Hierarchical near–EEP Codedesign for improved decoding performance

Simon Huettinger and Johannes Huber
Lehrstuhl für Nachrichtentechnik II, Universität Erlangen-Nürnberg
Cauerstr. 7, D-91058 Erlangen, Germany
Tel.: +49-9131-852-7116, Fax: +49-9131-852-8919, Email: simon.huettinger@LNT.de

Abstract — **In this paper we propose random coding bounds as a tool to analyze concatenated codes constructed from convolutional inner and Reed–Solomon outer codes. Using theses bounds we are able to construct hierarchically concatenated codes with nearly equal error protection. These codes can provide a significantly higher power efficiency than a conventional design.**

I. Introduction

Concatenated codes have long been used as a practical means of combation noisy channel. With moderate complexity long block codes can be constructed, decoded and high power efficiencies can be achieved.

Up to now most attention is payed to extremely regular codes, as they protect all information symbols equally. Hence, codes are searched for, that have a high minimum hamming distance between the codewords. But as the suboptimal decoding of concatenated codes performed by iteratively using decoders for the component codes, can not realize the power efficiency that could be achieved if maximum likelihood decoding were applied, irregular code structures can be an advantage. Work on irregular Gallager codes (low density parity check codes) [5] and irregular Turbo–Codes [3] has shown that power efficiency can be improved significantly.

In this paper we investigate irregular concatenations of Reed–Solomon codes and convolutional codes. The irregularity is seen within a hierarchical model, in which the protection of the important information bits and the unimportant information bits is optimized in a way, that the error rate of the unimportant bits does not dominate the total block error rate, but is only slightly higher than the error rate of the important information bits. This is called near–EEP.

The reference code, the well known ESA standard code for telemetry transmission [1], is investigated in section II. In section III an irregular hierarchical version is introduced, analyzed by means of random coding error exponents and a code with significantly improved power efficiency is constructed.

II. The ESA Telemetry Channel Code

The ESA standard code for telemetry transmission [1] uses an interleaved concatenation of an outer Reed–Solomon code with a convolutional inner code.

The Reed–Solomon code RS($N=255$,$K=223$) over GF(2^8) consists of 223 information symbols with $q=8$ bits each. The error correcting capability is 16 symbol errors due to the 32 redundancy symbols. Equivalently, 32 erasures can be corrected.

Interleaving is done by a symbol oriented block interleaver over $I=8$ RS–Codewords. Hence, a frame comprises $I \cdot q \cdot K = 8 \cdot 8 \cdot 223 = 14272$ information bits.

The interleaved RS–symbols are convolutionally encoded. The used convolutional code (CC) is build by the generator polynomials $g_1 = 171_8$ and $g_2 = 133_8$. This code of rate $R_i = 1/2$ has a constraint length $\nu = 7$ and a free distance $d_{free} = 10$. The output of g_2 is inverted.

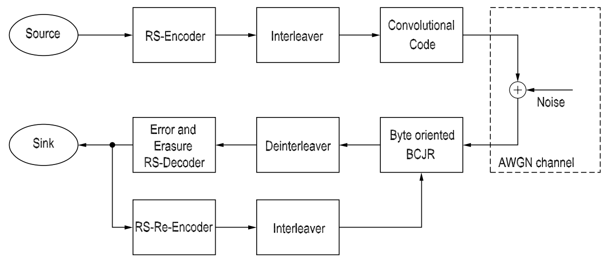

Figure 1: Blockdiagram of a Transmission scheme with the ESA telemetry code.

Optimum decoding of the inner convolutional code is done by BCJR–decoding [2] of a pseudo–code with virtually increased memory m. If $m = q$ the optimum soft–output for the RS–codesymbols of length q can be calculated from the state probabilities obtained during BCJR–decoding. The soft–values for the RS–codesymbols then can be used in an error–and–erasure decoder for the Reed–Solomon code. Using these component decoders iteratively already leads to power efficiencies only at about 1.7 dB apart from the Shannon bound for error free transmission [6].

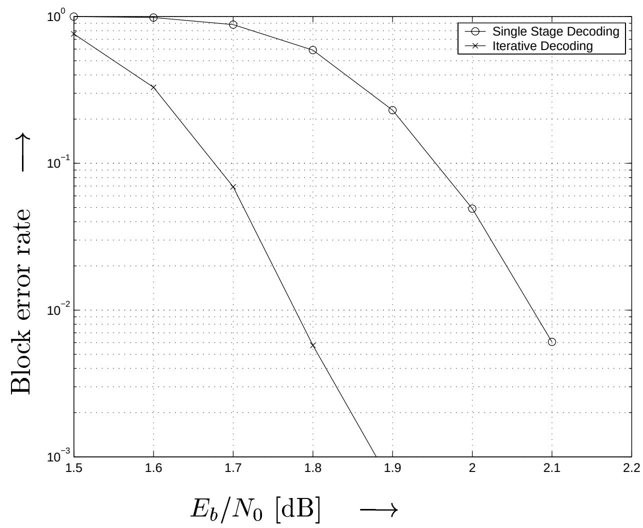

Figure 2: Performance of the ESA Telemetry Codes with soft–out BCJR–Decoding and Error and Erasure RS–Decoding.

III. Hierarchical Redesign

Simulation results (as presented e.g. in [4]) indicated that a construction based on RS–codewords with different amounts of redundancy can achieve an even higher power efficiency. To investigate this, a code with alternating highly protected RS–codewords and RS–codeword with only a small amount of redundancy as depicted in Fig. 3 is seen as a hierarchical channel code.

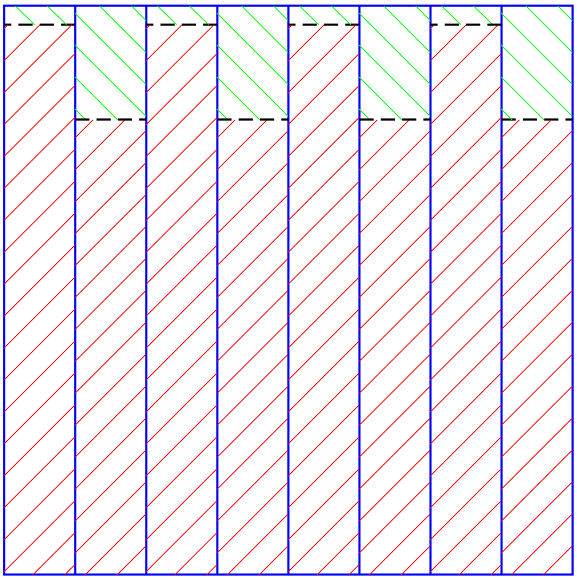

Figure 3: Interleaving for a hierarchically concatenated coding scheme (RS–codewords vertically, upper part is redundancy, lower part information).

Decoding then is performed in a hybrid iterative/multistage way. Firstly only for the highly protected RS–codeword within a frame that are called “Level 0” are decoded. The less protected RS–codewords on “Level 1” are decoded afterwards using the knowledge of Level 0 as a–priori information. But, during the decoding of Level 0 iterations between the inner convolutional code and the outer RS–code are performed. Iterative decoding of Level 1 is principally impossible within this coding scheme, as we will show later.

Hence, there can occur two types of frame errors. Either the Level 0 can not be decoded, which is the worst case, as it also leads to a decoding failure of Level 1 with very high probability — here assumed with probability one — or Level 1 results in a decoding failure, even if Level 0 was successfully decoded.

In the following the probabilities of both sources of error are estimated from random coding bounds, to easily be able to construct a hierarchical scheme that does not have a frame error rate dominated by failure of Level 1, but a nearly equal error protection of both levels. This is possible if the probability of failure $P_{W,L1}$ of Level 1 if Level 0 has been decoded successfully is much less than the probability of a failure while decoding Level 0 ($P_{W,L0}$).

The estimation of the block error rate of a concatenation of I interleaved RS–codewords RS(N,K) of GF(2^q) and a convolutional code with the parameters constraint length ν and rate R_i can be done quite accurate without knowledge on the used generator polynomials.

For random coding on a channel with input alphabet $\boldsymbol{X}$, being distributed according to $p_x(x)$, output alphabet $\boldsymbol{Y}$ and the transition probabilities $p_y(y|x)$ the Gallager error exponent is calculated by:

$$E(R) = \max_{0\leq\rho\leq 1} \{E_0(\rho) - \rho R\} \quad (1)$$

$$E_0(\rho) = -\log_2 \left(\int_{\boldsymbol{Y}} \left[\int_{\boldsymbol{X}} p_x(x) p_y(y|x)^{\frac{1}{1+\rho}} dx \right]^{1+\rho} dy \right) \quad (2)$$

For binary signaling with $\boldsymbol{X} = \{0,1\}$ Eq. (2) can be simplified to:

$$E_0(\rho) = -\log_2 \left[\int_{-\infty}^{\infty} \left(\frac{1}{2} p_y(y|1)^{\frac{1}{1+\rho}} + \frac{1}{2} p_y(y|0)^{\frac{1}{1+\rho}} \right)^{1+\rho} dy \right] \quad (3)$$

Starting from the Gallager–function (see Eq. 1), which is optimized to the signal to noise ratio of the channel by the best choice of $0 \leq \rho \leq 1$, the bit error probability P_b after Viterbi–decoding a convolutional code can be bounded as follows [7]:

$$P_b \leq \frac{2^{-\nu E(R)/R}}{\left[1 - 2^{-E(R)/R-\rho}\right]^2} \quad (4)$$

To achieve an upper bound on the word error probability P_W of the RS–codes the worst case of ideal bit–interleaving is assumed. This leads to:

$$P_s = 1 - (1 - P_b)^q \quad (5)$$

$$P_W = \sum_{i=t+1}^{N} \binom{N}{i} P_s^i (1 - P_s)^{N-i} \quad (6)$$

The performance of iterative decoding generally is very difficult to estimate, but in the investigated coding scheme a very obvious upper bound on the word error rate coincides with the results found in simulations. If the first decoding attempt or the first iteration does not lead to any correctly decoded RS–codeword, there is no extrinsic information that can be passed back to the inner convolutional code. Hence, the decoding can be stopped, as all further iterations will lead to exactly the same result as the first one. As the number of words on Level 0 is $I/2$ a block failure occurs if decoding fails for all $I/2$ RS–codewords.

$$P_B \geq (P_W)^{I/2} \quad (7)$$

The bound of Eq. (7) is very sharp and hence, with the framework given above, the performance of the ESA telemetry code as well as the performance of Level 0 of a hierarchically concatenated scheme can be estimated. The comparison between simulation and estimation is shown in Fig. 4 for the ESA telemetry code (RS(255,223)) and two other concatenated codes that are based on the same convolutional inner code, but use the outer codes RS(255,197) and RS(255,249). Even if the estimation shows a strong offset of nearly 1 dB in E_b/N_0 the relative position of the waterfall regions of the concatenated coding schemes is predicted very well. Furthermore the steepness of declension of the block error rate is estimated quite accurately.

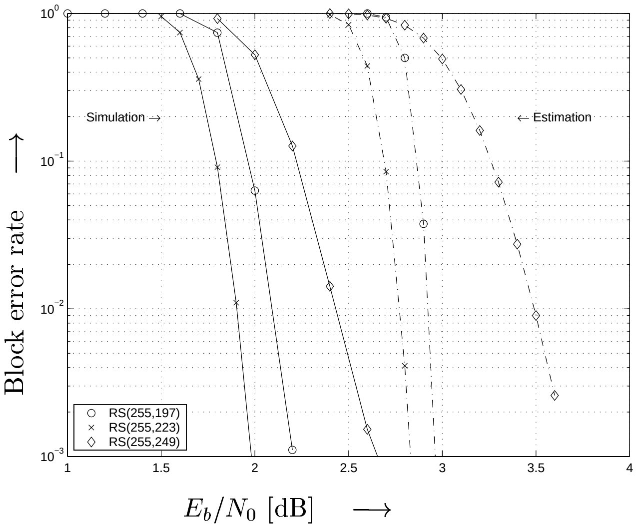

Figure 4: Comparison of Estimation and Simulation of the concatenation of RS(255,197), RS(255,223) and RS(255,249) with a inner convolutional code using Viterbi–decoding of the inner code.

The estimation of the probability of block errors caused by errors on Level 1 after a correct decoding of Level 0 can be estimated quite similar. But as the decoding of the convolutional code can be performed for each RS–codesymbol of Level 1 independently, if Level 0 is known, iterative decoding on Level 1 can not achieve any gain and hence can be neglected.

The knowledge of Level 0 within the convolutional code leads to separated codes for every RS–codesymbol on Level 1, if $\nu - 1 \leq q$. Due to the fact, that an increased constraint length only improves the power efficiency of the concatenated coding scheme, if the RS–code can correct longer error bursts, the case $\nu - 1 > q$ is of no practical importance. As the q information bits preceding the RS–codesymbol are known, the code starts in a predefined state. And as the q known subsequent information bits can be seen as a termination, the code is blocked within two RS–codesymbols of Level 0 and one of Level 1. This is depicted in Fig. 5.

This short terminated convolutional code is a block code of rate $R = \frac{R_i q}{\nu - 1 + q}$ and length $n = \frac{\nu - 1 + q}{R_i}$. It's error rate that corresponds to a symbol error rate on Level 1 can also be estimated using random coding bounds:

$$P_s \quad \leq \quad 2^{-nE_0(R)} \tag{8}$$

Any word error of this short block code leads to a symbol error of a RS–codeword on Level 1 This leads to an RS–codeword error rate on Level 1 of :

$$P_{W,L1} \quad = \quad \sum_{i=t+1}^{N} \binom{N}{i} P_s^i (1 - P_s)^{N-i} \tag{9}$$

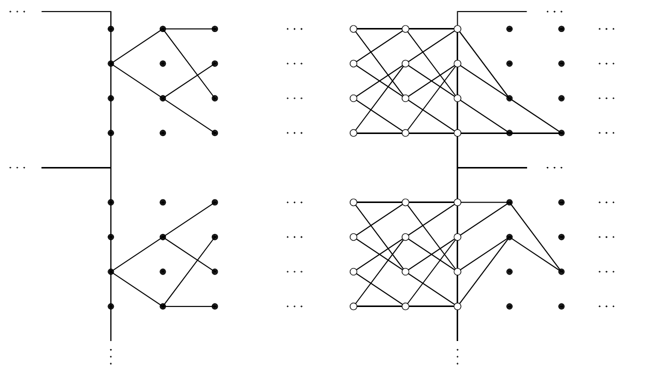

Figure 5: Trellis of a RS–symbol on Level 1 if Level 0 is known.

As the RS–codesymbols on Level 1 are protected by independent inner codes, iterative decoding can not be applied to the decoding of Level 1. Furthermore the symbol error events as well as the RS-codeword error events on Level 1 are uncorrelated. Hence, the block error rate $P_{B,L1}$ on Level 1 can be calculated by:

$$P_{B,L1} \quad = \quad 1 - (1 - P_{W,L1})^{I/2} \tag{10}$$

With the estimations given above the error rates of both sources can be calculated for every distribution of redundancy between the Levels. In Fig.6 this is done for a signal to noise ratio $E_b/N_0 = 1.5dB$ fixing the total code rate $R = (R_{L0} + R_{L1})/2 = 0.875$

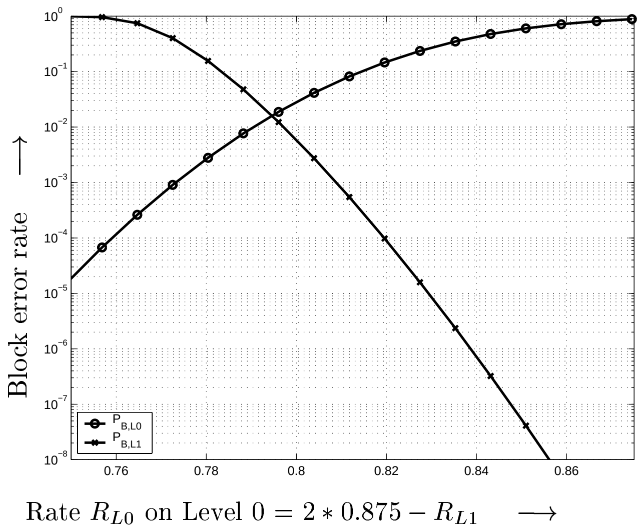

Figure 6: Plot of error rates for both levels for $E_b/N_0 = 2.2dB$.

In the region, where the curve for $P_{B,L1}$ is below the curve for $P_{B,L0}$ the overall block error rate is dominated by Level 0. Hence we call a code design that fulfills Eq. (11) near–EEP. Even if it still can happen, that after a decoding success of Level 0 an block error occurs due to an failure of Level 1, this case is not very likely to happen.

$$P_{W,L1} \quad < \quad P_{W,L0} \tag{11}$$

The code design that can achieve the minimum mean block error rate is if R_{L0} is chosen to be 0.77. Then the codes RS(255,203) and RS(255,243) are used as outer codes in the near–EEP hierarchically concatenated coding scheme instead of the RS(255,223) of a truly EEP coding scheme.

Fig. 7 shows that the hierarchical design outperforms the conventional EEP code design significantly. Without any changes in complexity a gain of about 0.3 dB is achieved.

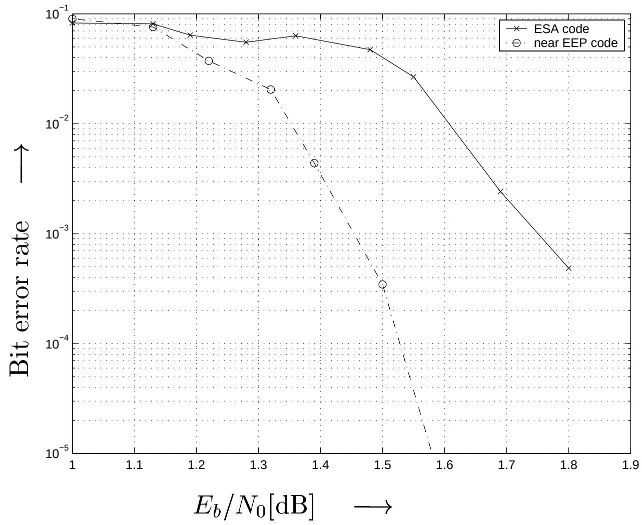

Figure 7: Performance–comparison of the EEP and hierarchical near–EEP code–concatenation.

IV. Conclusion

With the exemplary design of a near–EEP hierarchical channel code using the parameters of the ESA code for telemetry transmission, we have shown that it is possible to find codes that significantly outperform an EEP–codedesign in the average error rate. Furthermore a design guided by random coding bounds proved to be very practical, as the optimum rate distribution between the Levels could be determined without any simulation.

References

[1] European Space Agency. *PSS-04-103 Telemetry Channel Coding Standard.* Sept. 1989.

[2] L.R. Bahl, J. Cocke, F. Jelinek, and J. Raviv. Optimal decoding of linear codes for minimizing symbol error rate. *IEEE Transactions on Information Theory*, IT-20:pp.284–287, 1974.

[3] B. Frey and D. MacKay. Irregular turbocodes. In *Proc. of the 37th Allerton Conference on Communication, Control and Computiong*, Allerton House, Illinois, 1999.

[4] A. Klindt and E. Paaske. *Generalized Concatenated Coding System in Relation to CCSDS Standard.* Technical Report ISSN 0105–8541 Report IT–132, Institute of Circuit Theory and Telecommunication, Aug. 1992.

[5] M. Luby, M. Mitzenmacher, M. Shokrollahi, and D. Spielmann. Improved low–density parity–check codes using irregular graphs and belief propagation. In *Proceedings of International Symposium on Information Theory*, 1998.

[6] C.E. Shannon. A mathematical theory of communication. *Bell System Technical Journal*, vol.27:part I, pp. 379–427; part II, pp. 623–656, 1948.

[7] A. J. Viterbi. Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. *IEEE Transactions on Information Theory*, IT-13:pp.260–269, 1967.