[\mu], \ldots, a_K[\mu])$ stand for the users' spreading sequences and fading amplitudes, respectively. Further, supposing that for the number of users in the system holds $K \gg 1$, each user can be assigned an amplitude $a_k[\mu]$, which is drawn randomly from the chi–square distribution $f_{a,L}(\alpha) = \frac{2}{(\sigma_h^2)^L (L-1)!} \alpha^{2L-1} \mathrm{e}^{-\alpha^2/\sigma_h^2} U(\alpha)$. Here $U(\alpha)$ denotes the unit step function. Note, like for the design of the actual receiver, the dependencies between different time slots and users resulting from iterative decoding are neglected in the following. So, adapting Eq. (4) to the equivalent transmission model yields

$$\begin{aligned}\mathrm{SNR}_k^m[\mu] \quad = \quad & \sigma_{x_k}^2 a_k^2[\mu] \hat{\boldsymbol{s}}_k^H[\mu] \left(\sum_{\kappa<k} \hat{\boldsymbol{s}}_\kappa[\mu] a_\kappa^2[\mu] \sigma_{x_\kappa^m}^2[\mu] \left(\hat{\boldsymbol{s}}_\kappa[\mu]\right)^H \right. \\ & \left. + \sum_{\kappa>k} \hat{\boldsymbol{s}}_\kappa[\mu] a_\kappa^2[\mu] \sigma_{x_\kappa^{m-1}}^2[\mu] \left(\hat{\boldsymbol{s}}_\kappa[\mu]\right)^H + \sigma_n^2 \boldsymbol{I} \right)^{-1} \hat{\boldsymbol{s}}_k[\mu]. (6)\end{aligned}$$

For the following it turns out to be convenient to get rid of the dependence on $a_k^2[\mu]$. To achieve this, the normalized signal to noise ratio $\overline{\mathrm{SNR}}_k^m[\mu] \triangleq \mathrm{SNR}_k^m[\mu] / a_k^2[\mu]$ is defined. Based on the presuppositions made above and keeping the load β constant the signal to noise ratio $\overline{\mathrm{SNR}}_k^m[\mu]$ converges in the limit $N \to \infty$ in probability to

$$\begin{aligned}\overline{\mathrm{SNR}}_k^m[\mu] \sigma_n^2 + \frac{1}{N} & \left(\sum_{\kappa<k} \frac{\overline{\mathrm{SNR}}_k^m[\mu] \sigma_{x_k}^2 \, \sigma_{x_\kappa^m}^2 a_\kappa^2[\mu]}{\sigma_{x_k}^2 + \overline{\mathrm{SNR}}_k^m[\mu] \sigma_{x_\kappa^m}^2[\mu] a_\kappa^2[\mu]} + \right. \\ & \left. \sum_{\kappa>k} \frac{\overline{\mathrm{SNR}}_k^m[\mu] \sigma_{x_k}^2 \, \sigma_{x_\kappa^{m-1}}^2[\mu] a_\kappa^2[\mu]}{\sigma_{x_k}^2 + \overline{\mathrm{SNR}}_k^m[\mu] \sigma_{x_\kappa^{m-1}}^2[\mu] a_\kappa^2[\mu]} \right) = \sigma_{x_k}^2. \qquad (7)\end{aligned}$$

With the assumption that K/N is fixed for a rising spreading factor N, the limit of the normalized signal to noise ratio is equal for all time slots regardless of the actual channel state and actual soft decision of a single user. This is a consequence of the huge multiuser interference diversity for $K \to \infty$. Hence, the time index μ can be dropped in Eq. (7), where $\overline{\mathrm{SNR}}_k^m[\mu]$ is solved iteratively from . In order to proceed, we regard that post-multiplication of the MMSE–filter output $f_k^m[\mu] = x_k[\mu] + n_k^m[\mu]$ by $a_k[\mu]$ leads on the one hand to

$$\overline{f}_k^m[\mu] \quad \triangleq \quad a_k[\mu] x_k[\mu] + \overline{n}_k^m[\mu], \qquad (8)$$

while on the other hand the signal to noise ratio remains unchanged $\mathrm{SNR}_k^m[\mu] = a_k^2[\mu] \sigma_{x_k}^2 / \sigma_{\overline{n}_k^m}^2[\mu]$. Rewriting the last equation we get $\sigma_{\overline{n}_k^m}^2[\mu] = \sigma_{x_k}^2 / \overline{\mathrm{SNR}}_k^m$, and it reveals that the variance $\sigma_{\overline{n}_k^m}^2[\mu]$ of the zero mean Gaussian noise $\overline{n}_k^m[\mu]$ is equal for all time slots. At this point, we would like to emphasize that by multiplication of $f_k^m[\mu]$ by the known scalar $a_k[\mu]$ the decoder's soft output for user k is not changed. Of course, this multiplication is considered appropriately in calculation of the decoders soft inputs. So, employing the model described by Eq. (8) and equipped with the variance $\sigma_{\overline{n}_k^m}^2[\mu]$ it is possible to solve the resulting Bit Error Ratio $\mathrm{BER}_k^m = f_{\mathrm{BER}}(\sigma_{x_k}^2, \sigma_{\overline{n}_k^m}^2, L)$ of user k in iteration m. The function $f_{\mathrm{BER}}()$ describes the so–called single user bound (SUB). It is obtained by measuring the BER achievable by a single user transmitting with power $\sigma_{x_k}^2$ over a channel with weights drawn according to $f_{a,L}(\alpha)$ and being interfered by additive Gaussian noise with variance $\sigma^2_{\overline{n}_k^m}[\mu]$ for a specific code employed. Beside BER_k^m the distribution of the soft decisions $\hat{x}_k^m[\mu]$ provided by the MAP–SSE is required. This distribution is completely determined by the probability density function $f_{\hat{x}(\sigma^2_{x_k},\sigma^2_{\overline{n}_k^m},a_k[\mu])}(\alpha)$ relying on the signal and noise power as well as the channel state. Like $f_{\mathrm{BER}}()$, it can be derived by measurement of the decoder's soft outputs for various signal and noise variances and fading channels. Equipped with $\sigma^2_{x_k}, \sigma^2_{\overline{n}_k^m}, a_k[\mu]$ $\hat{x}_k^m[\mu]$ can be randomly chosen from $f_{\hat{x}(\sigma^2_{x_k},\sigma^2_{\overline{n}_k^m},a_k[\mu])}(\alpha)$ and the power $\sigma^2_{x_k^m}[\mu]$ remaining after cancellation is solved as $\sigma^2_{x_k} - |\hat{x}_k^m[\mu]|^2$.

Another possibility to obtain $\sigma^2_{x_k^m}[\mu]$ would be to solve the expectation which reads

$$\sigma^2_{x_k^m}[\mu] = \mathcal{E}\left\{|x_\kappa[\mu] - \hat{x}_\kappa^m[\mu]|^2 | p^m_{\kappa,C}[\mu]\right\} \quad (9)$$

$$= \sigma^2_{x_k} - \int_0^\infty |\alpha|^2 f_{\hat{x}(\sigma^2_{x_k},\sigma^2_{\overline{n}_k^m},a_k[\mu])}(\alpha)\mathrm{d}\alpha. \quad (10)$$

However, it is found that both ways lead to identical analytical results. This seems to be a consequence of the large number of users assumed, too. In this way, the calculation is carried out for all users and iterations. Finally, the average Bit Error Ratio after the mth iteration is obtained as

$$\mathrm{BER}^m = \frac{1}{K}\sum_{k=1}^{K} \mathrm{BER}_k^m \overset{K\to\infty}{\longrightarrow} \mathcal{E}\{\mathrm{BER}_k^m\}. \quad (11)$$

V. Results

In order to show the tight agreement of the simulated BER as well as analytic solution some numerical results are presented. In the studies a rate $R_c = 1/2$ convolutional code with 64 states and randomly chosen interleavers for all users are applied.[2] Further, a QPSK signal constellation with Gray labeling is employed and equal transmit energy users are supposed so that $\sigma^2_{x_k} = \sigma^2_x$, $\forall k$.

In Figs. 3, 4 and 5 the measured BER versus the required energy per bit to noise ratio E_b/N_0 are depicted for $\beta = 1, 1.5, 2$, respectively. Here, N_0 denotes the one–sided power spectral density of the continuous–time additive white Gaussian channel noise. The maximum number of iterations carried out is 2 for $\beta = 1$ and 4 for $\beta = 1.5, 2$. In addition, the curves resulting from the analytical approach are included.

Further, $N = 64$ and $L = 1, 3$ are chosen so that the condition $N \gg L$ is well satisfied.

Finally, for comparison the single user bounds belonging to the BER of a single user transmitting over frequency selective channels with L paths and performing ideal maximum ratio combining at the receiver without any distortion due to intersymbol or interpath interference are depicted.

It can be seen from the figures that regardless of the load, the number of paths as well as the iteration cycle, the simulated and analytic curves for the Bit Error Ratio coincide almost completely. In particular the convergence of the multiuser receiver's performance to the single user bound can be anticipated exactly by the analytic solution. In addition, the assumptions made in the design of the receiver turn out to be

[2]In this manuscript, interleavers of length 10000 are used, so that the results for the overloaded system with $\beta = 2$ are slightly better than in [13]. There, interleavers of length 1000 were assumed.

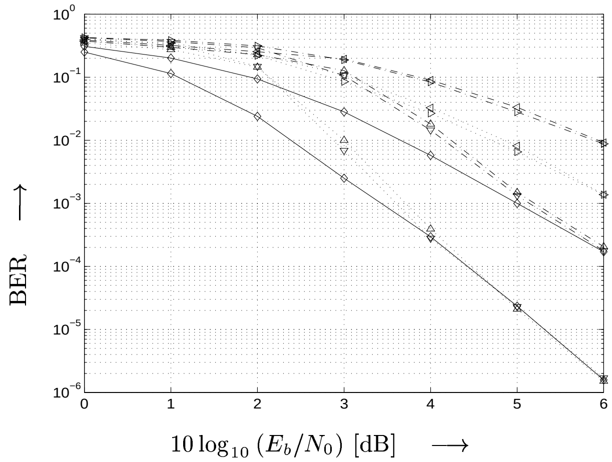

Figure 3: BER vs. $10\log_{10}(E_b/N_0)$ for $\beta = 1$ and $L = 1$ ($-\cdot$), $L = 3$ ($\cdots$) after 1st ($\triangleright$) and 2nd (∇) iteration for simulative results, after 1st ($\triangleleft$) and 2nd ($\triangle$) iteration for analytical results and corresponding SUB ($-\diamond$).

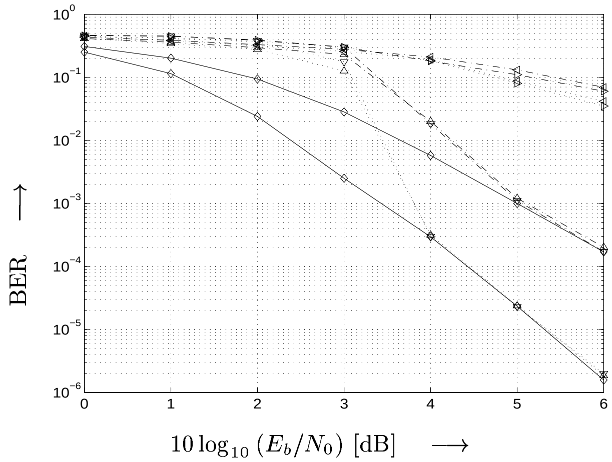

Figure 4: BER vs. $10\log_{10}(E_b/N_0)$ for $\beta = 1.5$ and $L = 1$ ($-\cdot$), $L = 3$ ($\cdots$) after 1st ($\triangleright$) and 4th (∇) iteration for simulative results, after 1st ($\triangleleft$) and 4th ($\triangle$) iteration for analytical results and corresponding SUB ($-\diamond$).

reasonable. So, in the theoretical approach the correlations between soft decisions of different users arising in course of the iterations have been ignored completely. From the good accordance of the analytical and simulated results it can be inferred that this holds for the actual receiver scheme to a large degree, too. Of course, these correlations depend heavily on the interleaver size chosen, as it is well–known for Turbo–Codes, and the system load β. Hence, we reckon that the small gaps appearing for $\beta = 2$ and $10\log_{10}(E_b/N_0) = 5$ dB and $L = 1$ as well as $10\log_{10}(E_b/N_0) = 4$ dB and $L = 3$ could be closed by choosing a larger interleaver.

On the other hand, our investigations showed that the Gaussian approximation is well satisfied already for $N = 64$. So, larger spreading factors did not provide a performance improvement. Moreover, it should be noted, that the derivation presented in Section IV is valid also for other randomly chosen spreading sequences as long as the chips are independent (see [11]). Thus, for sufficiently large spreading factor the performance of the system won't be affected by the chip distribution.

Next, the plots underline the advantages offered by

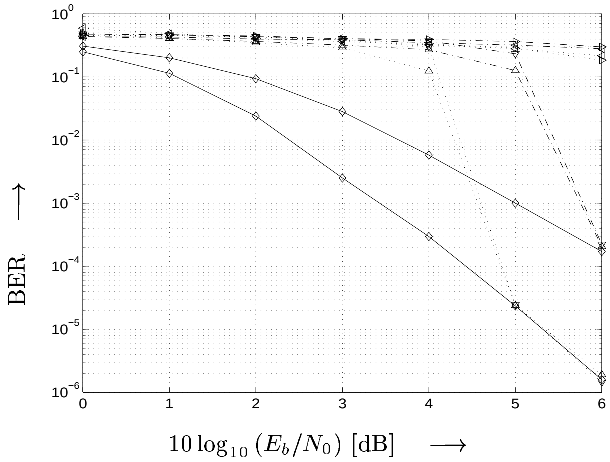

Figure 5: BER vs. $10\log_{10}(E_b/N_0)$ for $\beta = 2$ and $L = 1$ ($-\cdot$), $L = 3$ ($\cdots$) after 1st ($\triangleright$) and 4th ($\triangledown$) iteration for simulative results, after 1st ($\triangleleft$) and 4th ($\triangle$) iteration for analytical results and corresponding SUB ($-\diamond$).

nonorthogonal multiple access. To see this more clearly, let us consider the spectral efficiency Γ, which is defined as $\Gamma \triangleq K/N \cdot R_c \cdot \log_2(|\mathcal{X}|)$ (see [14]).[3] For the nonorthogonal DS–CDMA system at hand the spectral efficiency is, $\Gamma_{\not\perp} = K/N \cdot R_c \cdot \log_2(4) = \beta$. In contrast, assuming the same physical bandwidth and signal constellation for an orthogonal system the spectral efficiency is fixed to $\Gamma_{\perp} = R_c \cdot \log_2(4) = 1$. So, studying the curves provided we find that $\Gamma_{\not\perp} > \Gamma_{\perp}$ can be easily reached for $\beta > 1$ with the same power efficiency (E_b/N_0) and performance (BER). This means the other way around, to achieve for example $\Gamma_{\perp} = 2$ a 16–ary signal constellation has to be used by an orthogonal access scheme which in turn diminishes power efficiency. [4] Thus, if large spectral efficiency is the dominating performance parameter, nonorthogonal multiple access is highly preferable to orthogonal one. Unfortunately, additional computational complexity is required what in todays plannings is still a drawback.

Finally, it is noteworthy, that equal performance is achieved for all users transmitting at the same rate and same transmit power. While information theory yields that error free transmission of all users at the same rate and with equal power requires rate and power splitting [15, 16], i.e., each physical user has to be decomposed into at least two virtual users, iterative decoding and cancellation seems to overcome this requirement for the situation under study.

VI. Summary

In this work a multiuser receiver carrying out iterative multiuser detection employing MMSE–filters, iterative soft decision feedback interference cancellation and single user decoding is investigated and an analytic solution to the performance of this receiver for transmission over frequency selective fading channels is derived. It could be shown, that the actual and predicted BER's coincide for various system loads. Moreover, although, the theoretical derivation supposes infinite spreading factors, it was illustrated, that already for $N = 64$ the analytic result matches the simulations. In addition, supposing perfect knowledge of the instantaneous path weights it is confirmed that orthogonal multiple access schemes indeed can be outperformed in terms of spectral efficiency. Finally, having restricted in this paper to the MMSE receiver for sake of clarity, we can show that in the same way the performance of other iterative multiuser receivers for frequency selective fading channels with and without perfect channel state information can be predicted, too.

[3] $|\mathcal{X}|$ denotes the cardinality of the users' signal constellation.

[4] It has to be emphasized that using for example time division multiple access as orthogonal scheme the intersymbol interference occurring for $L \geq 2$ needs to be cancelled completely to achieve the single user bounds given.

References

[1] Matthew C. Valenti and Brian D. Woerner. Iterative multiuser detection for convolutionally coded asynchronous DS–CDMA. In *Proc. of Intern. Symp. on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Boston, MA, September 1998.

[2] Michael Moher. An iterative multiuser decoder for near–capacity communications. *IEEE Transactions on Communications*, 46(7):870–880, July 1998.

[3] Suman Das, Elza Erkip, Joseph R. Cavallaro, and Behnaam Aazhang. Iterative multiuser detection and decoding. In *Proc. of IEEE Global Telecommunications Conference (Globecom)*, pages 106–111, Sydney, Australia, December 1998.

[4] Mark C. Reed. Iterative multi–user detection with FEC on correlated multi–path channels. In *Proc. of 3rd ITG Conference on Source and Channel Coding*, pages 93–96, Munich, January 2000.

[5] Xiaodong Wang and H. Vincent Poor. Iterative (turbo) soft interference cancellation and decoding for coded CDMA. *IEEE Transactions on Communications*, 47(7):1046–1061, July 1999.

[6] Sergio Verdú. *Multiuser Detection.* Cambridge University Press, New York, 1998.

[7] David Tse and Stephen Hanly. Linear multiuser receivers: Effective interference, effective bandwidth and user capacity. *IEEE Transactions on Information Theory*, 45(2):641–657, March 1999.

[8] Stephan ten Brink. Iterative Decoding Trajectories of Parallel Concatenated Codes. In *Proc. of 3rd ITG Conference on Source and Channel Coding*, pages 75–80, Munich, January 2000.

[9] Alexander Lampe and Johannes Huber. On the Limits of Coded Transmission over Multipath Fding Channels with CDMA. In *Proc. of 3rd ITG Conference on Source and Channel Coding*, pages 313–318, Munich, January 2000.

[10] Peter Schramm and Ralf R. Müller. Spectral efficiency of CDMA systems with linear MMSE interference suppression. *IEEE Transactions on Communications*, 47(5):722–731, May 1999.

[11] Jack W. Silverstein. Strong convergence of the empirical distribution of eigenvalues of large dimensional random matrices. *Journal of Multivariate Analysis*, 55:331–339, 1995.

[12] Jamie S. Evans and David N.C. Tse. Linear multiuser receivers for multipath fading channels. In *Proc. of IEEE Information Theory Workshop (ITW)*, pages 30–32, South Africa, June 1999.

[13] Alexander Lampe. Coded transmission over multipath fading channels with cdma. *Submitted to IEEE Transactions on Vehicular Technology*, November 1999.

[14] Andrew J. Viterbi. *CDMA*. Addison–Wesley, 1995.

[15] Bixio Rimoldi and Rüdiger Urbanke. A rate–splitting approach to the Gaussian multiple–access channel. *IEEE Transactions on Information Theory*, 42(2):364–375, March 1996.

[16] Alexander Lampe, Ralf R. Müller, and Johannes B. Huber. Transmit Power Allocation for Gaussian Multiple Access Channels with Diversity. In *Proc. of IEEE Information Theory Workshop (ITW)*, page 101, South Africa, June 1999.