A Novel Iterative Multiuser Detector for Complex Modulation Schemes

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1 Introduction

In the past decade, the search for feasible multiuser detectors for direct sequence code–division multiple–access (DS–CDMA) systems has attracted high interest. Especially iterative multiuser detection schemes have gained attention due to their tractable complexity and good performance for moderate system loads $\beta = K/N$ (K denotes the number of users and N the spreading factor). So, various iterative receivers for uncoded as well as coded transmission have been proposed (see [1, 2, 3] and references therein).

In general, it is presumed that the residual interference in the inphase and the quadrature component of the filter output are mutually uncorrelated and have equal variance. Applying as interference suppression filter for example a matched filter or a minimum mean–squared error (MMSE) filter [4], it can be easily shown that this is true for long random spreading sequences, finite load, and bounded receive power from each user. However, investigating the interference at the input of these filters and assuming *real* channel symbols and complex spreading sequences, it turns out that the pseudocovariance matrix (for a definition see [5]) of the interference is nonzero, i.e., the multiuser interference represents a rotationally variant complex noise. This kind of noise is also referred to as improper noise [5] and, in this case, this property is simply a consequence of the real modulation scheme. The fact that the multiuser interference is for binary modulation rotationally variant was first used in [6] to present the optimum linear MMSE filter for DS–CDMA and in [7] the corresponding decorrelating filter was proposed. In both situations considerable gains can be obtained compared to the corresponding conventional linear receivers and we can report that this holds for the respective iterative algorithms, too. However, as real modulation schemes are generally less bandwidth efficient than their complex counterparts, we are particularly interested in the latter ones. Here, we face a completely different situation. If only rotationally invariant additive channel noise is present, for *complex* signal modulation with equal power in the inphase and quadrature component the pseudocovariance matrix of the received signal vanishes identically. Thus, before the first iteration the rotational invariance is given and the standard receiver need not to be modified. Nevertheless, after the feedback of soft decisions it turns out that the interference becomes rotationally variant. In order to benefit from this, we propose the application of a modified MMSE filter which accounts for the rotational variance of the residual interference in the signal. In this way, significant gains in terms of power efficiency and convergence speed are reached.

2 Transmission Model

In this paper, the transmission of K users employing DS–CDMA as multiple access technique to a common receiver, i.e., the uplink, is considered. For the sake of clarity, we restrict ourselves to a simplified discrete–time equivalent complex baseband transmission model which is mathematically described by $\boldsymbol{y}[\mu] = \sum_{k=1}^{K} \hat{\boldsymbol{s}}_k[\mu]\mathrm{e}^{\mathrm{j}\,\Theta_k[\mu]}x_k[\mu] + \boldsymbol{n}[\mu]$. The N dimensional vectors $\boldsymbol{y}[\mu] = (y_1[\mu], \ldots, y_N[\mu])^T$ and $\boldsymbol{n}[\mu] = (n_1[\mu], \ldots, n_N[\mu])^T$ represent the received signal and the additive rotationally invariant channel noise in the μth transmission interval, respectively. The independent and identically distributed (i.i.d.) samples $n_c[\mu], 1 \leq c \leq N$, are zero mean complex Gaussian random variables with variance σ_n^2. As modulation scheme QPSK with Gray mapping is employed, i.e., the channel symbols of user k are given by $x_k[\mu] \in \mathcal{X} = \{\pm 1 \pm \mathrm{j}\}, 1 \leq k \leq K$. The spreading sequence $\hat{\boldsymbol{s}}_k[\mu] = (\hat{s}_{1,k}[\mu], \ldots, \hat{s}_{N,k}[\mu])^T$ consists of randomly chosen elements $\hat{s}_{c,k}[\mu] \in \{(\pm 1 \pm \mathrm{j})/\sqrt{2N}\}, 1 \leq c \leq N$, and $\Theta_k[\mu] \in [-\pi, \pi), 1 \leq k \leq K$, represents the instantaneous phase–offset of user k. Furthermore, defining the effective spreading sequence $\boldsymbol{s}_k[\mu] \stackrel{\triangle}{=} \hat{\boldsymbol{s}}_k[\mu]\mathrm{e}^{\mathrm{j}\,\Theta_k[\mu]}$ and introducing $\boldsymbol{S}[\mu] = (\boldsymbol{s}_1[\mu], \ldots, \boldsymbol{s}_K[\mu])$, $\boldsymbol{x}[\mu] = (x_1[\mu], \ldots, x_K[\mu])^T$, the transmission model can also be written as

$$\boldsymbol{y}[\mu] \quad = \quad \boldsymbol{S}[\mu]\boldsymbol{x}[\mu] + \boldsymbol{n}[\mu]. \tag{1}$$

Since synchronous transmission of the users is assumed, the time index μ is dropped in the sequel. We note that the assumption of synchronous transmission is merely made for clarity and simplicity of presentation. Our receiver can be adapted easily to asynchronous transmission and/or frequency selective fading channels.

In the following sections, we outline the main steps performed for generation of soft estimates for the users' data symbols by means of iterated soft decision interference cancellation (ISDIC) using a bank of filters designed for minimization of the instantaneous mean–squared error. First, we briefly review the standard approach. Based on this, the novel receiver is derived. In general, there are two possible strategies for iterated soft decision interference cancellation - serial and parallel interference cancellation. Here, the parallel scheme is studied.

3 Standard ISDIC Based on MMSE

Considering iteration $m \geq 1$, user k is selected as user of interest. The aim is to obtain a soft estimate d_k^m for the transmitted data symbol x_k. The major parts of the decision chain are an MMSE filter $\boldsymbol{h}_k^m$ yielding the scalar output f_k^m and a nonlinear decision device providing the soft estimate $d_k^m = \mathcal{E}\{x_k | f_k^m\}$. Using parallel cancellation we introduce the kth user's estimation vector $\boldsymbol{d}_k^m \stackrel{\triangle}{=} \left(\ldots, d_{k-1}^{m-1}, 0, d_{k+1}^{m-1}, \ldots\right)^T$, where $d_1^m, ..., d_{k-1}^m$ and $d_{k+1}^{m-1}, ..., d_K^{m-1}$ are the soft decisions made for the interfering users in the $(m-1)$th iteration. As initialization $\boldsymbol{d}_k^0 \stackrel{\triangle}{=} (0, \ldots, 0)^T$, is chosen. Assuming that these soft estimates are reliable, the remodulated signal $\boldsymbol{S}\boldsymbol{d}_k^m$ is subtracted from $\boldsymbol{y}$ in order to reduce the multiuser interference. So, we get

$$\boldsymbol{y}_k^m = \boldsymbol{y} - \boldsymbol{S}\boldsymbol{d}_k^m = \boldsymbol{S}\left(\boldsymbol{x} - \boldsymbol{d}_k^m\right) + \boldsymbol{n}. \tag{2}$$

In order to suppress the remaining multiuser interference, $\boldsymbol{y}_k^m$ is passed through the filter $\boldsymbol{h}_k^m$ minimizing the mean–squared error $\mathcal{E}\{|x_k - (\boldsymbol{h}_k^m)^T \boldsymbol{y}_k^m|^2 | \boldsymbol{f}_k^{m-1}\}$ conditioned on $\boldsymbol{f}_k^{m-1}$. Here, the vector $\boldsymbol{f}_k^{m-1} = (\ldots, f_{k-1}^{m-1}, 0, f_{k+1}^{m-1}, \ldots)^T$ consists of the interference suppression filter outputs obtained in iteration $(m-1)$ and is initialized with $\boldsymbol{f}_k^0 = (0, \ldots, 0)^T$. So, $\boldsymbol{h}_k^m$ is solved as

$$(\boldsymbol{h}_k^m)^T \quad = \quad \sigma_x^2 \boldsymbol{s}_k^H \left(\boldsymbol{C}_k^m\right)^{-1} = \sigma_x^2 \boldsymbol{s}_k^H \left(\boldsymbol{S}\boldsymbol{E}_k^m \boldsymbol{S}^H + \sigma_n^2 \boldsymbol{I}_{N\times N}\right)^{-1}, \tag{3}$$

where $\boldsymbol{I}_{N\times N}, \sigma_x^2 = \mathcal{E}\{|x_k|^2\} = 2, \forall k$, and $\boldsymbol{E}_k^m = \mathcal{E}\{\left(\boldsymbol{x} - \boldsymbol{d}_k^m\right)\left(\boldsymbol{x} - \boldsymbol{d}_k^m\right)^H | \boldsymbol{f}_k^{m-1}\}$ denote the $N \times N$ identity matrix, the variance of the data symbols and the correlation matrix of $\boldsymbol{x} - \boldsymbol{d}_k^m$ conditioned on $\boldsymbol{f}_k^{m-1}$, respectively. As the number of crosscorrelations between different users' soft decisions increases with K^2 while the absolute value of these expectations converges to zero, virtually uncorrelated decisions for the different users are supposed in the following for the sake of computational tractability (cf. [3]). Then, $\boldsymbol{E}_k^m$ can be obtained as $\boldsymbol{E}_k^m = \operatorname{diag}\left(\sigma_x^2 - \left|d_1^{m-1}\right|^2, \ldots, \sigma_x^2 - \left|d_{k-1}^{m-1}\right|^2, \sigma_x^2, \sigma_x^2 - \left|d_{k+1}^{m-1}\right|^2, \ldots, \sigma_x^2 - \left|d_K^{m-1}\right|^2\right)$.

Passing $\boldsymbol{y}_k^m$ through the filter $\boldsymbol{h}_k^m$ yields for user k in iteration m the output $f_k^m = (\boldsymbol{h}_k^m)^T \boldsymbol{y}_k^m = u_k^m x_k + i_k^m$, where i_k^m denotes the sum of interference caused by the users

$1 \leq \kappa \leq K, \kappa \neq k$, and channel noise. Further, $u_k^m = (\boldsymbol{h}_k^m)^T \boldsymbol{s}_k$ stands for the bias inherent to the MMSE filter. The best estimate d_k^m for x_k according to the minimum mean–squared error criterion is the conditional a–posteriori expectation value. Making use of the assumption that the interference plus noise term i_k^m is rotationally invariant and complex Gaussian distributed with zero mean and variance $(\sigma_k^m)^2 = \sigma_x^2 u_k^m (1 - u_k^m)$, after some calculations

$$d_k^m = \tanh\left(f_{k,I}^m/(1-u_k^m)\right) + \mathrm{j}\tanh\left(f_{k,Q}^m/(1-u_k^m)\right), \tag{4}$$

is obtained. Here, the subscripts I and Q refer to the inphase and quadrature component of f_k^m, respectively. Applying parallel cancellation the new estimate d_k^m replaces the old one in the next iteration.

4 Novel ISDIC Based on Modified MMSE Filter

In the derivation outlined in the previous section, two significant assumptions were made. First, the soft decisions for the inphase and quadrature components of the different users data symbols are assumed to be mutually independent. For uncoded transmission this presumption is sufficiently satisfied for moderate system loads and is exactly true for coded transmission and large interleavers. Thus, this first simplification made for computational feasibility does not limit the system performance in general.

The second assumption taken for granted is that the multiuser interference affecting the estimation is rotationally invariant. Of course, this holds for all users in iteration 1 as

$$\mathcal{E}\left\{ (x_k - d_k^0)^2 \middle| f_k^0 \right\} = 0, \quad 1 \leq k \leq K. \tag{5}$$

However, cancelling the assumed interference caused by user k, we get for iteration $m > 1$

$$\begin{aligned} \mathcal{E}\left\{ (x_k - d_k^{m-1})^2 \middle| f_k^{m-1} \right\} &= (d_{k,Q}^{m-1})^2 - (d_{k,I}^{m-1})^2 \\ &\quad + 2\mathrm{j}\left[\mathcal{E}\left\{ x_{k,I} x_{k,Q} \middle| f_k^{m-1} \right\} - d_{k,I}^{m-1} d_{k,Q}^{m-1} \right]. \end{aligned} \tag{6}$$

From the above it can be seen that the so–called pseudoautocorrelation is only zero if (i) the absolute values of the soft decisions for the inphase and quadrature component are equal and (ii) the soft decisions for the inphase and quadrature components are statistically independent. In general, this will only be true at the beginning of the iterative decision process as well as at its end, i.e., when the soft decisions $d_{k,I}^m$ and $d_{k,Q}^m$ are practically converged to 1 or -1 for all users k.

In order to exploit the rotational variance of the multiuser interference arising for $m > 1$, the MMSE criterion the interference suppression filter is designed for has to be modified. In other words, the rotational variance of the residual multiuser interference which is equivalent to the mutual correlatedness of $\boldsymbol{d}_k^m$ and $(\boldsymbol{d}_k^m)^*$ suggests to employ a filter processing not only $\boldsymbol{y}_k^m = \boldsymbol{y} - \boldsymbol{S}\boldsymbol{d}_k^m$ but also $(\boldsymbol{y}_k^m)^* = (\boldsymbol{y} - \boldsymbol{S}\boldsymbol{d}_k^m)^*$. This can be done by application of following optimization criterion [8]

$$\tilde{\boldsymbol{h}}_k^m = \underset{\hat{\boldsymbol{h}}_k^m}{\mathrm{argmin}}\ \mathcal{E}\left\{ \left| x_k - (\hat{\boldsymbol{h}}_k^m)^T \left((\boldsymbol{y}_k^m)^T, (\boldsymbol{y}_k^m)^H \right)^T \right|^2 \middle| \tilde{\boldsymbol{f}}_k^{m-1} \right\}, \tag{7}$$

with $\tilde{\boldsymbol{f}}_k^{m-1} = \left(\ldots, \tilde{f}_{k-1}^{m-1}, 0, \tilde{f}_{k+1}^{m-1}, \ldots \right)^T$. Here, $\tilde{f}_k^{m-1} = (\tilde{\boldsymbol{h}}_k^{m-1})^T \tilde{\boldsymbol{y}}_k^{m-1}$ denotes the filter output for user k in iteration $m-1$ and $\tilde{\boldsymbol{y}}_k^{m-1} \triangleq \left((\boldsymbol{y}_k^{m-1})^T, (\boldsymbol{y}_k^{m-1})^H \right)^T$ is the filter input. Observing that the above optimization problem is equivalent to the standard Wiener approach except that the filter input is now $\tilde{\boldsymbol{y}}_k^m$, the solution reads $\left(\tilde{\boldsymbol{h}}_k^m \right)^T =$

$\left(\sigma_x^2 \boldsymbol{s}_k^H \boldsymbol{B}_{k,1}^m, \sigma_x^2 \boldsymbol{s}_k^H \boldsymbol{B}_{k,2}^m\right)$, with the matrices $\boldsymbol{B}_{k,1}^m = \left(\boldsymbol{C}_k^m - \hat{\boldsymbol{C}}_k^m \left((\boldsymbol{C}_k^m)^*\right)^{-1} (\hat{\boldsymbol{C}}_k^m)^*\right)^{-1}$ and $\boldsymbol{B}_{k,2}^m = -\boldsymbol{B}_{k,1}^m \hat{\boldsymbol{C}}_k^m \left((\boldsymbol{C}_k^m)^*\right)^{-1}$. Furthermore, $\hat{\boldsymbol{C}}_k^m \triangleq \mathcal{E}\left\{\boldsymbol{S}(\boldsymbol{x} - \boldsymbol{d}_k^m)(\boldsymbol{x} - \boldsymbol{d}_k^m)^T \boldsymbol{S}^T | \tilde{\boldsymbol{f}}_k^{m-1}\right\}$ is chosen as

$$\hat{\boldsymbol{C}}_k^m = \boldsymbol{S}\mathrm{diag}\left(\ldots, (d_{k-1,Q}^{m-1})^2 - (d_{k-1,I}^{m-1})^2, 0, (d_{k+1,Q}^{m-1})^2 - (d_{k+1,I}^{m-1})^2, \ldots\right) \boldsymbol{S}^T, \quad (8)$$

while $\boldsymbol{C}_k^m$ is given in Eq. (3). As before the filter output can be decomposed into signal plus interference terms $\tilde{f}_k^m = \tilde{u}_{k,I}^m x_{k,I} + \mathrm{j}\, \tilde{u}_{k,Q}^m x_{k,Q} + \tilde{i}_k^m$. The bias of the inphase and quadrature component of the signal is obtained as $\tilde{u}_{k,I}^m = \tilde{\boldsymbol{h}}_k^m \left(\boldsymbol{s}_k^T, \boldsymbol{s}_k^H\right)^T = \sigma_x^2 \boldsymbol{s}_k^H \boldsymbol{B}_{k,1}^m \boldsymbol{s}_k + \sigma_x^2 \boldsymbol{s}_k^H \boldsymbol{B}_{k,2}^m \boldsymbol{s}_k^*$, and $\tilde{u}_{k,Q}^m = \tilde{\boldsymbol{h}}_k^m \left(\boldsymbol{s}_k^T, -\boldsymbol{s}_k^H\right)^T = \sigma_x^2 \boldsymbol{s}_k^H \boldsymbol{B}_{k,1}^m \boldsymbol{s}_k - \sigma_x^2 \boldsymbol{s}_k^H \boldsymbol{B}_{k,2}^m \boldsymbol{s}_k^*$. In general, the bias for the inphase and quadrature component is complex. However, it can be observed that for moderately long spreading sequences ($N = 16$ is sufficient) holds $|\boldsymbol{s}_k^H \boldsymbol{B}_{k,2}^m \boldsymbol{s}_k^*|^2 \ll (\boldsymbol{s}_k^H \boldsymbol{B}_{k,1}^m \boldsymbol{s}_k)^2$, so that the bias can be approximated by

$$\tilde{u}_k^m = \sigma_x^2 \boldsymbol{s}_k^H \boldsymbol{B}_{k,1}^m \boldsymbol{s}_k. \quad (9)$$

This result can be explained by the fact that $\mathcal{E}\left\{x_k \boldsymbol{y}_k^m | \tilde{\boldsymbol{f}}^{m-1}\right\} = \boldsymbol{0}$ is valid. More explicitly, the filter assumes that there is no information on the signal contained in $(\boldsymbol{y}_k^m)^*$ which can be extracted. Thus, the second half of the filter $\sigma_x^2 \boldsymbol{s}_k^H \boldsymbol{C}_{k,2}^m$ serves purely for interference suppression and the signal component $x_k^* \boldsymbol{s}_k^*$ is treated as an additional interferer being suppressed for sufficiently large N. Furthermore, it turns out that the interference $\tilde{i}_k^m$ which is independent of the biased signal can be well modeled as rotationally invariant with equal variance in the inphase and quadrature component without affecting the system's performance. Therefore, the imaginary component of the pseudoautocorrelation given in Eq. (6) can be neglected in Eq. (8). Using the above approximations the soft decisions are now given by

$$d_k^m = \tanh\left(\tilde{f}_{k,I}^m / (1 - \tilde{u}_k^m)\right) + \mathrm{j} \tanh\left(\tilde{f}_{k,Q} / (1 - \tilde{u}_k^m)\right). \quad (10)$$

5 Numerical Results

In this section, a comparison of the iterative receivers considered above (ISDIC with conventional MMSE filter (ISDIC1) and ISDIC with modified MMSE filter (ISDIC2)) is provided. First, the performance gain obtainable for varying system loads is illustrated in Fig 1. There, the bit error ratio (BER) versus average received energy per bit to noise ratio (E_b/N_0) is depicted for $N = 16$ and $K = 12, 16, 20$. Four iterations are carried out for all receiver schemes, after which the steady state is virtually achieved. Further, the so–called single user bound (SUB) is included as reference.

Fig. 1 shows that significant gains are reachable by application of the modified MMSE filter accounting for the pseudocovariance matrix which diverges from the all zero matrix during the iterations. It can be seen that the gain is almost negligible for small signal–to–noise ratios because of the dominance of the rotationally invariant channel noise. On the contrary, for decreasing channel noise variance the interference is dominated more and more by multiuser interference and the gap between the bit error ratios achieved by the conventional approach and the novel scheme rises with increasing system load. In order to illustrate the influence of the incorporation of the pseudocovariance into the filter design from another point of view, in Fig. 2 the bit error ratios resulting after iteration $m = 1, 2, 4$ for $K = N = 16$ are given. Of course, the BER is the same for both receivers after the first iteration since the multiuser interference in the received signal is rotationally invariant. Hence, nothing can be gained in the first iteration. But after the second iteration the benefit arising from exploitation of the rotational variance of the multiuser interference pays off in a considerable reduction of the BER for the novel receiver.

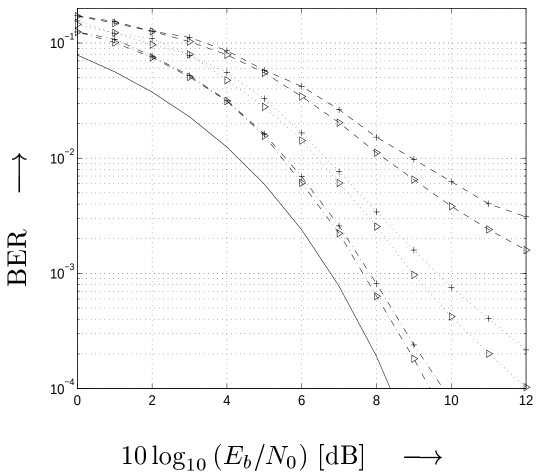

Figure 1: BER vs. $10\log_{10}(E_b/N_0)$ for $\beta = 0.75(-\cdot)$, $\beta = 1(\cdots)$ and $\beta = 1.25(--)$ for ISDIC1 (+), ISDIC2 ($\triangleright$), and SUB (−).

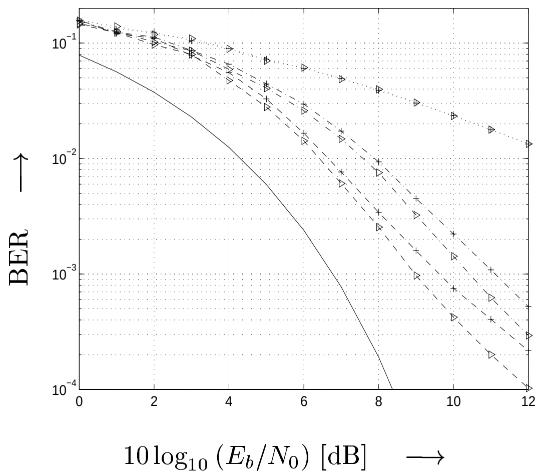

Figure 2: BER vs. $10\log_{10}(E_b/N_0)$ for $\beta = 1$ after 1st iteration $(\cdots)$, 2nd iteration $(-\cdot)$, and 4th iteration $(--)$ for ISDIC1 (+), ISDIC2 ($\triangleright$) and SUB (−).

6 Conclusions

We have shown that significant gains can be obtained for iterative soft decision interference cancellation based on a modified MMSE filter if the rotational variance of the interference arising in course of the iterations is regarded. Of course, these results can be extended to serial cancellation, frequency selective fading channels, and coded transmission as well. Applying channel coding the extrinsic information provided by the single user decoders can be used to determine the expected pseudocorrelation of the desired channel symbol. Beside this, also channel estimation can be incorporated into the iterative algorithm.

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