

# Differential Encoding Strategies for Transmission over Fading Channels

Robert F.H. Fischer, Lutz H.-J. Lampe, Stefano Calabrò

**Abstract** Transmission over Rician fading channels with unknown carrier phase is considered. Differential encoding of APSK constellations at the transmitter and non-coherent demodulation without channel state information/ carrier phase estimate at the receiver are applied. In this paper, various differential encoding strategies suited for multiple symbol detection are presented and compared. In particular, the encoding of the amplitude is considered. Moreover, a method for diversity enhancement is introduced. Regarding channel capacity as an appropriate performance measure when powerful channel coding is applied we show that the proposed differential encoding strategies have advantages over usual differential encoding for transmission over fading channels. The results obtained by numerical simulations are in great accordance with the prediction from information theory.

**Keywords** Fading channels, differential encoding, coded modulation, multilevel codes, channel capacity

## 1. Introduction and System Model

For a number of applications it is desirable to perform transmission over fading channels without requiring channel state information and/or reliable carrier phase estimation at the receiver. In these situations, differential encoding at the transmitter and non-coherent demodulation at the receiver are convenient. Furthermore, if high bandwidth efficient transmission is desired, mixed phase and amplitude modulation is advantageous. The straightforward extension of differential phase encoding is to transmit the information both in phase and in amplitude changes, which is known as differential amplitude and phase shift keying (DAPSK), e.g. [Sve95, Ada96, LCF99]. Moreover, high power efficiency is achieved by applying powerful channel coding schemes. If the underlying channel is slowly time-varying, demodulation and decoding is favourably based on blocks of  $N$  consecutive symbols, cf. e.g. [DS94]. This approach is called *multiple symbol detection*.

Unfortunately, it turns out that for the most relevant applications of DAPSK in practice, information carried by the amplitude changes shows only poor reliability. Hence, the investigation of alternative strategies to usual DAPSK is an appealing task. Based on the observation that even with entirely unknown channel state conveying information in the actual value of the amplitude is still possible, differential phase encoding with varying signal amplitudes is considered.

In this paper, various differential encoding strategies for transmission over Rician fading channels with unknown carrier phase are presented. Moreover, redundant mapping of data to phase changes and amplitude changes/absolute amplitudes, which can be regarded as diversity enhancement, is introduced. Regarding channel capacity as an appropriate performance measure when powerful channel coding is applied we show that the proposed differential encoding strategies have advantages over usual differential encoding for transmission over fading channels. The results obtained from information theory are verified by simulations.

The transmission scheme discussed in this paper is sketched in Figure 1. The actual channel is described by

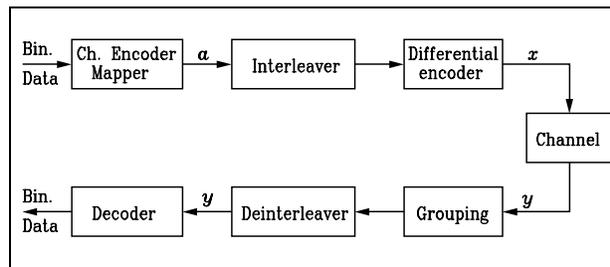


Fig. 1. System model.

its discrete-time model given in the equivalent low-pass domain, i.e., all quantities are complex [Pro95]. The channel is assumed to be a stationary, slowly time-varying, frequency non-selective (non-dispersive or flat) Rician fading channel with finite memory. As usual, channel state and carrier phase offset are expected to be constant over a block of at least  $N$  consecutive symbols. We focus on the situation where neither channel state information nor reliable estimation of the carrier phase are available at the receiver side.

The paper is organized as follows: In Section 2, the encoding strategies are presented and compared. Keeping the definitions of the encoder in mind, in Section 3, the capacity of the overall channel, including the actual channel and differential encoding/demodulation, is calculated. The complexity of the demodulation process is discussed. Sections 4 and 5 summarize numerical and simulation results, respectively. All observed phenomena well correspond to predictions from information theory. Finally, Section 6 gives some conclusions.

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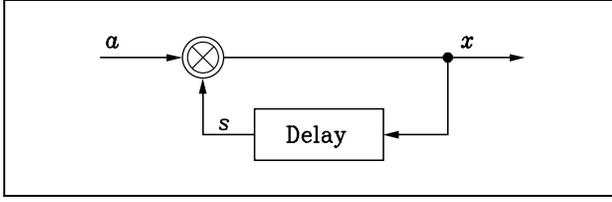


Fig. 2. Differential encoder.

## 2. Differential Encoding Strategies

In differential phase shift keying information is carried in *phase changes* rather than in the phase itself. For that purpose, a phase accumulation is performed at the transmitter. Choosing the PSK symbols equally spaced on the unit circle, this so-called *differential encoding* can be done by multiplying the data-carrying *differential symbol* (i.e., phase increments) by the previous transmit symbol, which also can be regarded as the *state* of the encoder [Bla90]. In this section, the concept of differential encoding is extended to APSK constellations and various encoding strategies are presented and discussed.

Let  $s$  be the current state of the encoder (reference symbol) and  $a \in \mathcal{A}$  a differential symbol drawn from the signal set  $\mathcal{A}$ . Then the encoder outputs the transmit symbol  $x \in \mathcal{X}$ , which also constitutes the next state of the differential encoder, i.e.,  $s \in \mathcal{X}$ . The relation between  $a$ ,  $s$ , and  $x$  will be formally specified by an operator “ $\otimes$ ”, namely

$$x = s \otimes a, \quad (1)$$

also sketched in Figure 2. In classical DPSK, “ $\otimes$ ” is the usual complex multiplication.

Either the same signal constellations  $\mathcal{A}$  and  $\mathcal{X}$  are chosen for the differential symbols  $a$  and for the transmit symbols  $x$  (cf. usual DPSK), respectively, or  $\mathcal{A}$  is a subset of  $\mathcal{X}$  like e.g. in  $\frac{\pi}{4}$ -DQPSK [Bak62, Rap96]. Thus, we always assume  $\mathcal{A} \subseteq \mathcal{X}$  and “ $\otimes$ ” to be a binary (possibly non-commutative) operation with

$$\otimes : \mathcal{X} \times \mathcal{A} \rightarrow \mathcal{X}. \quad (2)$$

For mixed phase and amplitude modulation over fading channels, especially constellations  $\mathcal{C}$  consisting of  $\alpha \cdot \beta$  points arranged in  $\alpha$  distinct concentric rings with radii  $r_i$ ,  $i = 0, \dots, \alpha - 1$ ,  $r_i < r_{i+1}$ , and  $\beta$  uniformly spaced phases  $\varphi_m = \frac{2\pi}{\beta}m$ ,  $m = 0, \dots, \beta - 1$ , are suitable [WHS91]:

$$\mathcal{C}_{\text{APSK}}(\alpha, \beta) \triangleq \left\{ c = r_i e^{j\frac{2\pi}{\beta}m} \mid \begin{array}{l} i \in \{0, 1, \dots, \alpha-1\} \\ m \in \{0, 1, \dots, \beta-1\} \end{array} \right\}. \quad (3)$$

We denote these constellations, which can support a maximum number of  $\log_2(\alpha \cdot \beta)$  bits per symbol, as  $\alpha\text{A}\beta\text{PSK}$ . For these constellations, subsequently possible definitions of the encoding operator “ $\otimes$ ” are discussed.

### 2.1 Differential Amplitude and Phase Encoding

In the case of differential encoding over phase and amplitude, the differential symbols  $a$  give phase *and* amplitude increments. We call this strategy *differential amplitude and phase shift keying (DAPSK)*.

Let the encoder state be  $s = r_i e^{j\varphi_n}$  and the differential symbol  $a = r_j e^{j\varphi_m}$ . For DAPSK with  $\mathcal{A} = \mathcal{X} = \mathcal{C}_{\text{APSK}}(\alpha, \beta)$  we can use the description of the differential encoder from [CFLM98, LCF99]. Here, the transmit symbol  $x$  (and, thus, implicitly the operator “ $\otimes$ ”) is given by

$$x = s \otimes a = r_i e^{j\varphi_n} \otimes r_j e^{j\varphi_m} \triangleq r_{(i+j) \bmod \alpha} e^{j\varphi_{(n+m) \bmod \beta}}. \quad (4)$$

Like the phase, the amplitude is incremented cyclicly, too.

### 2.2 Differential Phase and Absolute Amplitude Encoding

Designing a non-coherent transmission scheme, the carrier phase is assumed to be unknown and uniformly distributed. In order to resolve ambiguities, differential encoding over the phase is strictly necessary. In contrast to this, the amplitude of the received signal still provides information on the transmitted amplitude even if no channel state information is available at the receiver. This is because a) the channel gain is non-uniformly distributed and b) the observation of amplitude transitions enables some classification of the transmitted magnitude. The larger  $N$ , the length of the observation interval at the receiver, the more likely amplitude changes are perceivable and thus the more information is provided by the actual received amplitude.

Consequently, assigning information to the *phase change* and to the *actual amplitude* might be advantageous in some situations. Here, the definition (4) of the operator has to be modified to

$$x = s \otimes a = r_i e^{j\varphi_n} \otimes r_j e^{j\varphi_m} \triangleq r_j e^{j\varphi_{(n+m) \bmod \beta}}, \quad (5)$$

when again  $\mathcal{A} = \mathcal{X} = \mathcal{C}_{\text{APSK}}(\alpha, \beta)$  holds. Differential encoding in this manner will subsequently be called *absolute amplitude and differential phase shift keying (ADPSK)*.

### 2.3 Differential Phase Encoding with Constellation Expansion Diversity

A drawback of ADPSK is that if the channel is fading, part of the information carried by the current amplitude may be lost. In particular, even for high signal-to-noise ratios it is impossible to reliably transmit  $\log_2(\alpha \cdot \beta)$  bits per channel use.

This observation calls for strategies, which are capable to exploit the potentials of the amplitude modulation. A possible approach is to completely map the information onto phase changes of the transmit symbols. Additionally, the amplitude itself of the transmit symbols (partly) conveys the same information, which introduces *diversity*. The (differential) constellation  $\mathcal{A}$  is then required to have  $\alpha \cdot \beta$  possible phases. The complexity consideration in Section 3 shows that using again  $\alpha$  amplitudes does not change the computational effort. Among all approaches the most promising arrangement for the signal points is  $\mathcal{A} = \mathcal{C}_{\text{TAPSK}}(\alpha, \beta)$  with

$$\mathcal{C}_{\text{TAPSK}}(\alpha, \beta) \triangleq \left\{ c = r_{m \bmod \alpha} e^{j\frac{2\pi}{\alpha\beta}m} \mid m = 0, \dots, \alpha\beta - 1 \right\}, \quad (6)$$

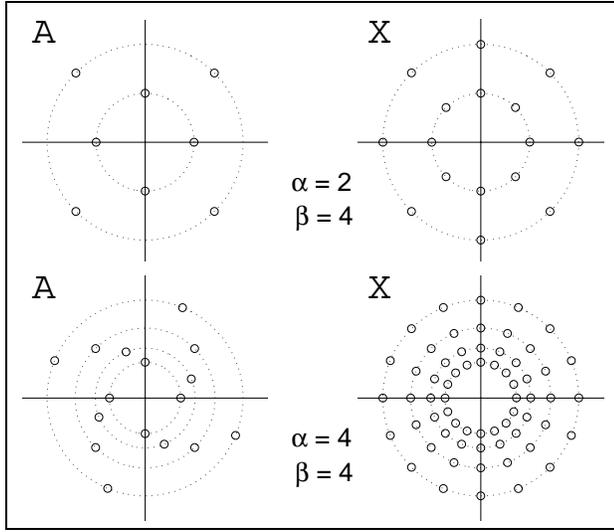


Fig. 3. Signal constellations  $\mathcal{A} = \mathcal{C}_{\text{TAPSK}}(\alpha, \beta)$  (left) according to (6) and  $\mathcal{X} = \mathcal{C}_{\text{APSK}}(\alpha, \alpha\beta)$  (right) for  $\alpha = 2, \beta = 4$  (top) and  $\alpha = 4, \beta = 4$  (geometric ring spacing, bottom).

because points in  $\mathcal{A}$  whose phase differ by the minimum value  $\frac{2\pi}{\alpha\beta}$  have different amplitudes. In the following such constellations will be denoted by *twisted  $\alpha A\beta$ PSK* ( $\text{T}\alpha A\beta\text{PSK}$ ).

It is easy to see, that now  $\mathcal{X}$  consist of again  $\alpha$  amplitudes but  $\alpha\beta$  phases, i.e.,  $\mathcal{X} = \mathcal{C}_{\text{APSK}}(\alpha, \alpha\beta)$ . Like in the  $\frac{\pi}{M}$ -shifted  $M$ -DPSK (e.g.,  $\frac{\pi}{4}$ -DQPSK [Rap96]) the sets  $\mathcal{A}$  is a true subset of  $\mathcal{X}$  and thus both sets have different cardinalities. But in contrast to shifted PSK at any time each transmit signal point is possible.

The operator “ $\otimes$ ” is now given by

$$x = s \otimes a = r_i e^{j\varphi_n} \otimes r_j e^{j\varphi_m} \triangleq r_j e^{j\varphi_{(n+m) \bmod \alpha\beta}}. \quad (7)$$

The action of the encoder can be described by a trellis (cf. [Bla90]). Here, the trellis has  $\alpha^2 \cdot \beta$  states and  $\alpha \cdot \beta$  branches leave each state. The succeeding state also gives the transmit symbol  $x$ . If usual APSK constellations are used, the trellis only has  $\alpha \cdot \beta$  states, and each state transition is possible. This reflects the non-redundant mapping.

For  $\alpha = 2, \beta = 4$  and  $\alpha = 4, \beta = 4$  the constellations  $\mathcal{A}$  and  $\mathcal{X}$  are shown in Figure 3. We call this differential encoding strategy *twisted absolute amplitude and differential phase shift keying* (*TADPSK*).

The presented approach is similar, but not identical, to the diversity enhancement by rotated constellations proposed e.g. in [Ker93, PE94, BV96, Fis96].

#### 2.4 Differential Amplitude and Phase Encoding with Twisted Constellations

Obviously,  $\text{T}\alpha A\beta\text{PSK}$  constellations can also be used in connection with the encoding strategy (4). One possible advantage of using these constellation in DAPSK is their higher minimum Euclidean distance. It is easy to verify, that here no constellation expansion takes place, and  $\mathcal{A} =$

$\mathcal{X} = \mathcal{A}_{\text{TAPSK}}$ . The operator “ $\otimes$ ” is given by

$$x = s \otimes a = r_i e^{j\varphi_n} \otimes r_j e^{j\varphi_m} \triangleq r_{(i+j) \bmod \alpha} e^{j\varphi_{(n+m) \bmod \alpha\beta}}. \quad (8)$$

This last strategy is denoted by *twisted differential amplitude and phase shift keying* (*TDAPSK*).

For an overview, in Table 1 the various encoding schemes are summarized. They are classified both by the encoding of the amplitude of the transmit symbol and by the use of APSK or TAPSK constellations.

In the following, when specifying the number of amplitudes and phases of the differential encoding scheme, the parameters of the differential constellation  $\mathcal{A}$  are given, i.e., the denominations  $\text{D}\alpha A\beta\text{PSK}$ ,  $\alpha \text{AD}\beta\text{PSK}$ ,  $\text{T}\alpha \text{AD}\beta\text{PSK}$  and  $\text{TD}\alpha A\beta\text{PSK}$ , respectively, are used.

### 3. Capacity, Decoding and Complexity

In multiple symbol detection, demodulation is based on blocks of  $N$  received symbols. Thus, at the receiver vectors  $\mathbf{y}$  of  $N$  consecutive output symbols  $y$ , overlapping each other by one symbol, are formed [DS94]. In order to get rid of statistical dependencies between the blocks  $\mathbf{y}$  ideal interleaving at the transmitter and deinterleaving at the receiver are performed.

Each receive vector  $\mathbf{y}$  corresponds to a block of  $N - 1$  (differential) symbols  $a[\kappa]$ ,  $\kappa = 1, 2, \dots, N - 1$ , combined into the vector  $\mathbf{a} = [a[1], a[2], \dots, a[N - 1]]$ . Due to ideal interleaving the channel between  $\mathbf{a}$  and  $\mathbf{y}$  is memoryless and can be entirely characterized by a single  $2N$ -dimensional probability density function (pdf)  $p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a})$  of  $\mathbf{y}$  given  $\mathbf{a}$ . In order to calculate this pdf we first have to consider the stationary, memoryless, non-dispersive and multiplicative Rician fading channel between the  $N$ -dimensional vector symbols  $\bar{\mathbf{x}} \triangleq [s, x[1], \dots, x[N - 1]] = [s, s \otimes a[1], s \otimes a[1] \otimes a[2], \dots, s \otimes a[1] \otimes \dots \otimes a[N - 1]]$  and  $\mathbf{y}$ :

$$\mathbf{y}[k] = g[k] \cdot e^{j\phi[k]} \cdot \bar{\mathbf{x}}[k] + \mathbf{n}[k]. \quad (9)$$

Here,  $k \in \mathbb{Z}$  is the discrete-time index of the vector channel.  $g[k]$  represents the complex Gaussian distributed channel gain, which is identical for all components of  $\bar{\mathbf{x}}[k]$  because of the assumption of slow fading. The average power of  $g[k]$  is normalized to one and the Rician parameter, equal to the ratio of direct to diffuse power, is denoted by  $K$ .  $\phi[k]$  is the carrier phase, which is uniformly distributed in  $[0, 2\pi)$  and independent of  $g[k]$ . The vector  $\mathbf{n}[k]$  denotes independent additive white Gaussian noise (AWGN) with variance  $\sigma_n^2 = N_0/T$  per complex component.

The pdf  $p_{\mathbf{Y}}(\mathbf{y}|\bar{\mathbf{x}})$  is given in [DS94, Appendix] ( $I_0(\cdot)$  is the modified Bessel function of order zero and  $\cdot^H$  denotes Hermite transposition):

$$p_{\mathbf{Y}}(\mathbf{y}|\bar{\mathbf{x}}) = \frac{1}{\pi^N (\sigma_n^2)^{N-1}} \cdot \frac{K+1}{|\bar{\mathbf{x}}|^2 + (K+1)\sigma_n^2} \cdot \exp\left(-\frac{1}{\sigma_n^2} \left[|\mathbf{y}|^2 + \frac{K}{K+1}|\bar{\mathbf{x}}|^2 - \frac{|\mathbf{y}\bar{\mathbf{x}}^H|^2 + \frac{K}{K+1}|\bar{\mathbf{x}}|^4}{|\bar{\mathbf{x}}|^2 + (K+1)\sigma_n^2}\right]\right)$$

Table 1. Classification of differential encoding strategies (denomination, encoding strategy, signal constellation  $\mathcal{X}$ ).

		Encoding of the amplitude	
		differential	absolute
Constellation	$\mathcal{A} = \mathcal{C}_{\text{APSK}}(\alpha, \beta)$	DAPSK Encoder (4), $\mathcal{X} = \mathcal{C}_{\text{APSK}}(\alpha, \beta)$	ADPSK Encoder (5), $\mathcal{X} = \mathcal{C}_{\text{APSK}}(\alpha, \beta)$
	$\mathcal{A} = \mathcal{C}_{\text{TAPSK}}(\alpha, \beta)$	TDAPSK Encoder (8), $\mathcal{X} = \mathcal{C}_{\text{TAPSK}}(\alpha, \beta)$	TADPSK Encoder (7), $\mathcal{X} = \mathcal{C}_{\text{APSK}}(\alpha, \alpha\beta)$

$$\cdot I_0 \left( \frac{2\sqrt{K(K+1)}}{|\bar{x}|^2 + (K+1)\sigma_n^2} |y\bar{x}^H| \right) . \quad (10)$$

Because  $\bar{x}$  is completely given by  $s$  and  $\mathbf{a}$ , it can be substituted by these quantities. Moreover, due to the uniformly distributed carrier phase the pdf only depends on the amplitude of  $s$ . Thus  $p_{\mathbf{Y}}(\mathbf{y}|\bar{x}) = p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a}, |s|)$  holds for all definitions of  $\otimes$ . Finally, averaging  $p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a}, |s|)$  over all possible amplitudes  $|s| = r$  yields [CFLM98]

$$p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a}) = \mathcal{E}_r \{p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a}, r)\} , \quad (11)$$

where  $\mathcal{E}_x \{\cdot\}$  denotes expectation with respect to  $x$ .

### 3.1 Capacity

As we are intended to employ turbo codes [BGT93, BG96] as channel coding scheme, which operates close to limits from information theory, capacity is considered as measure for performance assessment.

It is well-known that for capacity calculation an optimization over all free parameters has to be performed. On the one hand, the ring ratio of the (T)APSK constellation has to be chosen. On the other hand, the probabilities of the rings have to be regarded. Due to the rotational invariance of APSK constellations, uniformly distributed phases are optimal. If differential encoding of the amplitudes is used (encoding operators (4) and (8)), due to averaging, uniformly distributed amplitudes result regardless the probabilities of the differential signal points  $\mathbf{a}$ . In this case we choose the symbols  $\mathbf{a}$  to be uniformly, independently and identically distributed maximizing the throughput of the channel.

For absolute amplitude encoding an optimization of the amplitude distribution  $\Pr\{r_i\}$  can be performed. But it turns out that the gains are negligible. This is because uniformly distributed APSK constellations already exhibit some shaping. Like in warping [BCL94], a non-uniform density is achieved by optimizing the spacing of the signal points, rather than the probabilities. Hence, optimizing the ring ratio is almost identical to optimizing the amplitude distribution. The geometrical spacing of the rings leads to a discrete second order hyperbolic distribution of the two-dimensional points  $\mathbf{a}$ , which closely resembles a discrete Gaussian distribution. Moreover, despite the unavoidable uniformly distributed amplitudes, also for differential encoding of the amplitude some kind of shaping is active. Thus, in all situations we fix the symbols  $\mathbf{a}$  to be uniformly, independently and identically distributed.

Subsequently, we use the term ‘‘capacity’’ for the mutual information given the uniform distributed constellation. Then, the normalized channel capacity, measured in

bits per symbol, is calculated by [Gal68]

$$C(N) \triangleq \frac{1}{N-1} \cdot \mathcal{E}_{\mathbf{Y}, \mathbf{A}} \left\{ \log_2 \left( \frac{p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a})}{p_{\mathbf{Y}}(\mathbf{y})} \right) \right\} , \quad (12)$$

where  $p_{\mathbf{Y}}(\mathbf{y})$  is the average pdf of the channel output.

### 3.2 Demodulation and Complexity

The optimum receiver performing non-coherent demodulation without channel state information evaluates  $p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a})$  for all  $\mathbf{a}$ . In the uncoded case,  $\hat{\mathbf{a}} = \arg\max_{\mathbf{a}} p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a})$  is the hard decision estimate. For channel coding, the soft-information  $p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a})$  is directly passed to the channel decoder.

Rewriting (11), we have

$$p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a}) = \sum_{i=0}^{\alpha-1} \Pr\{r_i\} \cdot p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a}, r_i) . \quad (13)$$

Thus, the computational complexity for the calculation of the optimum demodulation values depends, a) on the size of the constellation  $\mathcal{A}$  and b) on the number of signalling amplitudes. For  $\alpha\mathbf{A}\beta\text{PSK}$  constellation we have  $|\mathcal{A}| = \alpha\beta$  and thus  $(\alpha\beta)^{N-1}$  possible vectors  $\mathbf{a}$ . Because for each  $\mathbf{a}$  the pdf (10) has to be evaluated  $\alpha$  times, the total number of pdf calculations is

$$\alpha^N \cdot \beta^{N-1} ,$$

where the number of amplitudes contributes more to the complexity than the number of phases.

Notice, that because the metric calculation primarily depends on the size of the constellation  $\mathcal{A}$ , constellation expansion with respect to phase does not required more effort. In particular, regardless the difference of one modulo addition at the encoder, all strategies discussed in Table 1 have the same complexity.

In coherent transmission over the AWGN channel, often the decoding complexity is reduced by regarding only the nearest neighbours of the receive point. Using this sub-optimal metric usually results only in marginally losses. Here, a similar approach can not be found. It seems to be impossible to obtain the radius corresponding to the dominant term in (13) by solely inspecting the arguments of the pdf (10).

## 4. Numerical Results

In this section we present capacity curves for differentially encoded transmission versus the (average) signal-

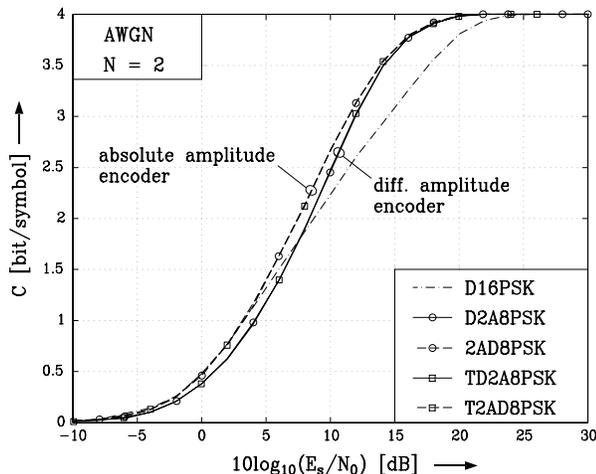


Fig. 4. Capacities for AWGN channel.  $N = 2$ .

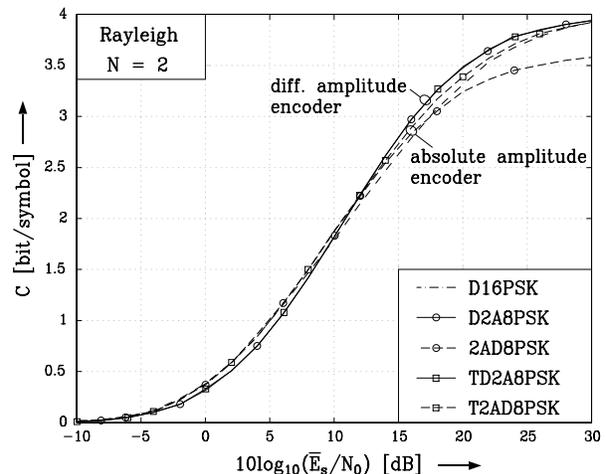


Fig. 5. Capacities for Rayleigh fading channel.  $N = 2$ .

to-noise ratio  $\bar{E}_s/N_0$  ( $\bar{E}_s$ : average energy per symbol,  $N_0$ : one-sided noise power spectral density). Thereby, we concentrate on the application of 16-ary modulation schemes with non-coherent detection and transmission over the AWGN channel ( $K \rightarrow \infty$ ) and the Rayleigh fading channel ( $K = 0$ ), which are the most interesting special cases of the Rician fading model.

Besides usual D16PSK, differentially encoded transmission using two signalling amplitudes is reported as a convenient scheme, cf. e.g. [Sve95, Ada96, CFLM98]. Consequently, we consider D2A8PSK, 2AD8PSK, TD2A8PSK, and T2AD8PSK constellations, respectively. According to [LF99] we choose the ring ratio  $r_1/r_0$  equal to 2.0, which is most appropriate for D2A8PSK transmission over fading channels with powerful channel coding. Note, optimally the ring ratio has to be optimized for each value of the signal-to-noise ratio.

For  $N = 2$ , Figures 4 and 5 show the capacity of D2A8PSK, 2AD8PSK, TD2A8PSK, and T2AD8PSK. Furthermore, the capacity of D16PSK is given for reference. Clearly, for the AWGN channel where the amplitude transmit factor is constant, modulation of the actual transmit amplitude is superior to differential encoding over the amplitude. But in the case of Rayleigh fading and  $N = 2$  DAPSK performs significantly better than ADPSK. TADPSK does not undergo the severe degradation as ADPSK, but it is outperformed by DAPSK for capacities larger than 2.2 bits/symbol. Only for low rates, introducing diversity and using the absolute amplitudes is rewarding. The curves for DAPSK and TDAPSK are almost indistinguishable. Thus, for differential amplitude encoding the higher Euclidean distance of twisted constellations does not provide gains. This emphasizes the known fact, that Euclidean distance is not a suited parameter for differential encoding. D16PSK, which requires only half the demodulation complexity, can always (considerably) be outperformed by D2A8PSK with optimal ring ratio.

Figures 6 and 7 summarize the numerical results for the case  $N = 3$ . Again, on the AWGN channel ADPSK and TADPSK show almost identical performance. These strategies are superior to the schemes with differential amplitude encoding. For the Rayleigh fading channel the point of intersection of the capacity curves of DAPSK

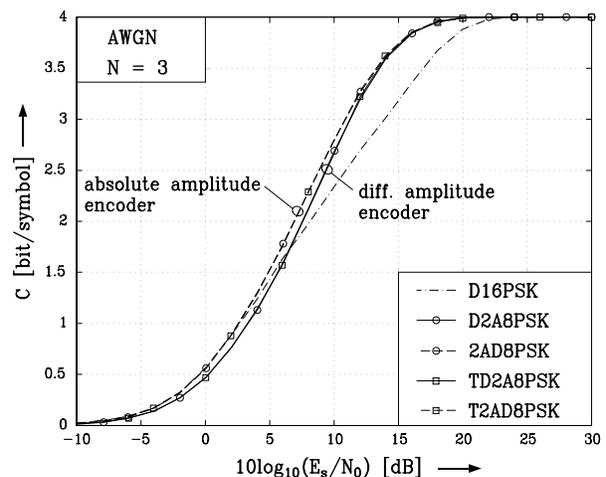


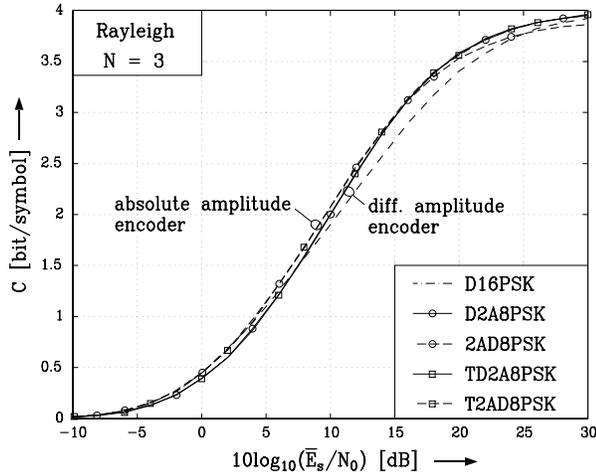
Fig. 6. Capacities for AWGN channel.  $N = 3$ .

and ADPSK moves towards higher capacity values (3.1 bits/symbol), as compared to  $N = 2$ . This is due to the fact that if  $N$  increases the use of both signal amplitudes within the observation interval becomes more probable. Hence, increasing  $N$ , ADPSK becomes more and more the favourite choice because amplitude classification gets more reliable. The best performance for the entire capacity range is achieved if diversity is introduced and TADPSK is applied. For high SNR, DAPSK, TADPSK and TDAPSK perform nearly the same. We can conclude from the results that for  $N > 2$  TADPSK constitutes the favorite strategy for multiplicative Rician fading.

The results can be summarized as follows:

**APSK constellation:** Modulation of the absolute amplitude is favorable for the AWGN channel, but for the Rayleigh fading channel a flattening at high SNRs occurs.

**TAPSK constellation:** TAPSK constellations do not provide any gain on the AWGN channel, and on the Rayleigh fading channel with differential amplitude en-

Fig. 7. Capacities for Rayleigh fading channel.  $N = 3$ .

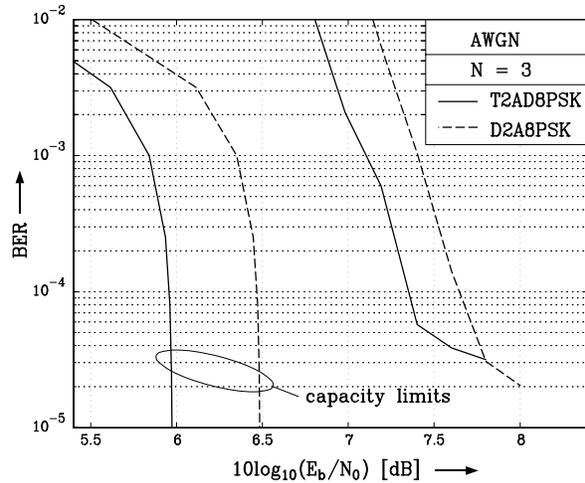
coding. But due diversity, TADPSK is the favorable choice when modulating the actual amplitude on the Rayleigh fading channel. The advantages grow as  $N$  increases. Table 2 gives some guidance for selecting the most appropriate differential encoding scheme.

## 5. Simulation Results

In order to further assess the proposed differential encoding strategies D2A8PSK and T2AD8PSK over the AWGN and the Rayleigh fading channel are numerically simulated. An observation interval  $N = 3$  is chosen, and again no channel state information is available at the receiver.

As coded modulation scheme we apply multilevel coding (MLC) with multistage decoding (MSD) [IH77] and binary component codes. At the transmitter  $N - 1 = 2$  consecutive transitions are determined by 2 differential symbols taken from an  $\alpha\beta = 16$ -ary signal set. For channel coding, these symbols are encoded in one step. Using Ungerböck's labeling,  $\ell = (N - 1) \cdot \log_2(\alpha\beta) = 8$  binary symbols are thus mapped to the vector symbols  $\mathbf{a}$ . In this paper, the rates  $R^i, i = 0, \dots, \ell - 1$  of the  $\ell$  binary component codes are always designed according to the capacities  $C^i$  of the equivalent channels of MLC/MSD with the sum equal to the desired target rate. Noteworthy, this procedure is optimum for capacity approaching codes [WFH99] and we do not regard parameters such as minimum squared Euclidean distance, minimum Hamming distance or product distance. For short to medium code lengths, other design rules, e.g. based on error exponent or on the error rate of the individual levels are imaginable. Thereby, heuristically, the error propagation from level to level can be taken into account. For details on code design rules and a tutorial review on MLC see [WFH99].

Turbo codes [BG96] (parallel concatenated convolutional codes with 16 states each) are employed as component codes. The systematical decoder concept according to [WH95] is used. Rate is adjusted by symmetric puncturing of parity symbols, cf. [WH95]. The interleavers of the turbo codes are randomly generated, i.e., no optimization has been performed. The decoders are allowed

Fig. 8. Simulation results. AWGN channel.  $N = 3$ .

to perform a maximum number of 6 iterations. The code lengths of the binary component codes are chosen equal to 3000, i.e., the overall delay of the transmission equals  $(N - 1) \cdot 3000 = 6000$  channel symbols.

In Figure 8 the bit error rates over  $E_b/N_0$  ( $E_b$ : energy per information bit) of both D2A8PSK and T2AD8PSK with a rate of 3.0 bits/channel use over the AWGN channel are plotted. The target rate 3.0 bits/channel use divides optimally into the individual rates  $R^i = C^i$  with

$$C^0/\dots/C^7 = 0.33/0.44/0.89/0.90/1.00/1.00/0.67/0.77$$

for D2A8PSK and

$$C^0/\dots/C^7 = 0.25/0.35/0.85/0.89/1.00/1.00/0.83/0.83$$

for T2AD8PSK. Here,  $R^0, R^2, R^4, R^6$  and  $R^1, R^3, R^5, R^7$  correspond to the first and to the second component of  $\mathbf{a}$ , respectively;  $R^6$  and  $R^7$  determine the amplitude change (D2A8PSK) and the absolute amplitude (T2AD8PSK) of the transmit signal, respectively. As reference, the capacity limits derived in the last section and now taking the finite error rate into account ("rate-distortion capacities") [BM74] are shown.

The curves in Figure 8 verify that the advantage of T2AD8PSK over D2A8PSK predicted by information theory can be utilized in practice. Almost the whole gain which is quantified by the rate-distortion capacities can be observed in the simulation. For BER's around  $10^{-4}$  the gap between the required signal-to-noise ratios and the capacity limits is about 1.3...1.4 dB. As no optimization of the interleavers of the turbo codes has been performed some flattening of the error curves can be recognized.

Next, Figure 9 presents the simulation results for transmission over the Rayleigh fading channel with a rate of 2.5 bits/channel use. Here, the optimal individual rates are given by

$$C^0/\dots/C^7 = 0.36/0.44/0.70/0.75/0.87/0.89/0.45/0.54$$

for D2A8PSK and

$$C^0/\dots/C^7 = 0.29/0.37/0.68/0.70/0.88/0.89/0.57/0.62$$

Table 2. Recommended differential encoding strategies.

		Encoding of the amplitude	
		differential	absolute
Constellation $\mathcal{A}$	APSK	DAPSK Rayleigh, $N = 2$	ADPSK AWGN
	TAPSK	TDAPSK —	TADPSK Rayleigh, $N > 2$

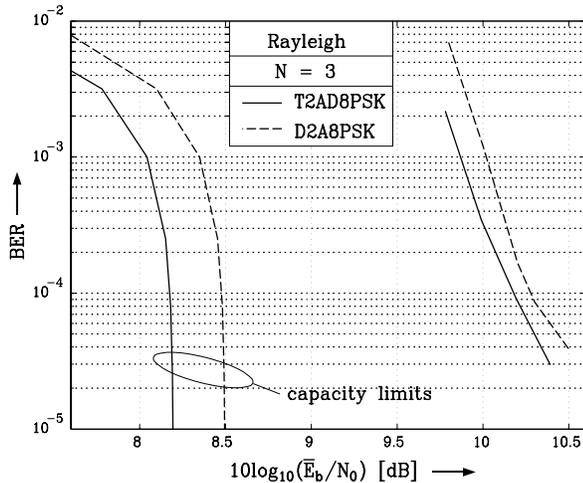


Fig. 9. Simulation results. Rayleigh fading channel.  $N = 3$ .

for T2AD8PSK. Again, T2AD8PSK outperforms D2A8PSK as expected by capacity arguments. The achieved gain in signal-to-noise ratio is in the order of the theoretical gain. For BER's around  $10^{-4}$  a gap between 1.8 and 2.0 dB to capacity limit results.

## 6. Conclusions

Differentially encoded transmission over non-dispersive fading channels and non-coherent reception without channel state information is considered. As alternative strategies to usual differential encoding over amplitude and phase, differential phase encoding with absolute amplitude encoding is regarded. Furthermore, redundant encoding of the information into phase changes and absolute amplitudes using twisted differential constellations is introduced.

In order to assess the performance of the different schemes the corresponding channel capacities are calculated as a function of the average signal-to-noise ratio. The numerical results for 16-ary modulation show that diversity differential phase encoding with absolute amplitudes almost always outperforms usual differential encoding. In particular, diversity differential encoding constitutes the most favourable choice if multiple symbol detection ( $N > 2$ ) is applied. Differential encoding of only the phase of APSK constellations is an interesting solution in the case of Rician fading with relatively large mean value and/or if multiple symbol detection with relatively long observation interval is used. The performed simulations

employing multilevel coding with turbo codes as component codes verify the theoretic results.

It turns out, that the gain attainable by the modified differential encoding strategies is for free, since it does not require any increment in the coding/decoding complexity when used together with channel coding.

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