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Transmit Diversity in DS–CDMA Systems

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Abstract

In this paper we investigate the synchronous transmission of K users each of them equipped with T_{tot} antennas employing Direct Sequence Code Division Multiple Access (DS–CDMA) to a common, single antenna receiver. Assuming ideal knowledge of the time–varying Rayleigh–fading channels, use of MMSE–SIC as well as random spreading sequences, we find that the system's spectral efficiency is maximized if each user transmits different codewords spread by distinct signature sequences over its antennas. In contrast, for imperfect channel knowledge the optimum choice of spreading sequences as well as codewords allocated to the different antennas depends on T_{tot} and the spectral efficiency Γ desired. This holds also for perfect channel state information if no successive cancellation is applied.

1. Introduction

The application of multiple antennas at receiver and/or transmitter has attracted considerable attention within the last years. One field for application of multiple antenna systems is the use of space–time codes [1]. Here, codes are tailored for a given number of antennas at transmitter and receiver to reduce the bit error ratio of single user transmission by exploitation of space–time diversity. Due to these possibilities the application of multiple antennas is also a basic feature of forthcoming third generation mobile communication systems.

Extensive simulations are carried out to evaluate the reachable performance in terms of bit error ratio versus power efficiency for various numbers of antenna elements and several receiver schemes [2, 3]. In this work, we study analytically the application of multiple transmit antennas. That is, we suppose that each user can transmit with T_{tot} antennas to a common receiver over fading channels. Compared to the situation of one transmit and multiple receive antennas discussed in [4] additional degrees of freedom arise for transmitter design, as it is not mandatory that a user transmits with the same spreading sequence the same codeword over all T_{tot} antennas. Assuming given powers we can show, that for perfect channel state information (CSI) at the receiver, Gaussian channel symbols and application of an MMSE–filter for multiuser interference suppression combined with single user decoding and successive cancellation (MMSE–SIC) an optimum scheme for the allocation of spreading sequences and codewords to the transmit antennas exists, which maximizes the system's spectral efficiency. In contrast, for imperfect CSI it turns out that it depends on the desired spectral efficiency how the spreading sequences and codewords have to be allocated to the various transmit antennas. The same holds if only linear multiuser interference suppression is applied at the common receiver. Demanding that each user can transmit reliably at a desired rate R, solutions on the required transmit powers for perfect and imperfect CSI and MMSE–SIC leading to minimum total transmit power are given in the limit $N \to \infty$.

The paper is organized as follows. In Section 2, the transmission model is given. Based on this model, the capacity for CDMA employing MMSE–SIC at the receiver is derived for various allocation schemes and perfect as well as imperfect CSI in Section 3. Finally, Section 4 points out conclusions.

2. Transmission Model

We consider synchronous CDMA transmission of K users over time–varying fading channels to a common receiver with spreading factor N. Assuming that each user employs a transmitter with T_{tot} antennas, we get the equivalent complex baseband model depicted in Fig. 1.

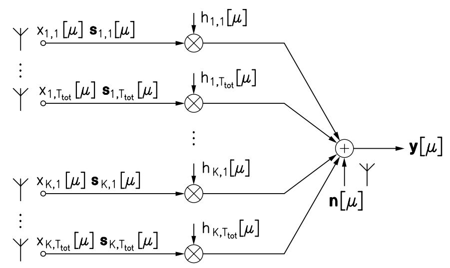

Figure 1: Transmission model of multiple–antenna single receiver system.

The transmission scenarios considered in the following can in general be described in matrix notation by

$$\boldsymbol{y}[\mu] \quad = \quad \boldsymbol{S}[\mu]\boldsymbol{H}[\mu]\boldsymbol{D}\tilde{\boldsymbol{X}}[\mu] + \boldsymbol{n}[\mu]. \qquad (1)$$

Here, the N dimensional vectors $\boldsymbol{y}[\mu] = (., y_j[\mu], .)^T$ as well as $\boldsymbol{n}[\mu] = (., n_j[\mu], .)^T$ represent the received signal and the additive channel noise in the μth transmission interval, respectively. The i.i.d. samples $n_j[\mu], 1 \leq j \leq$

N, are zero mean complex Gaussian random variables with variance σ_n^2. (Note, that the superscript T denotes the transpose, whereas $T_{\rm tot}$ stands for the number of antennas.) $\tilde{\boldsymbol{X}}[\mu] = \text{diag}(., \tilde{\boldsymbol{X}}_k[\mu], .)^T$ consists of the $T_{\rm tot} \times T_{\rm tot}$ diagonal matrices $\tilde{\boldsymbol{X}}_k[\mu] = \text{diag}(., \tilde{x}_{k,t}[\mu], .)^T$, where the (virtual) users' channel symbols $\tilde{x}_{k,t}[\mu]$, $1 \leq t \leq T_{\rm tot}, 1 \leq k \leq K$, are the product of symbols chosen from the set $\mathcal{X}$ being multiplied by $\sqrt{\tilde{P}_k}$ such that $\mathcal{E}\{|\tilde{x}_{k,t}[\mu]|^2\} = \tilde{P}_k$. $\boldsymbol{S}[\mu] = (., \boldsymbol{S}_k[\mu], .)$ is formed by the $N \times T_{\rm tot}$ dimensional matrices $\boldsymbol{S}_k[\mu] = (., \boldsymbol{s}_{k,t}[\mu], .)$. The N dimensional column vector $\boldsymbol{s}_{k,t}[\mu]$ has randomly chosen elements $s_{c,k,t}[\mu] \in \{(\pm 1 \pm \text{j})/\sqrt{2N}\}, 1 \leq c \leq N$. In addition, $\boldsymbol{H}[\mu] = \text{diag}(., \boldsymbol{H}_k[\mu], .)$ is the path weight matrix with submatrices $\boldsymbol{H}_k[\mu] = \text{diag}(., h_{k,t}[\mu], .)$. The path weight $h_{k,t}[\mu]$ represents the instantaneous short term fading affecting the transmission from antenna t of user k to the common receive antenna. In the sequel we suppose that $h_{k,t}[\mu], \forall t, \forall k$, is a zero mean Gaussian random variable with power $1/T_{\rm tot}$ independent from all other path weights. That is, a sufficient spatial seperation of the antenna elements is supposed. The kth user's signal gain caused by long term fading is represented by d_k and the matrix $\boldsymbol{D}$ is diagonal with $\boldsymbol{D}_{jj} = d_k, (k-1)T_{\rm tot} + 1 < j \leq kT_{\rm tot}$. In the following we assume that the fading processes are ergodic, the receiver knows all instantaneous d_k and omit for sake of readability the time index μ.

3. Optimum Diversity Allocation

Applying $T_{\rm tot}$ transmit antennas for transmission over single path fading channels to one receive antenna, various strategies for exploitation of the provided diversity can be discussed and two extremes can be regarded: one possibility is to transmit over each antenna the same information, i.e., the same codeword; the other way is to split the whole information stream of a single user into $T_{\rm tot}$ substreams and to transmit over each antenna one of the encoded substreams. For $N = 1$, i.e., no spreading it is well–known that the second possibility leads to a higher spectral efficiency on account of additional receiver complexity. Whether and when this holds for CDMA will be studied in the following. Here, we start with the investigation of the capacity achievable for given transmit powers $\tilde{P}_k$ assuming perfect CSI at the receiver, complex Gaussian channel symbols and $\boldsymbol{s}_{k,t} = \boldsymbol{s}_k, \forall t$. Supposing $\tilde{x}_{k,t} = \tilde{x}_k, \forall k$, the whole received signal is

$$\boldsymbol{y}_1 = \hat{\boldsymbol{S}}\text{diag}(., x_k \sum_t h_{k,t}, .) + \boldsymbol{n}, \qquad (2)$$

while for different codewords $\tilde{x}_{k,t} \neq \tilde{x}_{k,\tau}, t \neq \tau$, follows

$$\boldsymbol{y}_2 = \hat{\boldsymbol{S}}\text{diag}(., \sum_t h_{k,t} x_{k,t}, .) + \boldsymbol{n}. \qquad (3)$$

Here, we defined for sake of simplicity $x_{k,t} \triangleq d_k \tilde{x}_{k,t}$ and denote in the following the received power as $P_k = \mathcal{E}\{|x_{k,t}|^2\}$. Further, we have $\hat{\boldsymbol{S}} \triangleq (\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K)$. The total capacity of the scheme described by Eq. (2) is [5]

$$C_1 = \mathcal{E}_{\hat{\boldsymbol{S}},h}\{\log_2(\det(\boldsymbol{I} + \hat{\boldsymbol{S}}\text{diag}(., P_k|\textstyle\sum_t h_{k,t}|^2, .)(\hat{\boldsymbol{S}})^H))\},$$

while we get for the model of Eq. (3)

$$C_2 = \mathcal{E}_{\hat{\boldsymbol{S}},h}\{\log_2(\det(\boldsymbol{I} + \hat{\boldsymbol{S}}\text{diag}(., P_k \textstyle\sum_t |h_{k,t}|^2, .)(\hat{\boldsymbol{S}})^H))\}.$$

Thus, it follows $C_1 \leq C_2$ with equality iff $T_{\rm tot} = 1$.

In order to achieve these capacities no joint receiver for all users is necessary but it is well–known that the total capacities can be reached by application of MMSE–SIC. Now, we face two questions: i) Is C_2 achievable without transmission of a different codeword over each antenna and ii) is C_2 the ultimate limit for CDMA employing $T_{\rm tot}$ transmit antennas and varying $x_{k,t}$ as well as $\boldsymbol{s}_{k,t}$?

The first question can be answered considering that C_2 equals for $N \gg 1$ exactly the capacity derived for multipath fading channels with the help of an equivalent model in [6]. Defining $\boldsymbol{h}_k = (h_{k,1}, \ldots, h_{k,T})^T$ we have

Theorem 1: For rising spreading factor N, constant load β, application of different spreading sequences for each antenna element, i.e., $\boldsymbol{S}_k = (\boldsymbol{s}_{k,1}, \ldots, \boldsymbol{s}_{k,T_{\rm tot}})$, perfect CSI and transmission of the same channel symbol $\tilde{x}_{k,t} = \tilde{x}_k, \forall t$, holds

$$C_3 = \mathcal{E}_{\hat{\boldsymbol{S}},h}\left\{\log_2\left(\det(\boldsymbol{I} + \boldsymbol{S}\text{diag}(., P_k \boldsymbol{h}_k \boldsymbol{h}_k^H, .)\boldsymbol{S}^H)\right)\right\} \stackrel{N\to\infty}{\longrightarrow} C_2.$$

$\boldsymbol{I}$ denotes an $N \times N$ identity matrix. This convergence results from the fact, that the nonzero eigenvalues of $\text{diag}(., P_k \boldsymbol{h}_k^H \boldsymbol{h}_k, .)$ and $\text{diag}(., P_k \boldsymbol{h}_k \boldsymbol{h}_k^H, .)$ are equal and that their distribution determines completely the capacity for $N \to \infty$ [7]. In order to answer question ii) again the insight gained from investigation of multipath fading channels can be used and it turns out

Theorem 2: Choosing $\boldsymbol{s}_{k,t} \neq \boldsymbol{s}_{k,\tau}, \tilde{x}_{k,t} \neq \tilde{x}_{k,\tau}, \forall t \neq \tau$, the capacity of the whole system for perfect CSI is

$$C_4 = \mathcal{E}_{\boldsymbol{S},h}\left\{\log_2\left(\det(\boldsymbol{I} + \boldsymbol{S}\text{diag}(., P_k|h_{k,1}|^2, \cdot, P_k|h_{1,T_{\rm tot}}|^2, .)\boldsymbol{S}^H\right)\right\},$$

and it holds $C_4 \geq C_2 \geq C_3 \geq C_1$, with equality iff $T_{\rm tot} = 1$.

Thus, for the optimum CDMA–receiver it is not only necessary to transmit either with different spreading sequences or channel symbols to allow an independent resolution of the path weights, but the combination of both methods turns out to be superior. This can be shown for interference suppression by means of matched filters, too. In contrast, applying a decorrelating filter it is no longer optimum to split each user into $T_{\rm tot}$ subusers with its own spreading sequence due to the noise enhancement caused by the decorrelator if $KT_{\rm tot} \to N$. In Fig. 2a)-b) the spectral efficiencies $\mathit{\Gamma} = C/N$ for perfect CSI, $P_k = P, \forall k$, and $N \to \infty$ are depicted versus power efficiency $10\log_{10}(\bar{E}_b/N_o) = 10\log_{10}(P/\sigma_n^2 \cdot \beta/\mathit{\Gamma})$ for $\beta = 0.25, 1$ and $T_{\rm tot} = 2, 3$.

By now, ideal CSI at the receiver has been considered, however in practical situations it is to be expected that (severe) channel estimation errors occur. In the

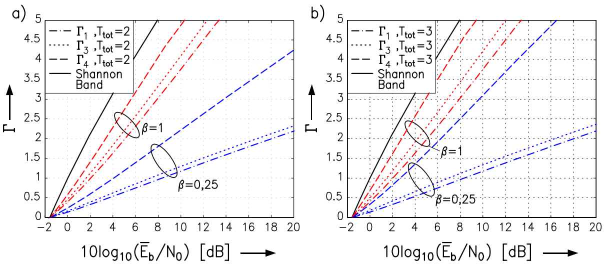

Figure 2: Spectral efficiency vs. power efficiency for MMSE–SIC, $T_{\rm tot}=2,3$ and load $\beta=0.25,1$.

following we assume independent channel estimates being modeled at receiver site as $h_{k,t}=\hat{h}_{k,t}+\hat{n}_{k,t}$. Here, $\hat{h}_{k,t}$ and $\hat{n}_{k,t}$ denote the zero mean Gaussian distributed path gain estimate with variance $1/T_{\rm tot}-J_{T_{\rm tot}}$ and the orthogonal zero mean Gaussian estimation error with power $J_{T_{\rm tot}}$, respectively. Beside this, we suppose that channel estimation is performed with equal accuracy for all users, paths and time slots. Demanding that the signal to interference ratio (SIR) at the output of an MMSE–filter is the decisive parameter determining the achievable capacity also for nonideal CSI, M–ary PSK and equal received powers P are chosen for all users.

A consequence of imperfect CSI is that the multiuser interference cannot be cancelled completely. Considering a system where the users are decoded in decreasing order of their indices, the signal employed for recovery of the kth user's information is for $\boldsymbol{s}_{k,t}=\boldsymbol{s}_k, x_{k,t}=x_k, \forall t$, given as $\boldsymbol{y}^k_{1,imp}=\sum_{\kappa=1}^{k}\boldsymbol{s}_\kappa(\hat{h}_\kappa+\hat{n}_\kappa)x_\kappa+\sum_{\kappa=k+1}^{K}\boldsymbol{s}_\kappa\hat{n}_\kappa x^d_\kappa+\boldsymbol{n}$, where $x^d_{\kappa,t}$ denotes the perfectly known channel symbol after decoding of user $\kappa>k$. Note, employing equal spreading sequences and equal channel symbols for all antennas, at receiver site the sum of independent Gaussian path weights $\sum_t h_{k,t}$ with variance $1/T_{\rm tot}$ resembles one single path weight h_k with power 1. Hence, merely h_k has to be estimated. Based on this model and Theorem 3.1 of [7], we get for the SIR at the output of the kth user's MMSE–filter if $N\rightarrow\infty$

Theorem 3: For rising spreading factor and constant load $\beta=K/N$, the SIR of user $k=\alpha/N, \alpha\in[0,\beta]$, converges in probability to

$$\mathrm{SIR}_{1,imp}(\alpha,\beta)\quad=\quad|\hat{h}_\alpha|^2/(\eta_1(\alpha,\beta)+J_1), \qquad (4)$$

where $\eta_1(\alpha,\beta)$ is solution to $\eta_1(\alpha,\beta)=\sigma_n^2/P+(\beta-\alpha)\frac{\eta_1(\alpha,\beta)J_1}{\eta_1(\alpha,\beta)+J_1}+\alpha\int\limits_0^\infty\frac{\eta_1(\alpha,\beta)\zeta}{\eta_1(\alpha,\beta)+\zeta}f_{|\hat{h}|_1^2+J_1}(\zeta)\mathrm{d}\zeta$.

Here, $f_{|\hat{h}|_1^2+J_1}(\zeta)$ denotes the pdf of $|\hat{h}|^2+J$, where the subscript 1 stands for the inverse of the path weight's variance and $|x|$ denotes the absolute value of x. Inserting this solution into the well–known formula for the mutual information of MPSK [5] and summing up over all users, the total capacity $C_{1,imp}$ is obtained. Note that this solution is true regardless of $T_{\rm tot}$. For $\boldsymbol{s}_{k,t}\neq\boldsymbol{s}_{k,\tau}, x_{k,t}=x_{k,\tau}, \forall t\neq\tau$, the model for the information transmitted over the tth antenna by user k reads $\boldsymbol{y}^{k,t}_{4,imp}=\sum_{\tau=1}^{t}\boldsymbol{s}_{k,\tau}(\hat{h}_{k,\tau}+\hat{n}_{k,\tau})x_{k,\tau}+\sum_{\tau=t+1}^{T_{\rm tot}}\boldsymbol{s}_{k,\tau}\hat{n}_{k,\tau}x^d_{k,\tau}+\sum_{\kappa=1}^{k-1}\sum_{\tau=1}^{T_{\rm tot}}\boldsymbol{s}_{\kappa,\tau}(\hat{h}_{\kappa,\tau}+\hat{n}_{\kappa,\tau})x_{\kappa,\tau}+\sum_{\kappa=k+1}^{K}\sum_{\tau=1}^{T_{\rm tot}}\boldsymbol{s}_{\kappa,\tau}\hat{n}_{\kappa,\tau}x^d_{\kappa,\tau}+\boldsymbol{n}$, and we get

Theorem 4: For rising N and constant β the SIR of the tth virtual users belonging to user $k=\alpha/N, \alpha\in[0,\beta]$, converges in probability to

$$\mathrm{SIR}_{4,imp}(\alpha,\beta)\quad=\quad|\hat{h}_{\alpha,t}|^2/(\eta_{T_{\rm tot}}(\alpha,\beta)+J_{T_{\rm tot}}), \qquad (5)$$

where $\eta_{T_{\rm tot}}(\alpha,\beta)=\frac{\sigma_n^2}{P}+(\beta-\alpha)T_{\rm tot}\frac{\eta_{T_{\rm tot}}(\alpha,\beta)J_{T_{\rm tot}}}{\eta_{T_{\rm tot}}(\alpha,\beta)+J_{T_{\rm tot}}}+\alpha T_{\rm tot}\int\limits_0^\infty\frac{\eta_{T_{\rm tot}}(\alpha,\beta)\zeta}{\eta_{T_{\rm tot}}(\alpha,\beta)+\zeta}f_{|\hat{h}|^2_{T_{\rm tot}}+J_{T_{\rm tot}}}(\zeta)\mathrm{d}\zeta$.

The total capacity $C_{4,imp}$ results from summation over all $KT_{\rm tot}$ (virtual) users. Studying the above formulas two significant points turn out. First, regarding that in the last scheme each physical user has to estimate $T_{\rm tot}$ path weights (one for each antenna) with variance $1/T_{\rm tot}$, the residual interference is increased in a large range, even if $J_{T_{\rm tot}}<J_1$. This is due to the fact that $T_{\rm tot}$ uncorrelated interferers with power $J_{T_{\rm tot}}$ may be worse than one single interferer with power J_1. In addition for a rising number of antenna elements, the channel estimation performance of the last scheme will degrade more and more as the power allocated to estimation of one path weight decreases. This, will even outweigh the advantage provided by the resolution of more paths and combination of $T_{\rm tot}$ virtual users. In Fig. 3a) the resulting spectral efficiencies $\Gamma_{imp}=C_{imp}/N$ versus power efficiency are depicted for QPSK, load $\beta=0.25,1$ and $T_{\rm tot}=3$. Beside this, in Fig. 3b) the normalized channel estimation error $J_{T_{\rm tot}}T_{\rm tot}$ is given in dependence of $\mathrm{SNR}=P/\sigma_n^2$ for the underlying iterative MMSE channel estimation algorithm whose description is not presented here.

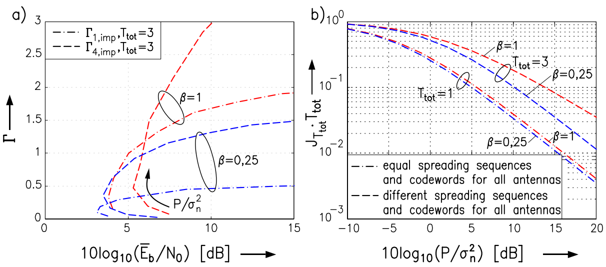

Figure 3: Spectral efficiency vs. power efficiency for $T_{\rm tot}=3$ and load $\beta=0.25,1$ in Fig. a) as well as normalized channel estimation error vs. signal to noise ratio in Fig. b).

Although, we have not considered the other two possibilities for sake of clarity, their performance can be evaluated along the same lines. For example, employing unequal spreading sequences for each antenna but transmitting the same codeword, it turns out that this scheme's spectral efficiency merges with $\Gamma_{4,imp}$ for low

SNR and with $\Gamma_{1,imp}$ for high SNR. It should be mentioned, that the necessity of an explicit optimization arises not only for imperfect channel estimation but also for ideal channel knowledge if no interference cancellation is employed. This is omitted here, due to space limitations.

By now, we have studied the rates achievable for given powers. In practical situations however, we may be interested in finding those unique transmit powers which minimize the total transmit power while allowing transmission at a desired rate tuple $(R_1, \ldots, R_K)$. Supposing perfect CSI and restricting ourselves to the equal–rate case, i.e., $R_k = R, \forall k$, as well as application of MMSE–SIC, we find that the optimum decoding order of the different (virtual) users depends solely on their long term gains due to the symmetry of the received power region (cf. [8, 9]). Thus, for $\boldsymbol{s}_{k,t} \neq \boldsymbol{s}_{k,\tau}, x_{k,t} \neq x_{k,\tau}, t \neq \tau$ we have: indexing the (virtual) users in increasing order of their long term signal gains such that $d_1 \leq, \ldots, \leq d_K$, the power $\tilde{P}_k = \tilde{P}_{\alpha=k/N}$ minimizing $T_{\text{tot}} \sum_{k=1}^{K} \tilde{P}_k$ while allowing reliable transmission at rate R by application of MMSE–SIC with $\beta, T_{\text{tot}} = \text{const.}$ is for $N \to \infty$ solution to $\overline{\text{SIR}}_4 \sigma_n^2 + \alpha T \int_0^{\alpha} \int_0^{\infty} \frac{\overline{\text{SIR}}_4 \tilde{P}_\alpha d_\alpha \gamma \zeta}{\tilde{P}_\alpha d_\alpha + \overline{\text{SIR}}_4 \gamma \zeta} f_{|h|^2_{T_{\text{tot}}}}(\zeta) \mathrm{d}\zeta f_P(\gamma) \mathrm{d}\gamma = \tilde{P}_\alpha d_\alpha$, where $\overline{\text{SIR}}_4$ is obtained from $R/T = \exp\left(T_{\text{tot}}/\overline{\text{SIR}}_4\right) \text{Ei}\left(-T_{\text{tot}}/\overline{\text{SIR}}_4\right) / \ln(2)$, and $f_P(\gamma)$ denotes the pdf of the users' required received powers $P = \tilde{P}d$. Note, that for $T_{\text{tot}}/N \to 0$ each of the T_{tot} virtual users of user k is able to transmit reliably at equal rate R/T_{tot} since the interference among these users vanishes for $N \to \infty$. While for perfect CSI the optimum diversity scheme is known, for imperfect channel knowledge the best choice of the spreading sequences and codewords depends in general on β and T_{tot}. Independently of this choice we can show that for equal channel estimation accuracy of all users, the optimum decoding order follows the users' long term signal gains. Then, the transmit powers required for $\boldsymbol{s}_{k,t} \neq \boldsymbol{s}_{k,\tau}, x_{k,t} \neq x_{k,\tau}, t \neq \tau$ are

Theorem 5: Supposing channel estimation error variance $J_{T_{\text{tot}}}$, $T_{\text{tot}} = \text{const.}$ transmit antennas, fixed load β and application of MMSE–SIC, the required transmit power $P_{\alpha=k/N}$ for each antenna of user $\alpha = k/N$ is for $N \to \infty$ in probability

$$\tilde{P}_\alpha = \eta_\alpha \overline{\text{SIR}}_{4,imp} / (1 - J_{T_{\text{tot}}} \overline{\text{SIR}}_{4,imp}). \tag{6}$$

where $\eta_\alpha = \sigma_n^2 + \alpha T_{\text{tot}} \int_0^{\alpha} \int_0^{\infty} \frac{\eta_\alpha \zeta f_{|\hat{h}|^2_{T_{\text{tot}}} + J_{T_{\text{tot}}}}(\zeta)}{\eta_\alpha + \gamma \zeta} f_P(\gamma) \mathrm{d}\zeta \mathrm{d}\gamma$ $+ (\beta - \alpha) T_{\text{tot}} \int_\alpha^{\infty} \int_0^{\infty} \frac{\eta_\alpha \gamma J_{T_{\text{tot}}}}{\eta_\alpha + \gamma J_{T_{\text{tot}}}} f_P(\gamma) \mathrm{d}\gamma$, **and for MPSK,** $\overline{\text{SIR}}_{4,imp}$ **is derived from**

$$R/T = \int_0^{\infty} \int_{\mathbb{C}} \sum_{m=0}^{M-1} \frac{\zeta \mathrm{e}^{-\zeta |\gamma - \mathrm{e}^{\mathrm{j}\, 2\pi m/M}|^2}}{\pi M \overline{\text{SIR}}_{4,imp}} \log_2 \left(\frac{\mathrm{e}^{-\zeta |\gamma - \mathrm{e}^{\mathrm{j}\, 2\pi m/M}|^2}}{\sum_{p=0}^{M-1} \mathrm{e}^{-\zeta |\gamma - \mathrm{e}^{\mathrm{j}\, 2\pi p/M}|^2}} \right) \times f_{|\hat{h}|^2_{T_{\text{tot}}}}(\zeta / \overline{\text{SIR}}_{4,imp}) \mathrm{d}\gamma \mathrm{d}\zeta.$$

The solution of the required transmit powers can be obtained numerically in an iterative way. For all other transmission schemes considered above an equivalent formula can be derived, too. So, in the special case of equal spreading sequences and codewords transmitted over the T_{tot} antennas, the powers are solved from Theorem 5 by choosing $T_{\text{tot}} = 1$.

4. Conclusion

In this work we studied the application of various transmission schemes in order to make use of transmit diversity in DS–CDMA. We found that for perfect CSI and successive cancellation the transmission of different codewords over each antenna spread by different signature sequence is optimum. In contrast, if the channel is not perfectly known or interference cancellation is not applied, it depends on the parameters chosen, which access procedure leads to the highest spectral efficiency for given power efficiency.

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