Performance Analysis and Design of STBC's for Frequency–Selective Fading Channels

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Abstract

In this paper, space–time block–coded transmission over frequency–selective fading channels is investigated. A lower bound for the pairwise error probability for optimum detection is given. Also an approximation for the bit error rate is derived and compared with simulation results for maximum–likelihood sequence estimation (MLSE) for the GSM/EDGE (Enhanced Data Rates for GSM Evolution) system. Furthermore, a novel design rule for space–time block codes (STBC's) for frequency–selective fading channels is provided. A corresponding code is designed and shown to yield higher performance than Alamouti's code. It is demonstrated that for fading channels with L independent impulse response coefficients, STBC's designed for the flat fading channel can achieve at most a diversity order of $(N_T + L - 1)N_R$ if N_T transmit antennas and N_R receive antennas are used. On the other hand, the maximum diversity order employing the proposed code design rule is LN_TN_R.

1 Introduction

Recently, it has been shown that space–time codes can significantly improve the achievable performance for transmission over *flat* fading channels [1, 2, 3]. In general, it is distinguished between space–time trellis codes (STTC's) [2] and space–time block codes (STBC's) [3]. Although most results in this paper may also be applied to STTC's, we concentrate on STBC's.

Unfortunately, wireless communication channels are usually *frequency–selective*. Since existing STBC's are designed for channels without intersymbol interference (ISI), it is not obvious how they perform in frequency–selective channels which suffer from ISI.

The performance of space–time codes in unequalized multipath fading has been investigated in [4]. Furthermore, an analysis for multipath fading and maximum–likelihood sequence estimation (MLSE) is given in [5]. The results in [4] and [5] show that multipath fading does not decrease the diversity order of space–time codes designed for flat fading, i.e., a diversity order of $N_T \cdot N_R$ can be achieved, if N_T transmit antennas and N_R receive antennas are used.

It is well known that the *matched filter bound* (MFB) is a simple but very useful tool to analyze the performance limits for transmission over fading ISI channels [6, 7]. Our performance analysis is related to the MFB, i.e., we consider the transmission of a single STBC word and evaluate the corresponding pairwise error probability (PEP) and the bit error rate (BER) for maximum–likelihood detection. However, for practical reasons we do not employ the optimum matrix matched filter (MF) as receive filter but identical fixed square–root Nyquist (SRN) filters for each receive branch, respectively.

2 Preliminaries

2.1 Notation

Bold upper case ($\boldsymbol{X}$) and lower case ($\boldsymbol{x}$) letters denote matrices and vectors, respectively. $\det(\cdot)$, $\mathrm{rank}(\cdot)$, $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$ refer to the determinant of a matrix, the rank of a matrix, transposition, Hermitian transposition, and complex conjugation, respectively. $\mathrm{diag}\{x_1, x_2, \ldots, x_M\}$ is a diagonal matrix with main diagonal elements $x_1, x_2, \ldots, x_M$, whereas $\boldsymbol{I}_M$, $\boldsymbol{0}_{M\times N}$, and $\boldsymbol{0}_{M\times 1} = \boldsymbol{0}_M$ are the $M \times M$ identity matrix, the $M \times N$ all–zero matrix, and the all–zero column vector of length M, respectively. $\Pr\{\cdot\}$, $\mathcal{E}\{\cdot\}$, $j \triangleq \sqrt{-1}$, $\Re\{\cdot\}$, $||\cdot||$, and $\otimes$ denote the probability of the event in brackets, expectation, the imaginary unit, the real part of a complex number, the L_2–norm of a vector, and the Kronecker product, respectively. Finally, $\delta_K[\cdot]$ and $\delta_D(\cdot)$ are the Kronecker and the Dirac delta functions, respectively. Throughout this paper, all signals are represented by their complex–baseband equivalents.

2.2 Transmission Model

We consider a transmission scheme using N_T transmit antennas and N_R receive antennas. In this paper, we focus on the transmission of a single $N_T \times N$ STBC matrix $\boldsymbol{C}$, whose element $c_{\nu k}$ is transmitted at time k, $1 \le k \le N$, from antenna ν, $1 \le \nu \le N_T$. Thereby, $\boldsymbol{C}$ is taken from a set $\mathcal{C}$ of 2^{N_b} different code matrices. N_b information bits are mapped to the elements of $\mathcal{C}$ and if we want to refer to a particular element of the set, we use a subscript α, $1 \le \alpha \le 2^{N_b}$ ($\boldsymbol{C}_\alpha \in \mathcal{C}$). In order to make the transmitted energy independent from the number of transmit antennas N_T, the normalization $\sum_{\nu=1}^{N_T} \sum_{k=1}^{N} \mathcal{E}\{|c_{\nu k}|^2\} = N \cdot E_c$ is adopted.

After T–spaced sampling (T denotes the symbol duration) the received signal at antenna μ can be written as [8]

$$r_\mu[k] = \sum_{\nu=1}^{N_T} \sum_{l=0}^{L-1} h_{\nu\mu}[l] c_{\nu\, k-l} + n_\mu[k], \qquad (1)$$

with the discrete–time overall channel impulse response (CIR) $h_{\nu\mu}[\cdot]$ from transmit antenna ν to receiver antenna μ and the noise process $n_\mu[\cdot]$. $h_{\nu\mu}[\cdot]$ is truncated to a suitable length L and includes the combined effects of transmit filter, multipath fading, receiver input filter, and sampling. As usual, we assume that the path weights of the wireless channel are complex Gaussian random variables (RV's). Therefore, the coefficients of the discrete–time overall CIR are (generally correlated) Gaussian RV's. Furthermore, a block fading channel model is adopted, i.e., $h_{\nu\mu}[\cdot]$ is assumed to be time–invariant within one block (and thus, for our analysis) but varies from block to block, e.g. due to frequency hopping.

As it is customary, we assume that the transmit filters and receiver input filters for all branches are identical, respectively. It can be expected that properly chosen fixed receiver input filters cause only a negligible loss in performance [9].

$n.[\cdot]$ is assumed to be spatially white. Since we employ fixed SRN receiver filters, $n.[\cdot]$ is also temporally white. Transmit and receive filters are normalized to guarantee a noise variance $\sigma_n^2 \triangleq \mathcal{E}\{|n_\mu[k]|^2\} = N_0/T$ and $E_h = \sum_{\nu=1}^{N_T} \sum_{\mu=1}^{N_R} \sum_{l=0}^{L-1} \mathcal{E}\{|h_{\nu\mu}[l]|^2\} = N_R N_T$, where N_0 denotes the single–sided power spectral density of the underlying continuous–time noise process.

3 Performance Analysis

Similar to the classical MFB (e.g. [6, 7]), we consider in the following the transmission of a single (matrix) symbol. The resulting PEP constitutes a lower bound for realizable receivers (employing the same transmit and receive filters).

3.1 Optimum Detector

Before the PEP can be analyzed, the optimum detection rule for the transmitted matrix $\boldsymbol{C}$ has to be derived. From Eq. (1) it can be observed that only the samples $r_\mu[k]$, $1 \le k \le N + L - 1$, depend on elements of $\boldsymbol{C}$. Hence, without loss of optimality, we may restrict our attention to the vector $\boldsymbol{r}_\mu \triangleq [r_\mu[1]\ r_\mu[2]\ \ldots\ r_\mu[N + L - 1]]^T$, $1 \le \mu \le N_R$. Regarding Eq. (1), $\boldsymbol{r}_\mu$ can be expressed as [8]

$$\boldsymbol{r}_\mu = \boldsymbol{C}'_m \boldsymbol{h}_\mu + \boldsymbol{n}_\mu. \qquad (2)$$

with the definitions $\boldsymbol{h}_\mu \triangleq [(\boldsymbol{h}^1_{1\mu})^T\ (\boldsymbol{h}^2_{1\mu})^T\ \ldots\ (\boldsymbol{h}^N_{N_T\mu})^T]^T$, $\boldsymbol{h}^k_{\nu\mu} \triangleq [\boldsymbol{0}^T_{k-1}\ \boldsymbol{h}^T_{\nu\mu}\ \boldsymbol{0}^T_{N-k}]^T$, $\boldsymbol{h}_{\nu\mu} \triangleq [h_{\nu\mu}[0]\ \ldots\ h_{\nu\mu}[L-1]]^T$, $\boldsymbol{C}'_m \triangleq [\boldsymbol{C}_{11}\ \boldsymbol{C}_{12}\ \ldots\ \boldsymbol{C}_{N_T N}]$, $\boldsymbol{C}_{\nu k} \triangleq c_{\nu k} \cdot \boldsymbol{I}_{N+L-1}$, and $\boldsymbol{n}_\mu \triangleq [n_\mu[1]\ \ldots\ n_\mu[N+L-1]]^T$. Now, we stack the signal vectors $\boldsymbol{r}_\mu$ of all receive antennas μ into a single vector $\boldsymbol{r} \triangleq [\boldsymbol{r}^T_1\ \boldsymbol{r}^T_2\ \ldots\ \boldsymbol{r}^T_{N_R}]^T$ and obtain

$$\boldsymbol{r} = \boldsymbol{C}_m \boldsymbol{h} + \boldsymbol{n}, \qquad (3)$$

where the definitions $\boldsymbol{h} \triangleq [\boldsymbol{h}^T_1\ \ldots\ \boldsymbol{h}^T_{N_R}]^T$, $\boldsymbol{n} \triangleq [\boldsymbol{n}^T_1\ \ldots\ \boldsymbol{n}^T_{N_R}]^T$, and $\boldsymbol{C}_m \triangleq \boldsymbol{I}_{N_R} \otimes \boldsymbol{C}'_m$ have been used.

Since we assume that the channel (i.e., $\boldsymbol{h}$) is perfectly known at the receiver and the elements of $\boldsymbol{n}$ are independent zero–mean complex Gaussian random variables with variance σ_n^2, respectively, the (optimum) maximum–likelihood detector determines the estimate $\hat{\boldsymbol{C}}$ for the transmitted matrix $\boldsymbol{C}$ (which is uniquely associated with $\boldsymbol{C}_m$) according to the decision rule

$$\hat{\boldsymbol{C}} = \underset{\boldsymbol{C}}{\mathrm{argmin}}\ \{||\boldsymbol{r} - \boldsymbol{C}_m \boldsymbol{h}||^2\}. \qquad (4)$$

3.2 Pairwise Error Probability

The PEP $P_e(\alpha, \beta)$ is the probability that $\hat{\boldsymbol{C}} = \boldsymbol{C}_\beta$ is detected if $\boldsymbol{C} = \boldsymbol{C}_\alpha$ ($\boldsymbol{C}_\alpha, \boldsymbol{C}_\beta \in \mathcal{C}$, $\alpha \ne \beta$) is transmitted. From Eq. (4) it is easy to see that PEP is given by

$$P_e(\alpha,\beta) = \Pr\{2\Re\{\boldsymbol{n}^H \boldsymbol{C}_m^{\beta|\alpha}\boldsymbol{h}\} > (\boldsymbol{C}_m^{\beta|\alpha}\boldsymbol{h})^H \boldsymbol{C}_m^{\beta|\alpha}\boldsymbol{h}\}, \tag{5}$$

with $\boldsymbol{C}_m^{\beta|\alpha} \triangleq \boldsymbol{C}_m^{\beta} - \boldsymbol{C}_m^{\alpha}$, where $\boldsymbol{C}_m^{\alpha}$ and $\boldsymbol{C}_m^{\beta}$ are obtained from $\boldsymbol{C}_\alpha$ and $\boldsymbol{C}_\beta$, respectively, in the same way as $\boldsymbol{C}_m$ from $\boldsymbol{C}$.

Since we assume a Ricean channel model, $\boldsymbol{h}$ may be written as $\boldsymbol{h} = \bar{\boldsymbol{h}} + \boldsymbol{h}_s$, where $\bar{\boldsymbol{h}} \triangleq \mathcal{E}\{\boldsymbol{h}\}$ and $\boldsymbol{h}_s$ denote the direct and the specular (Rayleigh) component of $\boldsymbol{h}$, respectively. The covariance matrix of $\boldsymbol{h}$ is given by $\boldsymbol{\Phi}_{h_s h_s} \triangleq \mathcal{E}\{\boldsymbol{h}_s \boldsymbol{h}_s^H\}$.

By introducing the vector $\boldsymbol{l} \triangleq [(\boldsymbol{C}_m^{\beta|\alpha}\boldsymbol{h})^T \ \boldsymbol{n}^T]^T$, Eq. (5) can be simplified to

$$P_e(\alpha,\beta) = \Pr\{\Delta(\alpha,\beta) < 0\}, \tag{6}$$

with $\Delta(\alpha,\beta) \triangleq \boldsymbol{l}^H \boldsymbol{F} \boldsymbol{l}$ and
$\boldsymbol{F} \triangleq \begin{pmatrix} \boldsymbol{I}_{(N+L-1)N_R} & -\boldsymbol{I}_{(N+L-1)N_R} \\ -\boldsymbol{I}_{(N+L-1)N_R} & \boldsymbol{0}_{(N+L-1)N_R \times (N+L-1)N_R} \end{pmatrix}$.

Since $\Delta(\alpha,\beta)$ is a quadratic form of Gaussian random variables, the Laplace transform of its pdf can be easily calculated to [10]

$$\Phi_{\Delta(\alpha,\beta)}(s) = \frac{\exp\left(-s\,\bar{\boldsymbol{l}}^H(\boldsymbol{F}^{-1} + s\,\boldsymbol{\Phi}_{ll})^{-1}\bar{\boldsymbol{l}}\right)}{\det(\boldsymbol{I}_{2(N+L-1)N_R} + s\,\boldsymbol{\Phi}_{ll}\boldsymbol{F})}, \tag{7}$$

where $\bar{\boldsymbol{l}} \triangleq [(\boldsymbol{C}_m^{\beta|\alpha}\bar{\boldsymbol{h}})^T \ \boldsymbol{0}^T_{(N+L-1)N_R}]^T$ and $\boldsymbol{\Phi}_{ll} \triangleq \begin{pmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{12}^T & \boldsymbol{A}_{22} \end{pmatrix}$ $[\boldsymbol{A}_{11} \triangleq \boldsymbol{C}_m^{\beta|\alpha}\boldsymbol{\Phi}_{h_s h_s}(\boldsymbol{C}_m^{\beta|\alpha})^H$, $\boldsymbol{A}_{12} \triangleq \boldsymbol{0}_{(N+L-1)N_R \times (N+L-1)N_R}$, $\boldsymbol{A}_{22} \triangleq \sigma_n^2 \cdot \boldsymbol{I}_{(N+L-1)N_R}]$ are the mean and the covariance matrix of vector $\boldsymbol{l}$, respectively. The PEP can be calculated directly from $\Phi_{\Delta(\alpha,\beta)}(s)$ [11]

$$P_e(\alpha,\beta) = \frac{1}{2\pi j}\int_{\gamma-j\infty}^{\gamma+j\infty} \Phi_{\Delta(\alpha,\beta)}(s)\frac{\mathrm{d}s}{s} \tag{8}$$

for $0 < \gamma < \Re\{s_1\}$, where s_1 denotes that pole of $\Phi_{\Delta(\alpha,\beta)}(s)$ which has minimum positive real part. The integral in Eq. (8) may be evaluated using efficient numerical methods (cf. [11]).

3.3 Approximation for BER

The exact calculation of BER is quite involved if not impossible. Therefore, we may resort to the union bound which is given by

$$P_b = \frac{1}{2^{N_b}} \sum_{\alpha=1}^{2^{N_b}} \sum_{\substack{\beta=1 \\ \beta\neq\alpha}}^{2^{N_b}} \frac{n(\alpha,\beta)}{N_b} \cdot P_e(\alpha,\beta), \tag{9}$$

where $n(\alpha,\beta)$ denotes the number of bit errors if $\boldsymbol{C}_\alpha$ is transmitted and $\boldsymbol{C}_\beta$ is detected. Since the union bound is an *upper* bound, P_b is not a lower bound for the achievable BER for space–time coded transmission. However, for moderate to large signal–to–noise ratios (SNR's) the union bound becomes tight and P_b is a very good approximation for the achievable BER of MLSE. A disadvantage of Eq. (9) is that the PEP's for all possible pairs of α and β have to be calculated. In order to decrease the computational load, we propose to approximate P_b by

$$P_b \approx \frac{N_{\max}}{N_b 2^{N_b}} \cdot P_{\max}, \tag{10}$$

with

$$P_{\max} \triangleq \max_{\substack{(\alpha,\beta) \\ \alpha\neq\beta}} \{P_e(\alpha,\beta)\}. \tag{11}$$

$N_{\max}$ denotes the number of pairs (α,β) for which $P_e(\alpha,\beta) = P_{\max}$ is true. Note that, for simplicity, in Eq. (10) it is assumed that each (matrix) symbol error causes only one bit error. For an efficient method to determine those pairs of (α,β) with $P_e(\alpha,\beta) = P_{\max}$ without calculating $P_e(\alpha,\beta)$ itself we refer to [8]. Once the relevant pairs (and their number $N_{\max}$) are available, P_b is obtained from Eqs. (10), (11). Despite the simplicity of the proposed approximation, comparisons with simulation results for MLSE show its accuracy and relevance.

4 Diversity Order and Code Design

For simplicity, in this section, we restrict ourselves to Rayleigh fading channels.

4.1 Bound on Diversity Order

Since the autocorrelation matrix (ACM) $\boldsymbol{\Phi}_{h_s h_s}$ of $\boldsymbol{h}_s$ is a positive semidefinite Hermitian matrix, there is a factorization (eigendecomposition) [12]

$$\boldsymbol{\Phi}_{h_s h_s} = \boldsymbol{M}\boldsymbol{\Lambda}\boldsymbol{M}^H = \boldsymbol{G}\boldsymbol{G}^H, \tag{12}$$

where $\boldsymbol{M}$ is a unitary matrix and $\boldsymbol{\Lambda} = \mathrm{diag}\{\lambda_1, \ldots, \lambda_{(N+L-1)NN_TN_R}\}$, $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{(N+L-1)NN_TN_R} \geq 0$, is a diagonal matrix whose elements are the eigenvalues of $\boldsymbol{\Phi}_{h_s h_s}$. $\boldsymbol{G}$ is given by $\boldsymbol{G} \triangleq \boldsymbol{M}\boldsymbol{\Lambda}^{1/2}$, with $\boldsymbol{\Lambda}^{1/2} = \mathrm{diag}\{\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_{(N+L-1)NN_TN_R}}\}$. Now $\boldsymbol{h} = \boldsymbol{h}_s$ may be expressed as

$$\boldsymbol{h} = \boldsymbol{G}\boldsymbol{h}_s', \tag{13}$$

where $\boldsymbol{h}_s'$ is a zero–mean complex Gaussian vector with $\boldsymbol{\Phi}_{h_s' h_s'} \triangleq \mathcal{E}\{\boldsymbol{h}_s'(\boldsymbol{h}_s')^H\} = \boldsymbol{I}_{(N+L-1)NN_TN_R}$.

For the following, we introduce the singular value decomposition [12]

$$\boldsymbol{C}_m^{\beta|\alpha}\boldsymbol{G} \triangleq \boldsymbol{V}^H\boldsymbol{\Sigma}\boldsymbol{W}, \tag{14}$$

where $\boldsymbol{V}$ and $\boldsymbol{W}$ are $(N+L-1)N_R \times (N+L-1)N_R$ and $(N+L-1)NN_TN_R \times (N+L-1)NN_TN_R$ unitary matrices, respectively. For the elements σ_{mn}, $1 \leq m \leq (N+L-1)N_R$, $1 \leq n \leq (N+L-1)NN_TN_R$, of matrix $\boldsymbol{\Sigma}$, $\sigma_{mn} = 0$, $m \neq n$, and $\sigma_{11} \geq \sigma_{22} \geq \ldots \geq \sigma_{(N+L-1)N_R\ (N+L-1)N_R} \geq 0$ holds. Taking the above considerations into account, Eq. (5) can be rewritten as [8]

$$P_e(\alpha,\beta) = \Pr\{2\Re\{\boldsymbol{n}'^H \boldsymbol{h}_s''\} > \boldsymbol{h}_s''^H \boldsymbol{h}_s''\}, \qquad (15)$$

with $\boldsymbol{n}' \triangleq \boldsymbol{V}\boldsymbol{n}$ and $\boldsymbol{h}_s'' \triangleq \boldsymbol{\Sigma}\boldsymbol{W}\boldsymbol{h}_s'$. Note that the elements of $\boldsymbol{n}'$ are independent zero–mean Gaussian random variables with variance σ_n^2, respectively. The $(N+L-1)N_R \times (N+L-1)N_R$ ACM of $\boldsymbol{h}_s''$ is given by $\boldsymbol{\Phi}_{h_s''h_s''} = \boldsymbol{\Sigma}\boldsymbol{\Sigma}^H = \mathrm{diag}\{\sigma_{11}^2, \ldots, \sigma_{qq}^2, 0, \ldots, 0\}$, i.e., $\boldsymbol{\Phi}_{h_s''h_s''}$ may be interpreted as the ACM of q independent Rayleigh fading processes with variances σ_{nn}^2, $1 \leq n \leq q$, respectively. Therefore, q is referred to as the *diversity order* of the STBC for a given fading ISI channel. $q = \mathrm{rank}(\boldsymbol{\Sigma}\boldsymbol{\Sigma}^H) = \mathrm{rank}(\boldsymbol{C}_m^{\beta|\alpha}\boldsymbol{G})$ can be bounded by [12]

$$q \leq \min\{\mathrm{rank}(\boldsymbol{C}_m^{\beta|\alpha}), \mathrm{rank}(\boldsymbol{G})\}. \qquad (16)$$

It can be shown easily that $\mathrm{rank}(\boldsymbol{C}_m^{\beta|\alpha}) \leq (N+L-1)N_R$ and $\mathrm{rank}(\boldsymbol{G}) = \mathrm{rank}(\boldsymbol{\Phi}_{h_sh_s}) \leq LN_TN_R$ is valid [8]. Thus, Eq. (16) can be rewritten as

$$q \leq N_R \cdot \min\{N+L-1,\ LN_T\}. \qquad (17)$$

Obviously, for $L = 1$ we obtain the well known result that $N = N_T$ is the best choice and the maximum diversity order is N_TN_R (cf. e.g. [3]). However, for $L > 1$ Eq. (17) indicates that choosing $N > N_T$ may be beneficial. If $N = N_T$ is adopted $q \leq (N_T+L-1)N_R$ holds, i.e., increasing the number of transmit antennas has the same effect as a longer channel impulse response. How the bound in Eq. (17) can be attained will be explained in the next section.

4.2 Design of STBC's

For the derivation of the proposed design rule for STBC's for fading ISI channels, we refer to [8]. Due to space limitations, we only give the final result here. Also, for simplicity, we restrict ourselves to the case where all coefficients $h_{\nu\mu}[l]$ of the discrete–time overall CIR's are mutually independent and have equal variances $1/L$. The general case, where the coefficients are correlated and have different variances is considered in [8].

We define the $(N+L-1) \times LN_T$ matrix

$$\boldsymbol{S}_{\beta|\alpha} \triangleq [\boldsymbol{S}_1^{\beta|\alpha}\ \ \boldsymbol{S}_2^{\beta|\alpha}\ \ \ldots\ \ \boldsymbol{S}_L^{\beta|\alpha}] \qquad (18)$$

with

$$\boldsymbol{S}_\nu^{\beta|\alpha} \triangleq \begin{pmatrix} \boldsymbol{0}_{(\nu-1)\times N_T} \\ \boldsymbol{C}_\beta^T - \boldsymbol{C}_\alpha^T \\ \boldsymbol{0}_{(L-\nu)\times N_T} \end{pmatrix}. \qquad (19)$$

It can be shown that for fading channels as specified above the diversity order for a given pair (α, β) is $q = N_R \cdot \mathrm{rank}(\boldsymbol{S}_{\beta|\alpha})$ [8]. Thus, it is obvious that we have to maximize the rank of matrix $\boldsymbol{S}_{\beta|\alpha}$ in order to maximize the diversity order. The maximum rank of $\boldsymbol{S}_{\beta|\alpha}$ is $\min\{N+L-1,\ LN_T\}$. We propose the following design rule for space–time block–coded transmission over fading ISI channels:

Design rule: *Select the code set $\mathcal{C}$ in such a way that matrix $\boldsymbol{S}_{\beta|\alpha}$ has rank $\min\{N+L-1,\ LN_T\}$ for $1 \leq \alpha,\ \beta \leq 2^{N_b}$, $\alpha \neq \beta$.*

Note that for $L = 1$ and uncorrelated fading coefficients our design rule is identical to the conventional design rule for frequency–nonselective channels given in [2, 3]. A secondary criterion for the search for code sets $\mathcal{C}$ which yield high performance is that the q singular values of $\boldsymbol{S}_{\beta|\alpha}$ should be evenly distributed and as large as possible for all pairs (α, β), $\alpha \neq \beta$ [3].

For mutually uncorrelated CIR coefficients, STBC's designed according to the given rule achieve a diversity order of $q = N_R \cdot \min\{N+L-1,\ LN_T\}$, i.e., the upper bound given in Eq. (17) is attained. Note that for $N = (N_T-1)L+1$, $q = LN_TN_R$ results and the diversity order grows linearly with L, N_T, and N_R, whereas for (full–rate) STBC's designed for flat fading channels $N = N_T$ holds and q grows linearly only with $N_T + L$ and N_R ($q = (N_T+L-1)N_R$).

4.3 Example: New Code for $N_T = 2$

In this section, again we assume that all coefficients of all discrete–time overall CIR's are mutually uncorrelated. We give an STBC for $N_T = 2$ which yields maximum diversity order for $L = 2$ [1]. For this, we assume that N binary phase–shift keying (BPSK) symbols $b_k \in \{1, -1\}$, $1 \leq k \leq N$, are mapped to matrix $\boldsymbol{C}$. For $L = 2$, $q = LN_T = 4$ can be achieved if $N = 3$ is chosen and $\boldsymbol{C}$ is properly designed ($N_R = 1$ is assumed). An example for a mapping which yields $q = 4$ is $c_{11} = 0.6\,b_1 + 0.4\,b_2 + 0.35\,b_3$, $c_{12} = 0.25\,b_2 - 0.2\,b_3$, $c_{13} = -0.35\,b_1 + 0.5\,b_2 + 0.6\,b_3$, $c_{21} = 0.6\,b_1 - 0.5\,b_2 + 0.4\,b_3$, $c_{22} = 0.25\,b_2 + 0.2\,b_3$, and $c_{23} = 0.4\,b_1 + 0.4\,b_2 - 0.6\,b_3$. This code has been found by a random search among codes with $q = 4$ and yields high performance. On the other hand, Alamouti's code

$$\boldsymbol{C} = \frac{1}{\sqrt{2}} \begin{pmatrix} b_1 & -b_2 \\ b_2 & b_1 \end{pmatrix} \qquad (20)$$

yields a diversity order of $q = N_T + L - 1 = 3$. Of course, the code is not orthogonal [13], however, orthogonality is of no importance for ISI channels.

In Fig. 1, the approximation for the BER according to Eq. (10) vs. $10\log_{10}(E_b/(N_0N_R))$ (E_b is the mean received energy per information bit) is shown for the

[1] Note that $L = 2$ with independent CIR taps is specified as test case for IS–136.

above STBC's ($N_R = 1$) and BPSK with $N_R = 2$ for different L. For $L = 1$, i.e., flat Rayleigh fading, $q = N_T = 2$ is valid for both STBC's and $q = N_R = 2$ holds for BPSK (BPSK with receive diversity and Alamouti's code yield identical BER's). However, for $L = 2$ Alamouti's code achieves $q = N_T + L - 1 = 3$, whereas $q = N + L - 1 = LN_T = 4$ and $q = LN_R = 4$ is realized by the new design and BPSK, respectively, i.e., the slope of the corresponding BER curves is larger. Similarly, for $L = 3$, $q = N_T + L - 1 = 4$ and $q = N + L - 1 = 5$ is realized by Alamouti's code and the new design, respectively. Note that a properly designed STBC with $N = 4$ could achieve $q = 6$ (cf. Eq. (17)), i.e., the same diversity order as BPSK with $N_R = 2$.

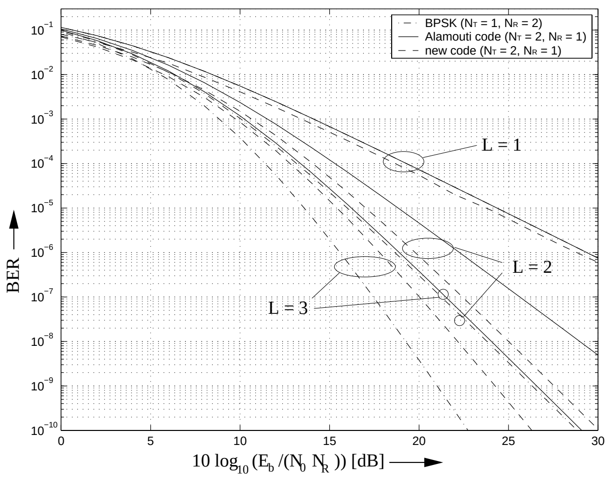

Figure 1: Approximation for BER vs. $10\log_{10}(E_b/(N_0 N_R))$ for the new code, Alamouti's code, and BPSK with receive diversity.

5 Numerical Results for GSM/EDGE

In this section, we investigate the performance of STBC's for (filtered) BPSK and 8PSK modulation. Filtered BPSK can be used to approximate Gaussian minimum–shift keying (GMSK) modulation which is employed for GSM. 8PSK is adopted for EDGE. For the numerical results two different power delay profiles with mutually independent paths are employed: rural area (RA) and typical urban area (TU). In contrast to the TU profile, the RA profile also contains a direct path. For each transmit and receive branch a linearized GMSK pulse and a square–root raised cosine filter with roll–off factor 0.3 are used, respectively. Furthermore, it is assumed that the CIR's between different antenna pairs are mutually independent. The novel STBC given in Section 4.3 did not yield a significant performance improvement over STBC's designed for flat fading for the investigated power delay profiles. On the other hand, the search for STBC's optimized for the GSM/EDGE system is beyond the scope of this paper and a topic for future research. Therefore, for simplicity, STBC's designed for flat fading are used exclusively in the following. For the numerical results BER is approximated by Eq. (10) and compared with simulation results for MLSE [14] wherever possible.

Fig. 2 shows BER vs. $10\log_{10}(E_b/(N_0 N_R))$ for BPSK transmission over a TU channel. BPSK with receive diversity, Alamouti's code, and the orthogonal STBC for $N_T = 4$ given in [13] are considered. It can be observed that for the TU channel a considerable gap between BPSK with receive diversity and the corresponding transmit diversity schemes remains, i.e., it can be expected that a significant gain can be achieved by STBC's optimized for frequency–selective fading channels. Remarkably, the BER approximation proposed in Section 3.3 is well confirmed by simulation results for MLSE.

8PSK transmission over an RA channel is considered in Fig. 3. Obviously, for the RA profile Alamouti's code is near optimum. This is not surprising since the RA channel is practically a flat Ricean fading channel. Also for 8PSK the BER results predicted by the proposed approximation coincide very well with simulation results for MLSE.

For further numerical results we refer to [8].

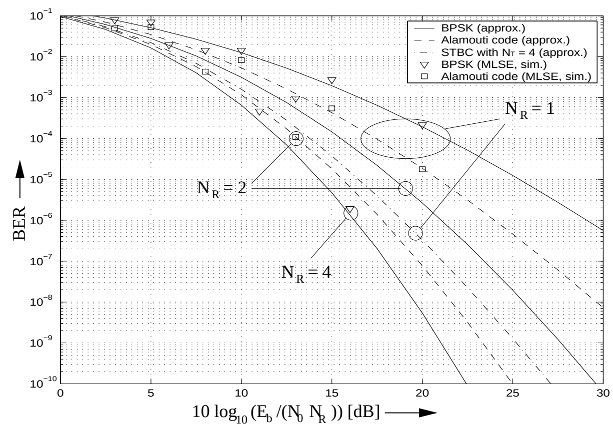

Figure 2: Approximation for BER vs. $10\log_{10}(E_b/(N_0 N_R))$ for BPSK transmission over TU channel. For comparison also simulation results for MLSE are shown.

6 Conclusions

In this paper, the performance of space–time block–coded transmission over frequency–selective fading channels has been investigated and a novel design rule for STBC's has been given. A lower bound on the pairwise error probability and an approximation for the BER have been derived. It has been shown that

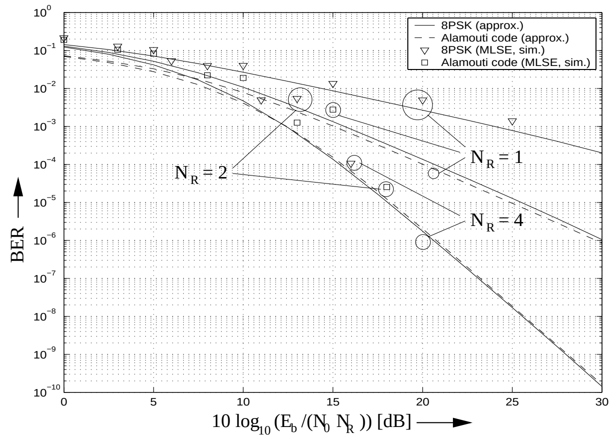

Figure 3: Approximation for BER vs. $10\log_{10}(E_b/(N_0 N_R))$ for 8PSK transmission over RA channel. For comparison also simulation results for MLSE are shown.

codes designed according to the novel rule can yield a higher diversity order and superior performance than STBC's optimized for flat fading channels. The maximum achievable diversity order for fading channels with L independent impulse response taps, N_T transmit and N_R receive antennas is LN_TN_R. Only one specific code has been given, however, the results of this paper motivate and enable the search for special STBC's matched to fading ISI channels.

The proposed approximation for BER is easy to compute and comparisons with simulation results for MLSE have shown its relevance. Finally, it has been shown that space–time block–coded transmission can be highly beneficial for the GSM/EDGE system. Since in most cases MLSE is too complex, our findings should also stimulate the search for low–complexity suboptimum equalizers for STBC's. A first step in this direction is done in [14].

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