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Multiuser Detection with Iterated Soft Decision Interference Cancellation for Multipath Fading Channels

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Abstract **In this paper we investigate the transmission over time-variant multipath Rayleigh-fading channels employing Direct Sequence Code Division Multiple Access (DS-CDMA). Assuming ideal knowledge of the actual channel state, we show that by application of Minimum Mean-Squared Error Equalization (MMSE-Equalization) combined with iterated soft decision interference cancellation (ISDIC) the multiple users' data symbols can be detected almost as reliable as in a single user system. This even holds for fully loaded systems. Further, using the simplifications made for derivation of the proposed receiver scheme an analytical solution to the achievable symbol error ratio (SER) is given.**

Keywords [redacted],[redacted],[redacted]

1. Introduction

One of the main disadvantages of CDMA systems is severe multiuser interference (MUI). Especially in the uplink which is addressed in this paper, MUI may cause huge drawbacks if it is treated as additional white Gaussian noise. On the other hand, using knowledge of the users' spreading sequences as well as their attenuations available at the receiver, it is possible to cancel the multiuser interference up to a certain degree. The optimum multiuser receiver with respect to minimum probability of detection error of the users' transmitted data symbols has been described by Verdú [1]. Since this receiver is too complex, numerous suboptimum linear receivers with and without decision-feedback have been proposed. Good results are achievable with iterated decision-feedback schemes which were presented by Varanasi and Aazhang [2, 3]. The main idea of these receiver concepts is to increase the reliability of the decision on a user's assumed data symbol by incorporating estimates obtained for other users' symbols in order to cancel their interference. First, such schemes employed hard decisions whereas it was shown by Miyajima et al. [4], Kechriotis and Manolakos [5], and Teich and Seidl [6] that feedback of soft decisions is highly preferable leading to the so-called iterated soft decision interference cancellation (ISDIC). The performance of ISDIC was further improved by Müller et al. [7] adapting the soft decision rule according to the decreasing multiuser interference power during the iterations being denoted as ISDIC-OSC where OSC stands for optimized soft-cancellation. All these proposals use the output of a bank of filters matched to the users' spreading sequences. Applying instead a bank of MMSE-filters being adapted according to the declining multiuser interference additional performance improvements are achievable. In [8] the author shows this for single-path channels assuming a synchronous CDMA system with randomly chosen spreading sequences. In this work transmitter synchronous communication over multipath Rayleigh-fading channels is the focus of attention. So, the performance achievable for serial and parallel cancellation is compared and the influence of the spreading gain is discussed. In addition to the simulation results an analytical solution to the SER yielding a tight approximation of the actual results is presented.

The paper is arranged as follows. In Sect. 2, the simplified transmission model is given. The proposed receiver scheme is introduced in Sect. 3.1 and simulation results are provided for performance evaluations. An analytical formula for the receiver's performance is derived in Sect. 3.2 based on the eigenvalue distribution of covariance matrices. Finally, Sect. 4 points out conclusions.

2. Transmission model

Considering the transmission of K users with CDMA to a single receiver and assuming that all users transmit synchronously the underlying discrete-time equivalent complex baseband transmission model is given in Fig. 1a (see also [9]).

Here, $x_k[\mu]$ and $\boldsymbol{s}_k[\mu] = (s_{1,k}[\mu], \ldots, s_{N,k}[\mu])^T$ denote the kth user's transmitted data symbol and his/her spreading sequence in the μth transmission interval, respectively. The users' data symbols $x_k[\mu], \forall k$, are chosen from the set $\mathcal{X} = \{\pm 1 \pm \mathrm{j}\}$ and the elements of the unit energy spreading sequence are drawn randomly as $s_{j,k}[\mu] \in \{(\pm 1 \pm \mathrm{j})/\sqrt{2N}\}, \forall j$. The kth user's modulated sequence $(x_k[\mu]s_{1,k}[\mu], \ldots, x_k[\mu]s_{N,k}[\mu])$ is sent over a channel with impulse response $\boldsymbol{h}_k[\mu]$. In order to account for the multipath propagation we use the well known tapped-delay-line channel model (see Fig. 1b) assuming L resolvable paths [9]. Here, T_c is the length of one chip interval. As usual the path weights $(h_{0,k}[\mu], \ldots, h_{L-1,k}[\mu]), \forall k$, are assumed to be independent and identically zero mean proper complex Gaussian distributed with autocorrelation function $\Phi_{hh}[\Delta\mu] =$

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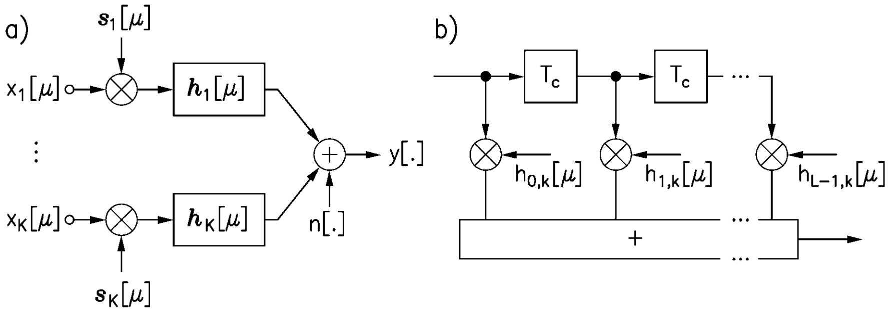

Fig. 1. (a) Transmission model of CDMA system with K users. (b) Tapped-delay-line channel model with L paths.

$\mathcal{E}\left\{h_{\lambda,\kappa}[\mu]h^*_{\lambda,\kappa}[\mu+\Delta\mu]\right\} = \frac{1}{L}J_0(2\pi f_D T_c \Delta\mu), \forall\lambda, \forall\kappa$, where f_D and J_0 denote the maximum Doppler frequency and the Bessel function of the first kind of order zero, respectively.[1] So, supposing that the tap weights are virtually constant over one transmission interval the received signal resulting from $x_k[\mu]$ can be written as $(x_k[\mu]s'_{1,k}[\mu], \ldots, x_k[\mu]s'_{N',k}[\mu])$. The kth user's effective spreading sequence $\boldsymbol{s}'_k[\mu] = \left(s'_{1,k}[\mu], \ldots, s'_{N',k}[\mu]\right)^T$ is the convolution of the actual spreading sequence $\boldsymbol{s}_k[\mu]$ and the channel's impulse response $\boldsymbol{h}_k[\mu]$. That is, $s'_{j,k}[\mu] = \sum_{\eta=1}^{N} s_{\eta,k}[\mu]h_{(j-\eta),k}[\mu]$, $1 \leq j \leq N' = N + L - 1$.

Next, for $N \gg L$ the influence of intersymbol interference on the systems performance vanishes compared to the effects of MUI and will therefore be neglected in this paper (cf. [10,11]). Thus, for assumption of synchronous transmission the time index $[\mu]$ may be dropped without loss of generality and the received signal reads in matrix notation

$$\boldsymbol{y} = \boldsymbol{S}'\boldsymbol{x} + \boldsymbol{n}. \tag{1}$$

The N' dimensional vectors $\boldsymbol{y} = (y_1, \ldots, y_{N'})^T$ and $\boldsymbol{n} = (n_1, \ldots, n_{N'})^T$ represent the received signal and the additive channel noise, respectively. The i.i.d. samples n_j, $1 \leq j \leq N'$, are drawn from a zero mean proper complex Gaussian distribution with variance σ_n^2. Further, $\boldsymbol{x} = (x_1, \ldots, x_K)$ and $\boldsymbol{S}' = \left(\boldsymbol{s}'_1, \ldots, \boldsymbol{s}'_K\right)$ consist of the K users' transmitted symbols and their effective spreading sequences as well.

3. Iterative multiuser MMSE-detection with Soft decision feedback

In this section, a generalized version of the proposed iterative multiuser detection algorithm is described first and based on this an analytical derivation of the resulting symbol error ratio confirming the suitability of the simplifications introduced for feasibility of the receiver scheme is provided afterwards.

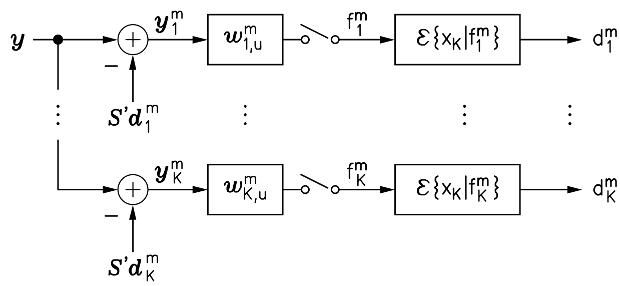

Fig. 2. Serial iterative multiuser MMSE-detection with soft decision-feedback.

3.1 General algorithm

Iterative cancellation can employ two possible strategies – serial and parallel decision-feedback. First, we outline the main steps carried out by the serial scheme illustrated in Fig. 2 for a general data symbol set $\mathcal{X}$ with $\mathcal{E}\{|x_k|^2\} = \sigma_x^2$, $\forall k$.

Let us consider the mth iteration and select the kth user as the user of interest. Our aim is to obtain a soft estimate d_k^m for his/her transmitted data symbol x_k. Then, the major parts of the decision chain being explained in the following are the unbiased interference suppression filter $\boldsymbol{w}_{k,\mathrm{u}}^m$ leading to the scalar output f_k^m and the subsequent soft decision device yielding the soft estimate $d_k^m = \mathcal{E}\left\{x_k|f_k^m\right\}$.[2] Using serial cancellation it is convenient to introduce the estimation vector $\boldsymbol{d}_k^m = \left(d_1^m, ..., d_{k-1}^m, 0, d_{k+1}^{m-1}, ..., d_K^{m-1}\right)^T$ where $d_1^m, ..., d_{k-1}^m$ and $d_{k+1}^{m-1}, ..., d_K^{m-1}$ are soft decisions made in the mth iteration for the preceding users $1, ..., k-1$ and in the $(m-1)$th iteration cycle for the succeeding users $k+1, ..., K$ respectively. These soft estimates are remodulated and $\boldsymbol{S}'\boldsymbol{d}_k^m$ is subtracted from $\boldsymbol{y}$ in order to reduce the multiuser interference. So, we get

$$\boldsymbol{y}_k^m = \boldsymbol{y} - \boldsymbol{S}'\boldsymbol{d}_k^m = \boldsymbol{S}'\left(\boldsymbol{x} - \boldsymbol{d}_k^m\right) + \boldsymbol{n}. \tag{2}$$

In order to suppress the remaining multiuser interference, $\boldsymbol{y}_k^m$ is fed into an interference suppression filter $\boldsymbol{w}_k^m$ min-

[1] $\mathcal{E}\{x\}$ and $|x|$ represent the expectation value and the absolute magnitude of x, respectively.

[2] $\mathcal{E}\{x|y\}$ denotes the expectation value of x conditioned on y.

imizing the mean-squared error $\mathcal{E}\{||x_k - (\boldsymbol{w}_k^m)^H \boldsymbol{y}_k^m||^2\}$.[3] Using well-known results the optimum solution for $\boldsymbol{w}_k^m$ is

$$(\boldsymbol{w}_k^m)^T = \mathcal{E}\left\{x_k (\boldsymbol{y}_k^m)^H\right\} \left(\mathcal{E}\left\{\boldsymbol{y}_k^m (\boldsymbol{y}_k^m)^H\right\}\right)^{-1}. \quad (3)$$

So, assuming $\mathcal{E}\{d_j^m x_\kappa^*\} \approx 0, \forall j \neq \kappa, j < k$, $\mathcal{E}\{d_j^{m-1} x_\kappa^*\} \approx 0, \forall j \neq \kappa, j > k$, due to vanishing correlations $\boldsymbol{s}_j^H \boldsymbol{s}_\kappa$, i.e., $\Pr\left(|\boldsymbol{s}_j^H \boldsymbol{s}_\kappa| > 0\right) \overset{N' \to \infty}{\longrightarrow} 0, \forall j \neq \kappa$, as well as decreasing multiuser interference in course of the iterations we get for the desired filter conditioning the expectations on the soft values $\boldsymbol{f}_k^m = (f_1^m, \ldots, f_k^m, 0, f_{k+1}^{m-1}, \ldots, f_K^{m-1})^T$

$$(\boldsymbol{w}_k^m)^T = \boldsymbol{s}'^H_k \sigma_x^2 \left(\boldsymbol{S}' \boldsymbol{E}_k^m \boldsymbol{S}'^H + \sigma_n^2 \boldsymbol{I}_{N'}\right)^{-1}. \quad (4)$$

where $\boldsymbol{I}_{N'}$ denotes the $N' \times N'$ identity matrix and

$$\boldsymbol{E}_k^m = \mathcal{E}\left\{ (\boldsymbol{x} - \boldsymbol{d}_k^m)(\boldsymbol{x} - \boldsymbol{d}_k^m)^H \middle| \boldsymbol{f}_k^m \right\} \quad (5)$$

$$= \mathrm{diag}\left(\ldots, \mathcal{E}\left\{ |x_{k-1} - d_{k-1}^m|^2 \middle| f_{k-1}^m \right\},\right.$$

$$\left. \sigma_x^2, \mathcal{E}\left\{ |x_{k+1} - d_{k+1}^{m-1}|^2 \middle| f_{k+1}^{m-1} \right\}, \ldots \right). \quad (6)$$

Employing the filter $\boldsymbol{w}_k^m$ would result in a biased estimate for x_k even if no interference were present. In order to avoid this, $\boldsymbol{w}_k^m$ is normalized by

$$u_k^m = \sigma_x^2 \boldsymbol{s}'^H_k \left(\boldsymbol{S}' \boldsymbol{E}_k^m \boldsymbol{S}'^H + \sigma_n^2 \boldsymbol{I}_{N'}\right)^{-1} \boldsymbol{s}'_k, \quad (7)$$

and the unbiased filter $\boldsymbol{w}_{k,\mathrm{u}}^m$ is obtained as

$$(\boldsymbol{w}_{k,\mathrm{u}}^m)^T = \frac{\boldsymbol{s}'^H_k \left(\boldsymbol{S}' \boldsymbol{E}_k^m \boldsymbol{S}'^H + \sigma_n^2 \boldsymbol{I}_{N'}\right)^{-1}}{\boldsymbol{s}'^H_k \left(\boldsymbol{S}' \boldsymbol{E}_k^m \boldsymbol{S}'^H + \sigma_n^2 \boldsymbol{I}_{N'}\right)^{-1} \boldsymbol{s}'_k}. \quad (8)$$

Thus, the output f_k^m of the unbiased interference suppression filter $\boldsymbol{w}_{k,\mathrm{u}}^m$ is

$$f_k^m = f_{k,I}^m + f_{k,Q}^m = (\boldsymbol{w}_{k,\mathrm{u}}^m)^T \boldsymbol{y}_k^m$$

$$= x_{k,I} + i_{k,I}^m + \mathrm{j}\left(x_{k,Q} + i_{k,Q}^m\right), \quad (9)$$

where the real valued scalars $i_{k,I}^m$ and $i_{k,Q}^m$ represent the interference caused by users $\kappa \neq k$ as well as the channel noise in the inphase (I) and quadrature (Q) part of f_k^m, respectively. Hence, modeling $i_{k,I}^m + \mathrm{j}\, i_{k,Q}^m$ as complex Gaussian distributed variable with zero mean and variance $(\sigma_k^m)^2$ [4] (see also [12]) as well as supposing equiprobable data symbols, the conditional a-posteriori expectation value $d_k^m = \mathcal{E}\{x | f_k^m\}$ for x_k in the mth iteration is given as

$$d_k^m = \frac{\sum_{x_k \in \mathcal{X}} x_k \exp\left(-\frac{|x_k - f_k^m|^2}{(\sigma_k^m)^2}\right)}{\sum_{x_k \in \mathcal{X}} \exp\left(-\frac{|x_k - f_k^m|^2}{(\sigma_k^m)^2}\right)}, \quad (10)$$

with $(\sigma_k^m)^2 = \frac{\sigma_x^2}{u_k^m} - \sigma_x^2$. Specializing the above equations to our particular signal set $\mathcal{X}$ and assuming Gray mapping we obtain after elementary calculations regarding $\sigma_x^2 = 2$

$$\boldsymbol{E}_k^m = \mathrm{diag}\left(2 - |d_1^m|^2, \ldots, 2 - |d_{k-1}^m|^2,\right.$$

$$\left. 2, 2 - |d_{k+1}^{m-1}|^2, \ldots, 2 - |d_K^{m-1}|^2\right).$$

since $\mathcal{E}\left\{|x_j - d_j^i|^2 | f_j^i\right\} = 2 - |d_j^i|^2$. Note, for signal constellations where $|x|^2$ is not equal for all $x \in \mathcal{X}$ this expectation value relies on the latest soft estimate d_j^i and has to be calculated explicitly as $\mathcal{E}\left\{|x_j|^2 | f_j^i\right\}$. Further, for the estimates of the inphase and quadrature parts of x_k in the mth iteration cycle it is solved

$$d_{k,I}^m = \tanh\left(\frac{u_k^m}{1 - u_k^m} f_{k,I}^m\right)$$

$$d_{k,Q}^m = \tanh\left(\frac{u_k^m}{1 - u_k^m} f_{k,Q}^m\right), \quad (11)$$

and $u_k^m = 2\boldsymbol{s}'^H_k \left(\boldsymbol{S}' \boldsymbol{E}_k^m \boldsymbol{S}'^H + \sigma_n^2 \boldsymbol{I}_{N'}\right)^{-1} \boldsymbol{s}'_k$. In the iterated decision and cancellation procedure, applying serial cancellation the new estimate d_k^m replaces the old one d_k^{m-1} and is used for obtaining successively the estimates $d_{k+1}^m, \ldots, d_{k-1}^{m+1}$. In contrast, employing parallel cancellation all estimates $d_k^m, \forall k$, are calculated at the same time based on $d_k^{m-1}, \forall k$, i.e., in this case the estimation vector for user k is $\boldsymbol{d}_k^m = (d_1^{m-1}, \ldots, d_{k-1}^{m-1}, 0, d_{k+1}^{m-1}, \ldots, d_K^{m-1})^T$. Afterwards the new estimates are used to update the estimation vectors for the $(m+1)$th iteration. Finally, after performing the desired number M of iteration cycles hard decisions are made as to get the final estimate.

Having derived the iterative receiver scheme its performance is illustrated by some numerical results to show on the one hand the reasonability of the assumptions made and to motivate the theoretical analysis given in the next section. First, the transmission over time-variant Rayleigh-fading channels with $L = 1, 2, 3, 5$ paths is considered. In Fig. 3 the SER's versus E_b/N_0 are depicted for serial and parallel cancellation and 4 iterations. E_b and N_0 denote the transmit energy per bit as well as the channel noise power spectral density, respectively, so that for quaternary phase-shift keying (QPSK) holds $2E_b/N_0 = \sigma_x^2/\sigma_n^2$. Further, for both cancellation schemes $K/N = 1$, i.e., fully loaded systems, with $N = 64$ were chosen.

[3] $||\boldsymbol{x}||$ represents the norm of vector $\boldsymbol{x}$ and the superscript H the conjugate transpose, respectively.

[4] It is assumed that $\boldsymbol{i}_{k,I}^m$ and $\boldsymbol{i}_{k,Q}^m$ are uncorrelated and have the same variance $(\sigma_k^m)^2/2$.

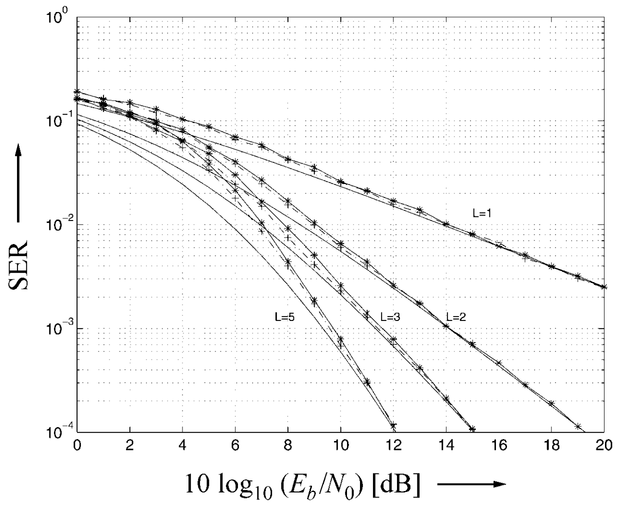

Fig. 3. SER vs. $10\log_{10}(E_b/N_0)$ for studied scheme with serial $(-+)$ and parallel $(-*)$ cancellation with parameter $K=N=64$ and SUB $(-)$ after four iterations.

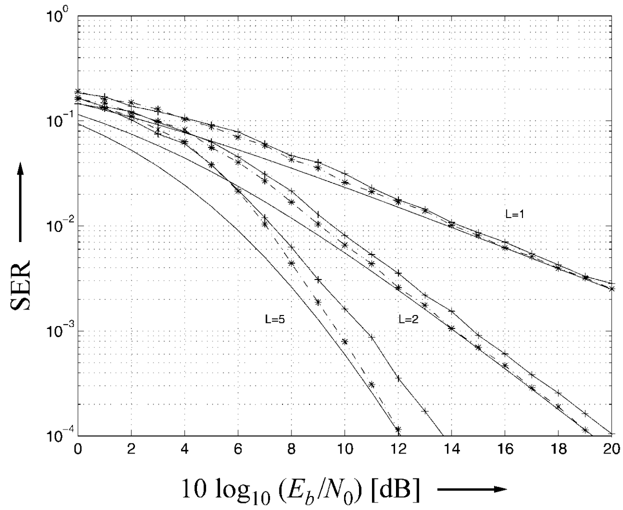

Fig. 4. SER vs. $10\log_{10}(E_b/N_0)$ for fully loaded parallel cancellation with $N=16$ $(-+)$ and $N=64$ $(-*)$ and SUB $(-)$ after four iterations.

In addition, for evaluation of the results the symbol error ratio achieved by a single user transmitting over the same multipath Rayleigh-fading channel is used. The so-called single user bound (SUB) is given as [13]

$$\mathrm{SER} = \frac{1-\rho}{2}\sum_{l=0}^{L-1}\binom{L-1+l}{l}\left(\frac{1+\rho}{2}\right)^{l}, \quad (12)$$

where $\rho=\sqrt{1/(1+LN_0/E_b)}$.

The figure shows that for rising E_b/N_0 the performance of the studied multiuser receiver reaches that of a single user transmission. This is due to the increasing reliability of the soft estimates used for interference cancellation. On the contrary, for very low SER's there is a gap of about 2 dB to the SUB. Further, we see that serial cancellation performs slightly better than parallel one for $\mathrm{SER}\approx 0.01$ and $L=3,5$ being a consequence of the instantaneous use of reliable decisions even within the first iteration. Moreover, the simulations show that there is no necessity to cancel the users in the order of their powers if an iterative procedure is applied.

Next, the influence of the spreading gain on the receiver's performance using parallel cancellation is investigated. Figure 4 provides a comparison of the results for processing gain $N=16$ and $N=64$ supposing a fully loaded system. Note, the system's spectral efficiency defined as $\Gamma=RK/N$ (see [14]) remains unchanged as for both processing gains $K/N=1$ and since the data rate $R=2$ bit/symbol is independent of N. Further, we assume that the physical bandwidth and therefore T_c are equal, too. As before four iterations were performed.

We see that for decreasing spreading factor N and rising number of propagation paths a gap to the SUB appears. Taking into account that all users have equal transmit power the explanation is found in the decreasing variance of the users' received powers resulting together with the shorter spreading sequences in i) lower signal to noise ratios at the filter's output since we can show that equal power users are the worst case for an MMSE-filter if the total average power is fixed, ii) larger correlations of the soft estimates, and iii) lower suitability of the Gaussian approximation for the interference at the output of the interference suppression filter.

3.2 Theoretical approximation of receiver performance

Having evaluated the performance of the proposed receiver scheme by means of simulations in this part of the work we tackle the problem to analyze this scheme allowing us to derive a closed expression for calculation of the achievable symbol error ratios. In addition, for $N \gg L$ it can be shown that from the viewpoint of the multiuser receiver's performance two multipath fading channels are identical if the path gains' powers being sorted in decreasing order are equal.

The main basis of this derivation is the asymptotic eigenvalue distribution of $N\times N$ dimensional random covariance matrices in the limit $N\to\infty$ given in [15]. This result was used in [16] to study the performance of a linear MMSE-filter for synchronous transmission over a single-path channel described by $\boldsymbol{y}=\boldsymbol{S}\boldsymbol{x}+\boldsymbol{n}$. Considering a specific matrix $\boldsymbol{S}$ representing the users' spreading sequences the signal to noise ratio for user 1 is

$$\mathrm{SNR}_1 = \sigma_{x_1}^2 \boldsymbol{s}_1^H\left(\boldsymbol{S}_1\boldsymbol{E}_1\boldsymbol{S}_1^H+\sigma_n^2\boldsymbol{I}_N\right)^{-1}\boldsymbol{s}_1, \quad (13)$$

where $\boldsymbol{S}_1=(\boldsymbol{s}_2,\ldots,\boldsymbol{s}_K)$ and $\boldsymbol{E}_1=\mathrm{diag}\left(\sigma_{x_2}^2,\ldots\sigma_{x_K}^2\right)$ ($\sigma_{x_k}^2=\mathcal{E}\left\{|x_k|^2\right\}$ denotes the kth user's transmit power). Of course, SNR_1 depends on $\boldsymbol{S}$. But, in the limit $N\to\infty$, SNR_1 converges to a non-random value even for randomly chosen spreading sequences and is solution to [16]

$$\mathrm{SNR}_1 = \frac{\sigma_{x_1}^2}{\sigma_n^2 + \frac{1}{N}\sum\limits_{k=2}^{K} \frac{\sigma_{x_1}^2 \sigma_{x_k}^2}{\sigma_{x_1}^2 + \mathrm{SNR}_1 \sigma_{x_k}^2}}. \tag{14}$$

Our aim is to obtain now an equivalent formula to approximate the SER of the proposed multiuser receiver taking into account the application of successive cancellation based on soft outputs as well as different iteration cycles. For this, we have to derive not only suitable analytical solutions for the signal to noise ratio SNR_k^m of the kth user after the mth iteration based on the soft output $d_k^m, \forall k, \forall m$, but also for the expected interference remaining after soft cancellation.

Before we can proceed we first have to simplify our transmission model. So, neglecting the first $L-1$ as well as last $L-1$ samples of each effective spreading sequence (being possible for $N \gg L$ [11]) all remaining elements of the kth user's effective spreading sequence can be modeled as $s'_{.,k} = \hat{s}_{.,k}\sqrt{\sum_{l=0}^{L-1} |h_{l,k}|^2}, \forall k$. With the assumption that the complex variables $\hat{s}_{.,k}$ are zero mean Gaussian distributed with variance $1/N$ the eigenvalue distribution of the covariance matrix $\boldsymbol{S}'_k \boldsymbol{E}_k^m \boldsymbol{S}'^H_k$ and therefore SNR_k^m are not changed if $N \gg 1$ (for conditions see [10, 15]). In fact, each users effective spreading sequence can be modeled as product $\hat{\boldsymbol{s}}_k\sqrt{\sum_{l=0}^{L-1} |h_{l,k}|^2} \triangleq \hat{\boldsymbol{s}}_k a_k$. Thus, we have now

$$\boldsymbol{y} = \hat{\boldsymbol{S}}\boldsymbol{A}\boldsymbol{x} + \boldsymbol{n}, \tag{15}$$

where $\hat{\boldsymbol{S}} = (\hat{\boldsymbol{s}}_1, \ldots, \hat{\boldsymbol{s}}_K)$ and $\boldsymbol{A} = \mathrm{diag}(a_1, \ldots, a_K)$ stand for the users' spreading sequences and fading amplitudes, respectively. Next, with the assumption $K \gg 1$ we can assign each user an amplitude a_k being drawn randomly from its pdf. The amplitudes' pdf can be solved as [17]

$$f_a(\alpha) = \frac{2}{\left(\sigma_h^2\right)^L (L-1)!}\alpha^{2L-1} \mathrm{e}^{-\alpha^2/\sigma_h^2} U(\alpha), \tag{16}$$

where $U(\alpha)$ denotes the unit step function. As before let us consider the kth user in the mth iteration and serial cancellation. So, our decision is based on the signal

$$\boldsymbol{y}_k^m = \hat{\boldsymbol{S}}\boldsymbol{A}\left(\boldsymbol{x} - \boldsymbol{d}_k^m\right) + \boldsymbol{n}. \tag{17}$$

which is fed into an appropriately adapted MMSE-filter. Then, based on the prerequisites given in the previous section for derivation of $\boldsymbol{w}_{k,u}^m$, in the limit $N \to \infty$ the signal to noise ratio SNR_k^m is solution to

$$\mathrm{SNR}_k^m = \frac{\sigma_{x_k}^2 a_k^2}{\sigma_n^2 + \frac{1}{N}\left(\sum\limits_{j<k} \frac{\sigma_{x_k}^2 a_k^2 (\sigma_{x_j}^m)^2 a_j^2}{\sigma_{x_k}^2 a_k^2 + \mathrm{SNR}_k^m (\sigma_{x_j}^m)^2 a_j^2} + \sum\limits_{j>k} \frac{\sigma_{x_k}^2 a_k^2 (\sigma_{x_j}^{m-1})^2 a_j^2}{\sigma_{x_k}^2 a_k^2 + \mathrm{SNR}_k^m (\sigma_{x_j}^{m-1})^2 a_j^2}\right)}.$$

In order to solve the above equation recursively, we have to calculate the expected power $(\sigma_{x_j}^m)^2 = \mathcal{E}\left\{|x_j - d_j^m|^2 | f_j^m\right\}$ remaining after soft interference cancellation. Making for $N \gg 1$ use of the model $f_j^m = x_j + i_j^m$ where i_j^m is complex Gaussian distributed with power $(\sigma_j^m)^2$ the pdf of f_j^m is

$$f_{f_j^m}(\alpha) = \sum_{x_j \in \mathcal{X}} \frac{1}{\pi\left(\sigma_j^m\right)^2} \exp\left(-|\alpha - x_j|^2 / \left(\sigma_j^m\right)^2\right). \tag{18}$$

Then, regarding that $\sigma_{x_j}^2/(\sigma_j^m)^2 = \mathrm{SNR}_j^m$ the desired expectation $\mathcal{E}\left\{|x_j - d_j^m|^2 | f_j^m\right\}$ and consecutively SNR_k^m can be calculated and the symbol error probability of the kth user in the mth iteration be solved. In general it is given as

$$\mathrm{SER}_k^m = \frac{1}{|\mathcal{X}|}\sum_{x_k \in \mathcal{X}} \Pr\left(d_k^m \notin \mathcal{D}_{x_k} | x_k \text{ transmitted}\right), \tag{19}$$

where $|\mathcal{X}|$ and $\mathcal{D}_{x_k}$ denote the cardinality of set $\mathcal{X}$ and the decision region belonging to $x_k \in \mathcal{X}$, respectively. Finally, the average symbol error ratio after the mth iteration is obtained as

$$\mathrm{SER}^m = \frac{1}{K}\sum_{k=1}^{K} \mathrm{SER}_k^m \overset{K\to\infty}{\longrightarrow} \mathcal{E}\left\{\mathrm{SER}_k^m\right\}. \tag{20}$$

To shed more light onto the above equations let us specialize to $\mathcal{X} = \{\pm 1 \pm \mathrm{j}\}$. Then, we have for the inphase and quadrature components of f_j^m

$$f_{f_{j,.}^m}(\alpha) = \frac{\exp\left(-(\alpha-1)^2/\left(\sigma_j^m\right)^2\right) + \exp\left(-(\alpha+1)^2/\left(\sigma_j^m\right)^2\right)}{2\sqrt{\pi}\sigma_j^m}. \tag{21}$$

So, using $2/(\sigma_j^m)^2 = \mathrm{SNR}_j^m$ we get

$$\begin{aligned} &\mathcal{E}\left\{\left(x_{j,.} - d_{j,.}^m\right)^2 | f_{j,.}^m\right\} = \mathcal{E}\left\{\left(x_{j,.} - \tanh\left(\mathrm{SNR}_j^m f_{j,.}^m\right)\right)^2\right\} \\ &= \frac{1}{\sqrt{2\pi}\sigma_j^m} \\ &\times \int_{-\infty}^{\infty} \left(1 + \tanh\left(\mathrm{SNR}_j^m \alpha\right)\right)^2 \exp\left(-(\alpha+1)^2 \mathrm{SNR}_j^m/2\right) \mathrm{d}\alpha. \end{aligned} \tag{22}$$

In the last step we took into account the symmetry of the pdf $f_{f_{j,.}^m}(\alpha)$ as well as the function $\tanh(x)$. The integral may be solved numerically. Next, for the chosen set $\mathcal{X}$ the hard decision based on the soft estimate $d_{k,.}^m = \tanh(u_k^m/(1-u_k^m) f_{k,.}^m)$ only relies on the sign of $f_{k,.}^m$ since $u_k^m/(1-u_k^m) > 0$ and $\tanh(x)$ is monotonically increasing. Hence, regarding the symmetry of $\mathcal{X}$ we get $\mathrm{SER}_k^m =$

$\Pr(f_{k,\cdot}^m < 0 | x_{k,\cdot} = 1)$. Now, using again $f_{k,\cdot}^m = x_{k,\cdot} + i_{k,\cdot}^m$, where $i_{k,\cdot}^m$ is the Gaussian distributed interference we get

$$\mathrm{SER}_k^m = \mathrm{Q}\left(\sqrt{\frac{\sigma_x^2}{\left(\sigma_k^m\right)^2}}\right) = \mathrm{Q}\left(\sqrt{\mathrm{SNR}_k^m}\right). \quad (23)$$

Here, we defined $\mathrm{Q}(t) = \int_x^\infty \frac{1}{2\pi} \exp\left(-\frac{t^2}{2}\right) \mathrm{d}t$.

The analytical curves resulting for serial cancellation are depicted in Fig. 5 for one iteration and in Fig. 6 for four iterations. In both plots $K/N = 1$ is chosen and the simulative results are obtained for $N = 64$. Further, the number of paths is assumed to be $L = 1, 2, 5$. For the sake of clarity we omitted the SUB's in Fig. 5.

The graphs show for the parameters under consideration a tight approximation of the simulations by the analytical results. Considering the plots we see the gap between the analytical and the numerical curves is less than 1 dB after the first iteration and gets smaller with increasing E_b/N_0. Moreover, carrying out four iterations the graphs of the analytical and simulated curves virtually merge for sufficiently large E_b/N_0. Furthermore we would like to stress that the analytical approximations reach the corresponding SUB's in the same way like the actual results. This indicates, that our approach to model the interference at the output of the MMSE-filter by a Gaussian distribution and to set the off diagonal elements of the matrix $\boldsymbol{E}_k^m$ to zero is reasonable for the parameters chosen. Finally, although we omitted the graphs for parallel cancellation the same results can be found there, too.

At the end of this section, let us turn to the question whether the main features of the receiver discussed above are also valid for arbitrary power delay profiles and not only equal gain multipath channels.

First, note that each arbitrary power delay profile can be regarded as linear superposition of equal gain profiles. More simply, representing the unequal gain power delay profile of interest having L' taps by $\boldsymbol{h}_{L'}^{\mathrm{int}} = \left(h_0^{\mathrm{int}}, \ldots, h_{L'-1}^{\mathrm{int}}\right)^T$ and the equal gain profile of same length but consisting of L consecutive nonzero taps only by $\boldsymbol{h}_L = (h_0, \ldots, h_{L-1}, 0, \ldots, 0)^T$, $\boldsymbol{h}_L \in \mathbb{C}^{L' \times 1}$, we can write in matrix notation

$$\boldsymbol{h}_{L'}^{\mathrm{int}} = \boldsymbol{\lambda}^T (\boldsymbol{h}_1, \ldots, \boldsymbol{h}_{L'}) = \boldsymbol{\lambda}^T \boldsymbol{H}. \quad (24)$$

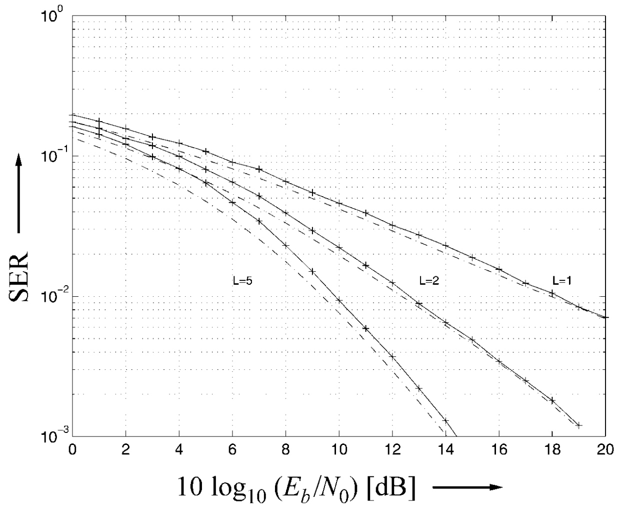

Fig. 5. SER vs. $10\log_{10}(E_b/N_0)$ for studied scheme with serial cancellation, parameter $K/N = 1$, simulated ($-+$) and analytical ($-\cdot$) results after first iteration.

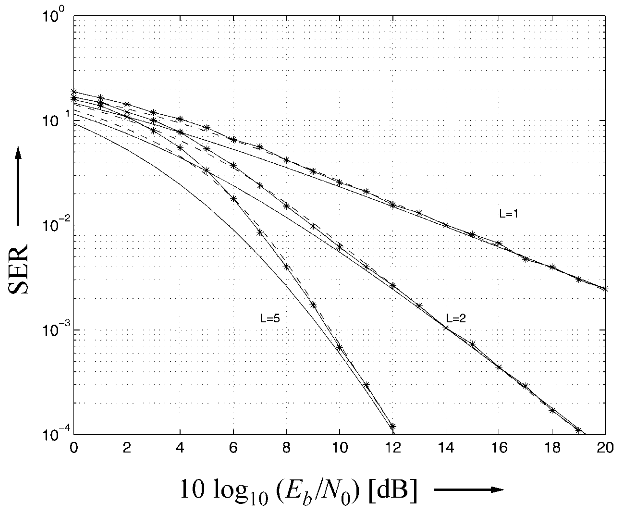

Fig. 6. SER vs. $10\log_{10}(E_b/N_0)$ for studied scheme with serial cancellation, parameter $K/N = 1$, simulated ($-*$) and analytical ($-\cdot$) results after four iterations.

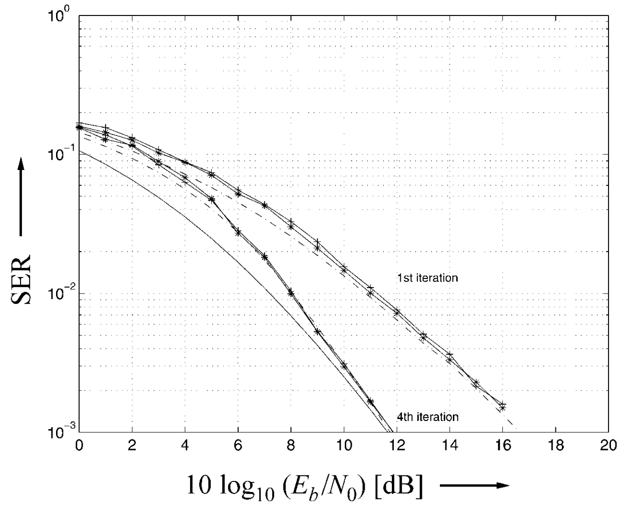

Fig. 7. SER vs. $10\log_{10}(E_b/N_0)$ for studied scheme with serial cancellation, parameter $K/N = 1$, three path ($-+$) as well as five path ($-*$) channel, analytical approximation ($-\cdot$) and SUB ($-$) after first and fourth iteration.

So, as all rows of $\boldsymbol{H}$ are linearly independent it is always possible to find a column vector $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_{L'})^T$ solving this equation.

Second, considering our above derivation the achievable symbol error ratios only depend on the pdf of the sum of squared absolute magnitudes of all taps. More explicitly, as long as the pdf $f_{a^2}(\alpha)$, where $a^2 = \sum_{\lambda=0}^{L-1} |h_{\lambda,\cdot}|^2$, is equal for two different multipath models, the receiver leads to the same SER^m. In Fig. 7 this is illustrated by comparison of a three path model with variances $\sigma_{0,h}^2 = 1/2, \sigma_{1,h}^2 = 1/3, \sigma_{2,h}^2 = 1/6$, where $\sigma_{\lambda,h}^2 = \mathcal{E}\{|h_{\lambda,\kappa}|^2\}, \forall\kappa$, to a five path channel with $\sigma_{0,h}^2 = 1/6$, $\sigma_{2,h}^2 = 1/3$, $\sigma_{4,h}^2 = 1/2$ as well as $\sigma_{1,h}^2 = \sigma_{3,h}^2 = 0$. (That is, the autocorrelation function of the gain of propagation path λ is now $\mathcal{E}\left\{h_{\lambda,\kappa}[\mu]h_{\lambda,\kappa}^*[\mu+\Delta\mu]\right\} = \sigma_{\lambda,h}^2 J_0(2\pi f_D T_c \Delta\mu), \forall\kappa$.) Further, we assumed $K = N = 64$.

The plot clearly shows, that transmission over both multipath channels leads to equal SER^m for given E_b/N_0. Furthermore, solving the pdf of the users' powers as (see [17])

$$f_{a^2}(\alpha) = \left(9\mathrm{e}^{-2\alpha^2} - 12\mathrm{e}^{-3\alpha^2} + 3\mathrm{e}^{-6\alpha^2}\right) U(\alpha), \quad (25)$$

we can derive the single user bound

$$\mathrm{SER} = \frac{9}{4}\left(1 - \frac{1}{\sqrt{1+2N_0/E_b}}\right) - 2\left(1 - \frac{1}{\sqrt{1+3N_0/E_b}}\right) + \frac{1}{4}\left(1 - \frac{1}{\sqrt{1+6N_0/E_b}}\right),$$

as well as the analytical approximation for the receiver's performance. Considering the corresponding curves depicted in the graph, we conclude that our results can be generalized to multipath channels with arbitrary power delay profiles if $N \gg L$.

Moreover, assuming $K \gg 1$ it can also be shown that the transmission with QPSK over a complex fading channel is equivalent to transmission with BPSK over a real valued fading channel if the users' fading amplitudes are chosen appropriately.

4. Conclusions

We have seen that data symbol detection can be carried out for multiple users almost as reliable as for a single user by means of iterative soft detection in conjunction with successive interference cancellation. This indicates that similar techniques should lead to good results also for (adaptive) multiuser channel estimation and/or decoding and even multiple antenna receivers, too. So, first near optimum results where recently reported in [11, 18, 19] for parallel interference suppression by means of a bank of matched filters and an appropriately adapted bank of MMSE-filters, respectively, combined with iterative single user decoding using a MAPSSE-algorithm. Here, it turns out that serial cancellation is by far preferable to parallel cancellation. This holds for the application of matched filters as well as adapted MMSE-filters. Finally, we found recently that coded transmission can be treated analytically in the same way merely replacing the soft decision function by the statistics of the code employed.

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