Decision–Feedback Differential Detection Based on Linear Prediction for 16DAPSK Signals Transmitted Over Flat Ricean Fading Channels

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Abstract

In this paper, prediction–based decision–feedback differential detection (DF–DD) for 16–level differentially encoded amplitude/phase–shift keying (16DAPSK) is proposed. Unlike previously reported DF–DD schemes, this scheme provides a performance gain over conventional differential detection (DD) under general Ricean fading conditions. A further important advantage of the novel scheme is that it is able to compensate a small carrier frequency offset. The linear predictor coefficients may be updated using the recursive least–squares (RLS) algorithm, which can start blind, i.e., without a priori knowledge about the channel statistics and without a training sequence. This makes the scheme attractive for application in mobile communications since the statistics of a nonstationary mobile channel can be tracked.

Keywords:

Decision–feedback differential detection (DF–DD), noncoherent detection, 16DAPSK, fading channels, BER analysis.

This work has been supported by TCMC (Technology Centre for Mobile Communications), Philips Semiconductors, Germany, and has been presented in part at the IEEE Vehicular Technology Conference (VTC), Amsterdam, The Netherlands, September, 1999.

1 Introduction

Recently, 16–level differentially encoded amplitude/phase–shift keying (16DAPSK) has gained high attention [1, 2, 3, 4, 5, 6, 7, 8] in research as well as in practice. A possible application field for 16DAPSK are future personal communication systems which will require bandwidth efficient modulation schemes because of an increasing demand for high data rate services. Especially for mobile communications, differential detection (DD) of 16DAPSK is very attractive because of its simplicity of implementation and its robustness against phase variations [1, 3]. The main drawback of conventional DD when applied to DAPSK signals is the significant loss in power efficiency under additive white Gaussian noise (AWGN) conditions in comparison to coherent detection (CD). Therefore, improved noncoherent detection schemes have been proposed in literature. Divsalar et al. [9], Suzuki et al. [2], and Machida et al. [7] reported three different multiple–symbol detection (MSD) schemes. However, these MSD schemes are not well suited for implementation because of their complexity. Complexity is reduced significantly by using decision–feedback differential detection (DF–DD). For 16DAPSK different DF–DD schemes have been proposed in literature [3, 5, 6, 8]. Although 16DAPSK is especially attractive for application in fading channels [1], these DF–DD schemes are designed for the AWGN channel and under fast fading conditions they perform even worse than conventional DD. For M–ary differential phase–shift keying (MDPSK) and continuous phase modulation (CPM) it has been shown that for Rayleigh fading, the maximum–likelihood (ML) receiver inherently employs linear prediction [10, 11, 12, 13, 14, 15]. DF–DD for these modulation formats and the Rayleigh fading channel has been proposed in [16, 17]. In [18, 19] it is demonstrated that prediction–based DF–DD for MDPSK is very efficient in general Ricean fading channels. Thus, here we propose to apply prediction–based DF–DD also to 16DAPSK.

2 Transmission Model

We assume transmission over a flat Ricean fading channel and assume that the discrete–time received signal samples can be written as

$$r[k] = \mathrm{e}^{j\Theta}\mathrm{e}^{j2\pi\Delta f T k} f[k]s[k] + n[k], \qquad (1)$$

where Θ, Δf, T, $f[\cdot]$, $s[k]$, and $n[\cdot]$ denote unknown constant phase shift, carrier frequency offset, symbol duration, Ricean fading coefficient, transmitted 16DAPSK symbol [1], and white Gaussian noise, respectively.

$s[k] = R[k]b[k]$ is the product of an amplitude symbol $R[k] \in \mathcal{A}_R = \{R_H, R_L\}$, $R_H/R_L \triangleq a_0 > 1$, and an 8PSK symbol $b[k]$. $R[k]$ and $b[k]$ are obtained via differential encoding: $R[k] = \Delta R[k]R[k-1]$, $b[k] = a[k]b[k-1]$. Here, $\Delta R[k]$ and $a[k]$ denote amplitude and phase difference symbols, respectively.

$f[\cdot]$ and $n[\cdot]$ are mutually uncorrelated. $f[k]$ has power $q_f^2 = \mathcal{E}\{|f[k]|^2\} = E_S/T$ and $n[k]$ has variance $\sigma_n^2 = \mathcal{E}\{|n[k]|^2\} = N_0/T$. Here, $\mathcal{E}\{\cdot\}$ denotes expectation and E_S is the mean received energy per symbol, whereas N_0 is the single–sided power spectral density of the underlying passband noise process. The Ricean fading process is characterized by the normalized fading bandwidth $B_f T$ of its scattered component, the normalized Doppler shift $f_D T$ of its line–of–sight component, and the Ricean factor K [20].

3 Prediction–Based DF–DD Scheme

In this section, prediction–based DF–DD for 16DAPSK is introduced. The received 16DAPSK signal sample $r[k]$ may be written as $r[k] = \mathrm{e}^{j\Theta}\mathrm{e}^{j2\pi\Delta f T k} f[k]s[k-1]a[k]\Delta R[k] + n[k]$. For optimum coherent detection (CD) of $a[k]$ and $\Delta R[k]$, the reference value

$$r_{\mathrm{ref}}[k-1] \triangleq \mathrm{e}^{j\Theta}\mathrm{e}^{j2\pi\Delta f T k} f[k]s[k-1] \tag{2}$$

has to be known. For conventional DD [1], $r[k-1]$ is used as an estimate for $r_{\mathrm{ref}}[k-1]$ and because of the noisy character of $r[k-1]$, a performance degradation compared to CD is inevitable. Here, we propose to estimate $r_{\mathrm{ref}}[k-1]$ not only from $r[k-1]$ but from the last $N-1$ observed symbols $r[k-\nu]$, $1 \le \nu \le N-1$, i.e., the decision is based on N received signal samples. The coefficients $p_\nu^s[k]$, $1 \le \nu \le N-1$, of the proposed linear estimator are obtained by minimizing the mean–squared error (MSE) between $r_{\mathrm{ref}}[k-1]$ and an *estimated* reference value

$$r_e[k-1] \triangleq \sum_{\nu=1}^{N-1} p_\nu^s[k] r[k-\nu]. \tag{3}$$

For calculation of the estimator coefficients we assume that $\Delta R[k]a[k]$ is known at the receiver. Thus, the resulting coefficients for estimation of $r_{\mathrm{ref}}[k-1]$ are also the solution to the problem of estimating $\Delta R[k]a[k]r_{\mathrm{ref}}[k-1]$ from $\Delta R[k]a[k]r_e[k-1]$ with minimum MSE. Since $n[\cdot]$ is an uncorrelated Gaussian random process, also the same estimator coefficients are obtained for estimation of $\Delta R[k]a[k]r_{\mathrm{ref}}[k-1] + n[k] = r[k]$ from $\Delta R[k]a[k]r_e[k-1]$, i.e., the estimator coefficients can be designed to minimize the error variance $\sigma_{\mathrm{MSE}}^2(\boldsymbol{s}[k]) = \mathcal{E}\{|r[k] - \Delta R[k]a[k]r_e[k-1]|^2\}$ ($\boldsymbol{s}[k] \triangleq [s[k]\ s[k-1]\ \ldots\ s[k-N+1]]^T$, $[\cdot]^T$ denotes transposition). If $p_\nu^s[k]$ is replaced by

$$p_\nu^s[k] \triangleq p'_\nu[k]s[k-1]/s[k-\nu] = p'_\nu[k]\prod_{\mu=1}^{\nu-1}\Delta R[k-\mu]a[k-\mu], \qquad 1 \le \nu \le N-1, \tag{4}$$

it can be shown that minimizing $\sigma_{\mathrm{MSE}}^2(\boldsymbol{s}[k])$ is identical to determining the coefficients $p'_\nu[k]$, $1 \le \nu \le N-1$, of a linear Wiener predictor [21] for the process

$$c[k] \triangleq \mathrm{e}^{j2\pi\Delta f T k} f[k] + \mathrm{e}^{-j\Theta} n[k]/s[k]. \tag{5}$$

In order to obtain time–invariant estimator coefficients p_ν, $1 \leq \nu \leq N-1$, from now on the $s[\cdot]$ in Eq. (5) is treated as a random variable. The additional averaging over $s[\cdot]$ is an important difference to the MDPSK case [18, 19]. The resulting time–invariant predictor (estimator) coefficients p_ν, $1 \leq \nu \leq N-1$, can be obtained from the *Yule–Walker equations* [21]. Alternatively, the recursive least–squares (RLS) algorithm can be applied using a similar approach as in [17] and [19] for CPM and MDPSK, respectively (cf. [22]).

In practice, the transmitted symbols $\Delta R[k-\nu]$, $a[k-\nu]$, $1 \leq \nu \leq N-2$, required in Eq. (4) are not known at the receiver and have to be replaced by previously detected symbols $\Delta\hat{R}[k-\nu]$, $\hat{a}[k-\nu]$, $1 \leq \nu \leq N-2$. Thus,

$$\hat{r}_e[k-1] = \sum_{\nu=1}^{N-1} p_\nu \prod_{\mu=1}^{\nu-1} \Delta\hat{R}[k-\mu]\hat{a}[k-\mu]r[k-\nu] \tag{6}$$

has to be used as an estimate for the reference value $r_{\mathrm{ref}}[k-1]$ instead of $r_e[k-1]$.

Using $\hat{r}_e[k-1]$, amplitude and phase decision are based on the decision variables $g_P[k] = r[k]\hat{r}_e^*[k-1]$ and $g_A[k] = |r[k]/\hat{r}_e[k-1]|$, respectively (cf. [22] for details).

4 BER Analysis for Genie–Aided DF–DD

The BER analysis is restricted to genie–aided prediction–based DF–DD since it is difficult to take into account the effect of error propagation. This means for the analysis presented here, $R[k-1]$ [1], $\Delta R[k-\nu]$, $1 \leq \nu \leq N-2$ (or equivalently $R[k-\nu]$, $1 \leq \nu \leq N-1$), and $a[k-\nu]$, $1 \leq \nu \leq N-2$, are assumed to be known at the receiver when a decision is made on $\Delta R[k]$ and $a[k]$, i.e., $r_e[k-1]$ (cf. Eqs. (3), (4)[2]) is used instead of $\hat{r}_e[k-1]$ (cf. Eq. (6)). Amplitude and phase detection are independent of each other and thus, the BERs for both cases can be determined separately.

Phase Detection

For calculation of the BER for the bits mapped to the phase difference symbols $a[\cdot]$, we use a similar approach to Adachi in [3, 4], i.e., we apply the results of Pawula et al. [23]. The total BER for the phase bits P_{pha}^{tot} is given by

$$P_{pha}^{tot} = \frac{1}{2^N} \sum_{\boldsymbol{R}[k]} P_{pha}(\boldsymbol{R}[k]), \tag{7}$$

where $\boldsymbol{R}[k]$ is defined as $\boldsymbol{R}[k] \triangleq [R[k]\ R[k-1]\ \ldots\ R[k-N+1]]^T$, and $P_{pha}(\boldsymbol{R}[k])$ is the BER for the phase bits for a given $\boldsymbol{R}[k]$. Without loss of generality, for the following, it is assumed that $b[k] = b[k-1] = 1$ (i.e., $a[k] = 1$) are transmitted. For convenience, the variables

$$x[k] \triangleq r[k]\Big|_{b[k]=1} = \mathrm{e}^{j\Theta}\mathrm{e}^{j2\pi\Delta fTk} f[k]R[k] + n[k], \tag{8}$$

[1]Knowledge of $R[k-1]$ is necessary for the amplitude decision [22].

[2]Note that in Eq. (4) $p'_\nu[k]$ has been replaced by p_ν, $1 \leq \nu \leq N-1$.

$$y[k] \;\stackrel{\triangle}{=}\; r_e[k-1]\Big|_{b[k-1]=1} = \sum_{\nu=1}^{N-1} p_\nu \left(\mathrm{e}^{j\Theta}\mathrm{e}^{j2\pi\Delta f T(k-\nu)} f[k-\nu] + \frac{n[k-\nu]}{s[k-\nu]} \right) R[k-1] \quad (9)$$

are introduced. Ideally, the phase difference $\Delta\Phi$ between $x[k]$ and $y[k]$ is zero because transmission of $a[k]=1$ is assumed. Thus, according to [4, 23] the probability that $\Delta\tilde{\eta} = \arg\{x[k]y^*[k]\} = \arg\{r[k]r_e^*[k-1]\}$ [3] lies between $\Delta\eta$ and $-\pi$ is given by

$$\Pr\{\Delta\eta \geq \Delta\tilde{\eta} \geq -\pi\} = F(\Delta\eta,\, \boldsymbol{R}[k]) - F(-\pi,\, \boldsymbol{R}[k]) + u(\Delta\eta), \quad (10)$$

with $u(t)=1$ if $t \geq 0$ and $u(t)=0$ otherwise. $F(\Delta\eta,\, \boldsymbol{R}[k])$ can be calculated from the first and second order moments of $x[k]$ and $y[k]$ (conditioned on $\boldsymbol{R}[k]$) as described in [4, 23]. Since the phase difference symbols $a[\cdot]$ are Gray encoded, $P_{pha}(\boldsymbol{R}[k])$ can be expressed as [4]

$$P_{pha}(\boldsymbol{R}[k]) = \frac{1}{3}\left(F(-\frac{\pi}{8},\, \boldsymbol{R}[k]) - F(\frac{\pi}{8},\, \boldsymbol{R}[k]) + F(-\frac{3\pi}{8},\, \boldsymbol{R}[k]) - F(\frac{3\pi}{8},\, \boldsymbol{R}[k]) \right). \quad (11)$$

Amplitude Detection

The BER for the amplitude bits may be calculated according to

$$P_{amp}^{tot} = \frac{1}{2^{N-2}} \sum_{\boldsymbol{R}_2[k]} P_{amp}(\boldsymbol{R}_2[k]), \quad (12)$$

with

$$P_{amp}(\boldsymbol{R}_2[k]) \stackrel{\triangle}{=} 1/4 \cdot \Big(P_{HH}(\boldsymbol{R}_2[k]) + P_{LH}(\boldsymbol{R}_2[k]) + P_{HL}(\boldsymbol{R}_2[k]) + P_{LL}(\boldsymbol{R}_2[k]) \Big), \quad (13)$$

where the definition $\boldsymbol{R}_2[k] \stackrel{\triangle}{=} [R[k-2]\; R[k-3]\; \ldots\; R[k-N+1]]^T$ is used. $P_{ij}(\boldsymbol{R}_2[k])$, $i,\, j \in \{L,\, H\}$, denotes the probability that an error is made when $R[k]=R_i$ and $R[k-1]=R_j$ are transmitted. Taking into account the amplitude decision variable $g_A[k] = |r[k]/r_e[k-1]|$ and using the results of [20, Appendix 4B], $P_{ij}(\boldsymbol{R}_2[k])$ can be expressed as

$$\begin{aligned} P_{ij}(\boldsymbol{R}_2[k]) \;&=\; \mathrm{Q_M}\Big(a_{ij}\left(\boldsymbol{R}_2[k]\right), b_{ij}(\boldsymbol{R}_2[k])\Big) - \frac{v_2^{ij}(\boldsymbol{R}_2[k])}{v_1^{ij}(\boldsymbol{R}_2[k]) + v_2^{ij}(\boldsymbol{R}_2[k])} \\ &\cdot \mathrm{I}_0\Big(a_{ij}(\boldsymbol{R}_2[k])\, b_{ij}(\boldsymbol{R}_2[k])\Big) \exp\left(-\frac{a_{ij}^2(\boldsymbol{R}_2[k]) + b_{ij}^2(\boldsymbol{R}_2[k])}{2} \right), \end{aligned} \quad (14)$$

where $\mathrm{Q_M}(\cdot,\cdot)$ and $\mathrm{I}_0(\cdot)$ denote the Marcum Q–function and the zeroth order modified Bessel function of the first kind, respectively. $a_{ij}(\boldsymbol{R}_2[k])$, $b_{ij}(\boldsymbol{R}_2[k])$, $v_1^{ij}(\boldsymbol{R}_2[k])$, and $v_2^{ij}(\boldsymbol{R}_2[k])$ are defined in [20, Appendix 4B] and depend only on the first and second order moments of $x[k]$ and $y[k]$.

Total BER

Since 3 bits are mapped to the phase difference symbols, while 1 bit is mapped to the amplitude ratio, the total BER is $P_b = 1/4 \cdot (3P_{pha}^{tot} + P_{amp}^{tot})$.

[3] $\arg\{\cdot\}$ denotes the phase angle of a complex number.

5 Simulation Results and Discussion

For all presented simulation results the predictor coefficients are adapted using an RLS algorithm with forgetting factor $w = 1.0$ (cf. [19, 22]). For the sake of brevity, in the following we will refer to this adaptive prediction–based DF–DD scheme as *adaptive DF–DD*. Adaptive DF–DD is compared with MSD–based DF–DD [6] and the detector proposed by Wei et al. [5]. Since conventional DD yields almost the same BER as the scheme of Wei et al. [5] with $N = 2$ no simulations for conventional DD are presented.

Fig. 1 shows BER vs. normalized frequency offset ΔfT for an AWGN channel with $10\log_{10}(E_b/N_0) = 15$ dB and a ring ratio of $a_0 = 1.8$. Adaptive DF–DD is unaffected by a constant frequency offset, whereas the other schemes degrade considerably for $\Delta fT > 0$. This degradation is the more severe, the larger N, i.e., there is a trade–off between performance under pure AWGN conditions and robustness against frequency offset. For adaptive DF–DD, however, this trade–off does not exist since the adapted predictor coefficients compensate the frequency offset.

Figs. 2 and 3 show BER vs. $10\log_{10}(E_b/N_0)$ for Rayleigh fading ($B_fT = 0.03$) and Ricean fading ($10\log_{10}(K) = 5$ dB, $f_DT = 0.015$, $B_fT = 0.03$), respectively. A ring ratio of $a_0 = 2.0$ and a fading simulator based on Jakes model [24] are used. The MSD–based scheme is not considered here since it performs poorly under fading conditions. Also the curves for adaptive DF–DD with $N = 2$ are omitted since they are almost identical with the curves for the scheme of Wei et al. with $N = 2$. It can be observed from Figs. 2 and 3 that for adaptive DF–DD the irreducible error floor decreases significantly with increasing N, whereas it increases for the scheme of Wei et al. For comparison also the BER curves for genie–aided adaptive DF–DD (theory and simulation) are shown. At high E_b/N_0 ratios, it can be assumed that no burst errors occur and thus, for $N > 2$ BER increases approximately by a factor of two if detected symbols are fed back instead of correct symbols. On the other hand, at low E_b/N_0 ratios, where burst errors occur, the factor is smaller than two. For $N = 2$ the factor is always smaller than two since in this case the phase decision does not require decision feedback.

Finally, it should be mentioned that in contrast to optimum ML–based receivers (cf. e.g. [12, 14]), for prediction–based DF–DD the mean of the Ricean fading process has not to be known. This leads to a lower complexity of the receiver at the expense of a loss in performance. This becomes apparent e.g. for transmission over an AWGN channel where for small values of N the prediction–based receiver is outperformed by both MSD–based DF–DD and the scheme of Wei et al. (cf. [22]).

6 Conclusions

DF–DD for 16DAPSK signals based on linear prediction has been proposed in this paper. The predictor coefficients may be updated efficiently using the RLS algorithm which makes the scheme particularly interesting for mobile communications. The BER performance for genie–aided prediction–based DF–DD has been analyzed and verified by simulations. The simulation results presented here and in [22] confirm that adaptive prediction–based DF–DD can significantly improve the performance of conventional DD under AWGN, frequency offset, Rayleigh, and Ricean fading conditions.

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Figures:

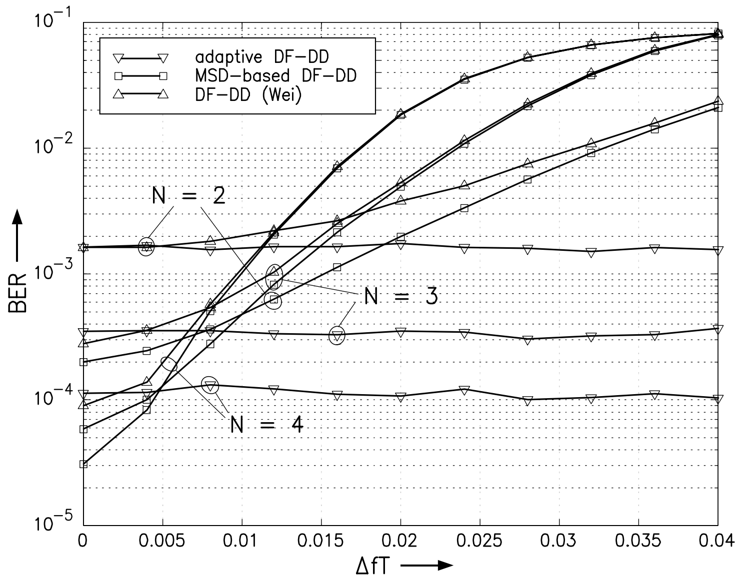

Figure 1: BER vs. ΔfT for DF–DD proposed by Wei et al., MSD–based DF–DD, and adaptive DF–DD. $10\log_{10}(E_b/N_0) = 15$ dB and $a_0 = 1.8$ are valid.

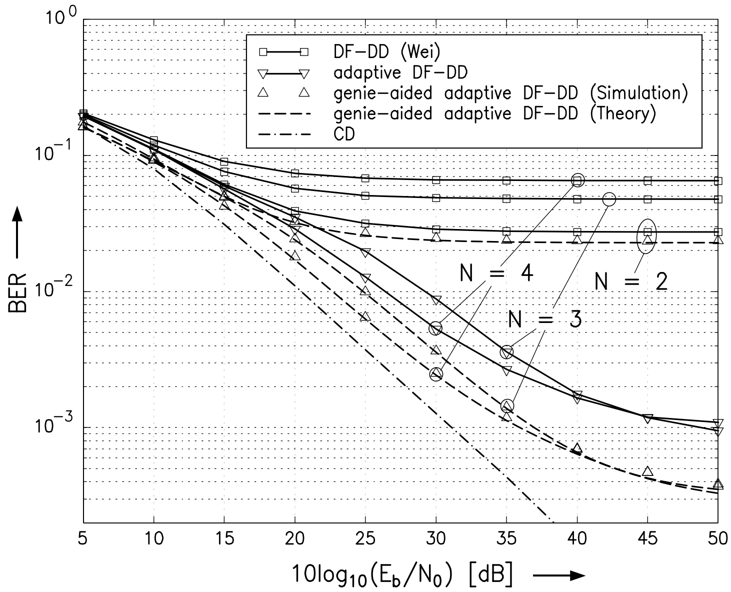

Figure 2: BER vs. $10\log_{10}(E_b/N_0)$ for DF–DD proposed by Wei et al., adaptive DF–DD, genie–aided adaptive DF–DD (simulation and theory), and CD for a Rayleigh fading channel ($B_fT = 0.03$, $a_0 = 2.0$).

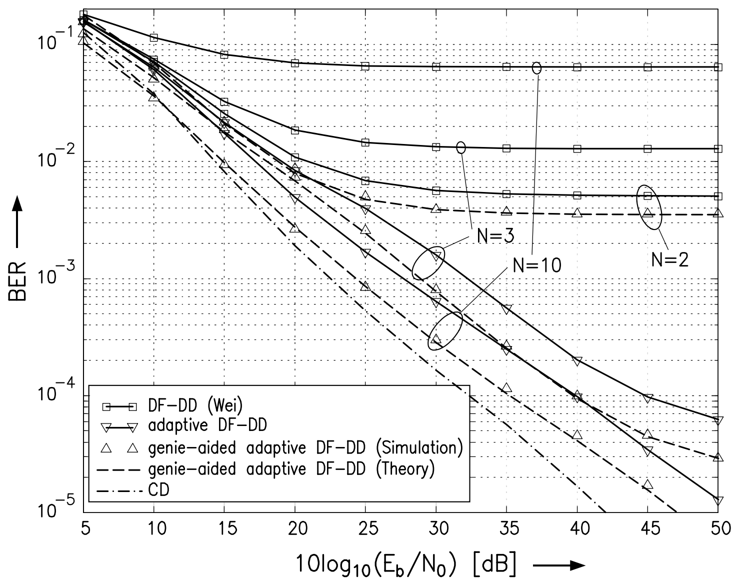

Figure 3: BER vs. $10\log_{10}(E_b/N_0)$ for DF–DD proposed by Wei et al., adaptive DF–DD, genie–aided adaptive DF–DD (simulation and theory), and CD for a Ricean fading channel ($10\log_{10}(K) = 5$ dB, $f_DT = 0.015$, $B_fT = 0.03$, $a_0 = 2.0$).