Adaptive Noncoherent DFE for MDPSK Signals Transmitted Over ISI Channels

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Abstract

In this paper, a novel noncoherent decision–feedback equalization (NDFE) scheme for M–ary differential phase shift–keying (MDPSK) signals transmitted over intersymbol interference (ISI) channels is presented. A suboptimum version with lower computational complexity and a noncoherent linear equalizer (NLE) are derived from the original NDFE scheme. Furthermore, the relation of the novel NLE to a previously proposed NLE is investigated. In contrast to known NDFE schemes, the novel scheme can approach the performance of coherent minimum mean–squared error (MMSE) DFE. For adaptation of the feedforward (FF) and feedback (FB) filters, efficient novel modified least–mean–square (LMS) and recursive least–squares (RLS) algorithms are presented. Finally, it is shown that the proposed adaptive NDFE scheme is robust against frequency offset.

Keywords:

(Noncoherent) decision–feedback equalization ((N)DFE), (noncoherent) linear equalization, noncoherent detection, intersymbol interference (ISI) channels, adaptive algorithms.

1 Introduction

The field of coherent linear and nonlinear equalization is a very mature one (see e.g. [1, 2, 3]). However, in the presence of fast phase variations, which might be introduced by a carrier frequency offset or fading, coherent equalizers degrade severely [4, 5, 6]. Thus, the application of robust noncoherent equalizers could be advantageous. Although many results for noncoherent receivers for M–ary differential phase–shift keying (MDPSK) signals transmitted over frequency–nonselective channels have been published in literature (see e.g. [7, 8, 9, 10, 11, 12, 13, 14, 15] and references therein), only few results are available for noncoherent equalization of frequency–selective channels. Noncoherent linear equalization for MDPSK has been introduced by Sehier et al. [4]. Recently, a novel noncoherent linear equalizer (NLE) which can approach the performance of the well–known coherent minimum mean–squared error (MMSE) linear equalizer [16] has been proposed by Schober et al. [17]. Like coherent LEs, NLEs degrade severely if the underlying discrete–time channel has spectral nulls.

Colavolpe et al. [18, 19] and Gerstacker et al. [20] proposed noncoherent equalization algorithms based on sequence estimation, which do not have this disadvantage. Like in the coherent case, for large M and long channel impulse responses, the computational complexity of noncoherent sequence estimation is considerably high. This problem is resolved in [18, 19] by employing reduced–state techniques. However, since no allpass prefilter is employed a favourable trade–off between performance and complexity can only be expected for minimum–phase channel impulse responses [21]. In addition, the scheme in [18, 19] is not adaptive and thus, cannot be used for time–varying intersymbol interference (ISI) channels. These problems are overcome by the adaptive noncoherent decision–feedback equalization (NDFE) scheme presented here.

NDFE schemes have been already proposed by Masoomzadeh–Fard et al. [5], Benvenuto et al. [22], and Jones et al. [6]. All these schemes apply a conventional differential demodulator [2] to remove the absolute phase of the received signal. In the next stage of the receiver, a DFE scheme is used to compensate for the nonlinear ISI caused by differential demodulation. Clearly, these schemes are suboptimum and cause a large loss in power efficiency (typically more than 4 dB) when compared with coherent DFE.

The novel NDFE scheme introduced here is derived from optimum noncoherent maximum–likelihood sequence estimation (MLSE) [19, 20] and can approach coherent MMSE–DFE [23, 24, 25, 26] arbitrarily close. A suboptimum version (referred to as 'suboptimum NDFE') with lower computational complexity can be derived directly from the proposed NDFE scheme. For adaptation of the feedforward (FF) and feedback (FB) filters efficient novel modified least–mean–square (LMS) and recursive least–squares (RLS) algorithms are provided. Simulations confirm that the proposed adaptive noncoherent equalization schemes are robust against frequency offset.

This paper is organized as follows. In Section 2, the discrete–time transmission model is introduced. The proposed NDFE scheme is derived in Section 3 and a suboptimum version is presented in Section 4. Also in Section 4, a NLE is derived from NDFE and its relation to a previously proposed NLE [17] is investigated. A modified LMS and a modified RLS algorithm for adaptation of the FF and FB filters are given in Section 5. In Section 6, some simulation results are provided, and some conclusions are drawn in Section 7.

2 Transmission Model

Fig. 1 shows a block diagram of the discrete–time transmission model. All signals are represented by their complex–valued baseband equivalents. For simplicity only T–spaced equalizers are considered here, however, our results can be extended to fractionally–spaced equalizers. At the transmitter, the MDPSK symbols $a[\cdot] \in \mathcal{A} = \{e^{j2\pi\nu/M} | \nu \in \{0, 1, \ldots, M-1\}\}$ are differentially encoded. The resulting MPSK symbols $b[\cdot]$ are given by

$$b[k] = a[k]b[k-1], \quad k \in \mathbb{Z}. \tag{1}$$

The discrete–time received signal, sampled at times kT (T is the symbol interval) at the output of the receiver input filter can be written as

$$r[k] = e^{j\Theta} \sum_{\nu=0}^{L_h-1} h_\nu b[k-\nu] + n[k], \tag{2}$$

where Θ denotes an unknown, constant, uniformly distributed phase. h_ν, $0 \leq \nu \leq L_h - 1$, are the coefficients of the combined discrete–time impulse response of transmit filter, channel, and receiver input filter; its length is denoted by L_h. We assume a square–root Nyquist frequency response for the receiver input filter[1], and thus, the zero mean complex Gaussian noise $n[\cdot]$ is white. Due to an appropriate normalization, the variance of $n[\cdot]$ is $\sigma_n^2 = \mathcal{E}\{|n[k]|^2\} = N_0/E_S$, where $\mathcal{E}\{\cdot\}$ denotes expectation. E_S and N_0 are the mean received energy per symbol and the single–sided power spectral density of the underlying passband noise process, respectively.

Eq. (2) may be rewritten to

$$r[k] = e^{j\Theta} r_{\mathrm{coh}}[k], \tag{3}$$

with

$$r_{\mathrm{coh}}[k] \triangleq \sum_{\nu=0}^{L_h-1} h_\nu b[k-\nu] + n_{\mathrm{coh}}[k], \tag{4}$$

$$n_{\mathrm{coh}}[k] \triangleq e^{-j\Theta} n[k]. \tag{5}$$

Since $n[k]$ and $n_{\mathrm{coh}}[k]$ have the same statistical properties, $r[k]$ may be viewed as the received signal $r_{\mathrm{coh}}[k]$ of a coherent receiver multiplied by $e^{j\Theta}$.

[1] This also contains the whitened matched filter [27] as a special case.

The noncoherent decision–feedback equalizer introduces a decision delay k_0 which should be optimized like in the coherent case (cf. e.g. [28]) since it can significantly affect performance. The estimated MPSK symbol is denoted by $\hat{b}[k-k_0]$. Finally, differential encoding has to be inverted, yielding the estimated MDPSK symbol $\hat{a}[k-k_0]$,

$$\hat{a}[k-k_0] = \hat{b}[k-k_0]\hat{b}^*[k-k_0-1], \tag{6}$$

where $(\cdot)^*$ denotes complex conjugation.

3 Noncoherent DFE

In this section, a novel NDFE structure is derived. In contrast to previously proposed schemes [5, 22, 6], our NDFE scheme does not use a conventional differential demodulator. Since coherent DFE might be interpreted as coherent reduced–state MLSE [29, 30, 21] (Viterbi algorithm [31] with only one state), we derive NDFE from noncoherent MLSE.

3.1 Noncoherent DFE Decision Rule

The optimum noncoherent MLSE metric for estimation of a block of $N \geq 2$ symbols $b[k-\nu]$, based on observation of N received symbols $r[k-\nu]$, $0 \leq \nu \leq N-1$ (the symbols $b[k-\nu]$, $\nu \geq N$, are assumed to be known at the receiver), is given in [20, 19] (cf. e.g. Eq. (7) in [20]). For moderate to high E_S/N_0 ratios, the optimum metric is well approximated by [20, 19]

$$\lambda[k] \triangleq \sum_{\nu=k-N+1}^{k} \left(|r[\nu]|^2 + |y[\nu]|^2\right) - 2\left|\sum_{\nu=k-N+1}^{k} r[\nu]y^*[\nu]\right|, \tag{7}$$

where the definition

$$y[k] \triangleq \sum_{\nu=0}^{L_h-1} h_\nu b[k-\nu] \tag{8}$$

is used. Note, that Eq. (7) was derived under the assumption of an additive white Gaussian noise (AWGN) channel with unknown, uniformly distributed phase Θ (cf. Fig. 1). Since DFE is to be applied, an FF filter $f_{\mathrm{F},\nu}[k]$[2], $0 \leq \nu \leq L_\mathrm{F}-1$, is introduced. The FF filter output signal $r_\mathrm{DFE}[k]$ and the combined impulse reponse of channel and FF filter $g_\nu[k]$, $0 \leq \nu \leq L_\mathrm{F}+L_h-2$, are given by

$$r_\mathrm{DFE}[k] = \boldsymbol{f}_\mathrm{F}^H[k]\boldsymbol{r}[k] \tag{9}$$

and

$$g_\nu[k] = \sum_{\mu=0}^{L_\mathrm{F}-1} f_{\mathrm{F},\mu}[k]h_{\nu-\mu}, \tag{10}$$

[2]The coefficients $f_{\mathrm{F},\nu}[k]$ are defined as time–variant in order to include filter adaptation as a special case (cf. Section 5).

respectively ($[\cdot]^H$ denotes Hermitian transposition). Here, the FF filter coefficient vector $\boldsymbol{f}_{\mathrm{F}}[k]$ and the vector $\boldsymbol{r}[k]$ of received signal samples are defined as

$$\boldsymbol{f}_{\mathrm{F}}[k] \triangleq [f_{\mathrm{F},0}[k]\ f_{\mathrm{F},1}[k]\ \dots\ f_{\mathrm{F},(L_{\mathrm{F}}-1)}[k]]^H, \tag{11}$$

$$\boldsymbol{r}[k] \triangleq [r[k]\ r[k-1]\ \dots\ r[k-L_{\mathrm{F}}+1]]^T \tag{12}$$

($[\cdot]^T$ denotes transposition). In principle, the DFE–FF filter should minimize the precursors $g_\nu[k]$, $0 \le \nu \le k_0 - 1$, and the remaining coefficients $g_\nu[k]$, $k_0 \le \nu \le L_{\mathrm{F}} + L_h - 2$, should form a minimum–phase sequence[3]. If precursors and channel noise are neglected, the useful component of $r_{\mathrm{DFE}}[k]$ may be modelled as

$$y_{\mathrm{DFE}}[k] = \sum_{\nu=0}^{L_{\mathrm{B}}-1} f_{\mathrm{B},\nu}[k] b[k-k_0-\nu], \tag{13}$$

where $f_{\mathrm{B},\nu}[k]$ are the coefficients of the DFE–FB filter, which have to be optimized in the following. For simplicity, here $L_{\mathrm{B}} = L_{\mathrm{F}} + L_h - 1 - k_0$ is assumed. The adjustment of the FF and FB filter coefficients will be discussed in Section 3.2. Since the FF and FB filters introduce an additional degree of freedom, either $f_{\mathrm{F},\nu}[k]$, $0 \le \nu \le L_{\mathrm{F}} - 1$, or $f_{\mathrm{B},\nu}[k]$, $0 \le \nu \le L_{\mathrm{B}} - 1$, has to be constrained in order to avoid the trivial solution $f_{\mathrm{F},\nu}[k] \equiv 0$, $0 \le \nu \le L_{\mathrm{F}} - 1$, $f_{\mathrm{B},\nu}[k] \equiv 0$, $0 \le \nu \le L_{\mathrm{B}} - 1$, $\forall k$. Here, we use the constraint

$$f_{\mathrm{B},0}[k] \equiv 1, \qquad \forall k, \tag{14}$$

which is also used for coherent DFE. Although the noise component of $r_{\mathrm{DFE}}[k]$, in general, is not exactly white nor Gaussian (because of uncancelled precursors), for simplicity, the metric according to Eq. (7), which is now suboptimum, still might be used. This leads to

$$\lambda_{\mathrm{DFE}}[k] = \sum_{\nu=k-N+1}^{k} \left(|r_{\mathrm{DFE}}[\nu]|^2 + |y_{\mathrm{DFE}}[\nu]|^2\right) - 2\left|\sum_{\nu=k-N+1}^{k} r_{\mathrm{DFE}}[\nu] y^*_{\mathrm{DFE}}[\nu]\right|. \tag{15}$$

If we interpret NDFE as noncoherent reduced–state MLSE with only one state, in Eq. (15) the unknown symbols $b[k-k_0-\nu]$, $\nu \ge 1$, have to be replaced by previously decided symbols $\hat{b}[k-k_0-\nu]$, $\nu \ge 1$. The resulting metric to be minimized with respect to $b[k-k_0]$ is

$$\begin{aligned}\hat{\lambda}_{\mathrm{DFE}}[k] \;=\; & \sum_{\nu=k-N+1}^{k} |r_{\mathrm{DFE}}[\nu]|^2 + \left|b[k-k_0] + \boldsymbol{f}_{\mathrm{B}}^H[k]\hat{\boldsymbol{b}}[k-k_0-1]\right|^2 + \sum_{\nu=k-N+1}^{k-1} |\hat{y}_{\mathrm{DFE}}[\nu]|^2 \\ & -2\left|r_{\mathrm{DFE}}[k]\left(b[k-k_0] + \boldsymbol{f}_{\mathrm{B}}^H[k]\hat{\boldsymbol{b}}[k-k_0-1]\right)^* + \sum_{\nu=k-N+1}^{k-1} r_{\mathrm{DFE}}[\nu]\hat{y}^*_{\mathrm{DFE}}[\nu]\right|,\end{aligned} \tag{16}$$

with the definitions

$$\hat{y}_{\mathrm{DFE}}[k] \triangleq \sum_{\nu=0}^{L_{\mathrm{B}}-1} f_{\mathrm{B},\nu}[k]\hat{b}[k-k_0-\nu], \tag{17}$$

$$\boldsymbol{f}_{\mathrm{B}}[k] \triangleq [f_{\mathrm{B},1}[k]\ f_{\mathrm{B},2}[k]\ \dots\ f_{\mathrm{B},(L_{\mathrm{B}}-1)}[k]]^H, \tag{18}$$

$$\hat{\boldsymbol{b}}[k-k_0-1] \triangleq [\hat{b}[k-k_0-1]\ \hat{b}[k-k_0-2]\ \dots\ \hat{b}[k-k_0-L_{\mathrm{B}}+1]]^T. \tag{19}$$

[3]For MMSE–DFE with FIR filters the minimum–phase property is not necessarily strictly fulfilled, cf. [25].

If additive terms in Eq. (16) which do not influence the decision are omitted, the resulting DFE decision rule is

$$\hat{b}[k-k_0] = \underset{b[k-k_0]}{\operatorname{argmin}} \, \{\hat{\lambda}'_{\mathrm{DFE}}[k]\}, \tag{20}$$

where the definitions

$$\hat{\lambda}'_{\mathrm{DFE}}[k] \triangleq \left| b[k-k_0] + \boldsymbol{f}_{\mathrm{B}}^{H}[k]\hat{\boldsymbol{b}}[k-k_0-1] \right|^2 - 2\left| r_{\mathrm{DFE}}[k]\left(b[k-k_0] + \boldsymbol{f}_{\mathrm{B}}^{H}[k]\hat{\boldsymbol{b}}[k-k_0-1]\right)^* + \hat{q}_{\mathrm{nrec}}^{N-1}[k-1] \right| \tag{21}$$

$$\hat{q}_{\mathrm{nrec}}^{N-1}[k-1] \triangleq \sum_{\nu=k-N+1}^{k-1} r_{\mathrm{DFE}}[\nu] \, \hat{y}_{\mathrm{DFE}}^{*}[\nu] \tag{22}$$

are used. Eqs. (9) and (20)–(22) show that the decision does not depend on the channel phase Θ (cf. Eq. (2)) which is mandatory for a noncoherent receiver. Consequently, differential encoding is necessary since the absolute phase of the estimated symbol sequence $\hat{b}[\cdot]$ may differ from the absolute phase of the transmitted symbol sequence $b[\cdot]$ by $2\pi\nu/M$, $1 \leq \nu \leq M-1$. The NDFE decision rule according to Eq. (20) is directly derived from noncoherent MLSE and it should be mentioned, that coherent DFE can be obtained in a similar way from coherent MLSE [27]. Moreover, in Section 3.3, it will be shown that if the FF and FB filters are adjusted according to Section 3.2, for $N \to \infty$ this NDFE scheme approaches coherent MMSE–DFE. The structure of the above NDFE scheme is shown in Fig. 2. In a coherent DFE scheme, the decision is based on the location of a decision variable in the two–dimensional signal space. In principle, this is also possible for the proposed NDFE scheme, however, the shape of the related decision regions is difficult to obtain. Thus, we prefer to calculate M metrics per symbol decision (cf. Eq. (20)). In Section 4, a suboptimum NDFE scheme which estimates the transmitted symbol in a similar way like coherent DFE will be provided.

The phase of $\hat{q}_{\mathrm{nrec}}^{N-1}[k-1]$ might be interpreted as estimate for the phase difference between $r_{\mathrm{DFE}}[k]$ and $\hat{y}_{\mathrm{DFE}}[k]$. Our simulations show that power efficiency of NDFE improves as N increases. On the other hand, in this case, the number of terms in Eq. (22) increases, too. To avoid this problem, a recursively generated phase reference symbol

$$\hat{q}_{\mathrm{rec}}[k-1] \triangleq \alpha\hat{q}_{\mathrm{rec}}[k-2] + r_{\mathrm{DFE}}[k-1]\hat{y}_{\mathrm{DFE}}^{*}[k-1] \tag{23}$$

may be used in Eq. (20) instead of the nonrecursively generated symbol $\hat{q}_{\mathrm{nrec}}^{N-1}[k-1]$. α, $0 \leq \alpha < 1$, is a forgetting factor. Note, that $\hat{q}_{\mathrm{nrec}}^{N-1}[k-1]$ and $\hat{q}_{\mathrm{rec}}[k-1]$ are identical for the special cases $N=2$, $\alpha=0$ and $N \to \infty$, $\alpha \to 1$.

3.2 Filter Adjustment

In a coherent MMSE–DFE scheme, the optimum FF and FB filters minimize the variance of the difference between detector input signal and transmitted symbol. For NDFE, a simple filter design criterion based on minimization of an error variance cannot be found. On the other hand, if we assume perfect knowledge of the transmitted symbol sequence $b[\cdot]$, $\lambda_{\text{DFE}}[k]$ may be used for (noncoherent) data–aided estimation of the FF and FB filter coefficients. Although a (noncoherent) ML filter coefficient estimator would only be obtained if the noise at the output of the FF filter was white and Gaussian (which is only approximately true in general), this is a promising approach. Therefore, we use $\mathcal{E}\{\lambda_{\text{DFE}}[k]\}$ as (noncoherent) cost function for calculation of the FF and FB filters. The optimum[4] filter settings $\boldsymbol{f}_{\text{F}}[k] = \boldsymbol{f}_{\text{F}}$, $\forall\, k$, and $\boldsymbol{f}_{\text{B}}[k] = \boldsymbol{f}_{\text{B}}$, $\forall\, k$, can be calculated from

$$\frac{\partial}{\partial \boldsymbol{f}^*_{\text{X}}} \mathcal{E}\{\lambda_{\text{DFE}}[k]\} = \mathbf{0}_{S_{\text{X}}}, \qquad \text{X} = \text{F, B}, \tag{24}$$

where $\mathbf{0}_{S_{\text{X}}}$ denotes the all zero vector with S_{X} ($S_{\text{F}} \triangleq L_{\text{F}}$, $S_{\text{B}} \triangleq L_{\text{B}} - 1$) rows. For finite N, calculation of $\boldsymbol{f}_{\text{F}}$ and $\boldsymbol{f}_{\text{B}}$ from Eq. (24) is difficult because of the magnitude operator in Eq. (15). Therefore, in Section 5, novel modified LMS and RLS algorithms are derived to find the desired filter settings. A similar approach was proposed in [4, 5, 17], however, there a different cost function was used and thus different adaptive algorithms were obtained. The steady–state solution of these adaptive algorithms can be obtained from[5]

$$\mathcal{E}\left\{\frac{\partial}{\partial \boldsymbol{f}^*_{\text{X}}[k]} \lambda_{\text{DFE}}[k]\right\} = \frac{\partial}{\partial \boldsymbol{f}^*_{\text{X}}[k]} \mathcal{E}\{\lambda_{\text{DFE}}[k]\} = \mathbf{0}_{S_{\text{X}}}, \qquad \text{X} = \text{F, B}. \tag{25}$$

Note, that for differentiation with respect to $\boldsymbol{f}_{\text{X}}[k]$ (X = F, B), $\boldsymbol{f}_{\text{X}}[\nu]$, $\nu \leq k-1$, are treated as constants in Eqs. (25).

For all channels investigated, the adaptive algorithms presented in Section 5 converged to the correct FF and FB filter settings. However, for $N < \infty$ no analytical proof for the absence of spurious local minima in the cost function could be found so far. For $N \to \infty$, closed–form results can be obtained. The steady–state solution of the adaptive algorithms for this case will be provided in Section 3.3.

3.3 Limiting Performance for $N \to \infty$

In this section, the limiting performance of the proposed NDFE is investigated. First, the steady–state solutions for the FF and FB filters obtained by the adaptive algorithms described in Section 5 are calculated.

[4]The filter settings are optimum in the sense that they minimize the underlying cost function.

[5]For example, Eq. (25) (X = F) follows from Eq. (61) since under steady–state conditions $\mathcal{E}\{\boldsymbol{f}_{\text{F}}[k+1]\} = \mathcal{E}\{\boldsymbol{f}_{\text{F}}[k]\}$ is valid.

3.3.1 Steady–State Solution for the FF and FB Filters

Using Eqs. (9) and (13), Eq. (15) may be rewritten as

$$\begin{aligned}\lambda_{\mathrm{DFE}}[k] &= |\boldsymbol{f}_{\mathrm{F}}^{H}[k]\boldsymbol{r}[k]|^2 + \sum_{\nu=k-N+1}^{k-1} |r_{\mathrm{DFE}}[\nu]|^2 + \left|b[k-k_0] + \boldsymbol{f}_{\mathrm{B}}^{H}[k]\boldsymbol{b}[k-k_0-1]\right|^2 \\ &\quad + \sum_{\nu=k-N+1}^{k-1} |y_{\mathrm{DFE}}[\nu]|^2 - 2\left|\boldsymbol{f}_{\mathrm{F}}^{H}[k]\boldsymbol{r}[k](b[k-k_0] + \boldsymbol{f}_{\mathrm{B}}^{H}[k]\boldsymbol{b}[k-k_0-1])^* \right. \\ &\quad \left. + q_{\mathrm{nrec}}^{N-1}[k-1]\right|, \end{aligned} \tag{26}$$

with the definitions

$$\boldsymbol{b}[k-k_0-1] \triangleq [b[k-k_0-1]\; b[k-k_0-2]\; \ldots\; b[k-k_0-L_{\mathrm{B}}+1]]^T, \tag{27}$$

$$q_{\mathrm{nrec}}^{N-1}[k-1] \triangleq \sum_{\nu=k-N+1}^{k-1} r_{\mathrm{DFE}}[\nu] y_{\mathrm{DFE}}^*[\nu]. \tag{28}$$

Using the method for complex differentiation described in [3], Appendix B, and the rule

$$\frac{\partial}{\partial \boldsymbol{w}^*}|z(\boldsymbol{w})| = \frac{\partial}{\partial \boldsymbol{w}^*}\frac{\partial(z(\boldsymbol{w})z^*(\boldsymbol{w}))}{\partial(z(\boldsymbol{w})z^*(\boldsymbol{w}))}\sqrt{z(\boldsymbol{w})z^*(\boldsymbol{w})} = \frac{1}{2}\frac{1}{|z(\boldsymbol{w})|}\frac{\partial(z(\boldsymbol{w})z^*(\boldsymbol{w}))}{\partial \boldsymbol{w}^*}, \tag{29}$$

where $z(\boldsymbol{w}) \neq 0$ denotes a complex–valued function of a complex–valued vector $\boldsymbol{w}$, the derivatives $\partial\lambda_{\mathrm{DFE}}[k]/\partial\boldsymbol{f}_{\mathrm{F}}^*[k]$ and $\partial\lambda_{\mathrm{DFE}}[k]/\partial\boldsymbol{f}_{\mathrm{B}}^*[k]$ may be calculated to:

$$\frac{\partial}{\partial \boldsymbol{f}_{\mathrm{F}}^*[k]}\lambda_{\mathrm{DFE}}[k] = \left(r_{\mathrm{DFE}}[k] - \frac{q_{\mathrm{nrec}}^{N}[k]}{|q_{\mathrm{nrec}}^{N}[k]|}y_{\mathrm{DFE}}[k]\right)^* \boldsymbol{r}[k], \tag{30}$$

$$\frac{\partial}{\partial \boldsymbol{f}_{\mathrm{B}}^*[k]}\lambda_{\mathrm{DFE}}[k] = \left(y_{\mathrm{DFE}}[k] - \frac{(q_{\mathrm{nrec}}^{N}[k])^*}{|q_{\mathrm{nrec}}^{N}[k]|}r_{\mathrm{DFE}}[k]\right)^* \boldsymbol{b}[k-k_0-1]. \tag{31}$$

Note, that $r_{\mathrm{DFE}}[\nu]$, $\nu \leq k-1$, and $q_{\mathrm{nrec}}^{N-1}[k-1]$, which depend only on former $\boldsymbol{f}_{\mathrm{X}}[\nu]$, $\nu \leq k-1$, must be considered as constants for differentiation with respect to $\boldsymbol{f}_{\mathrm{X}}[k]$, X = F, B (cf. [4, 5]).

Now, Eq. (25) can be rewritten to

$$\mathcal{E}\{r_{\mathrm{DFE}}^*[k]\boldsymbol{r}[k]\} = \mathcal{E}\left\{\frac{(q_{\mathrm{nrec}}^{N}[k])^*}{|q_{\mathrm{nrec}}^{N}[k]|}y_{\mathrm{DFE}}^*[k]\boldsymbol{r}[k]\right\}, \tag{32}$$

$$\mathcal{E}\{y_{\mathrm{DFE}}^*[k]\boldsymbol{b}[k-k_0-1]\} = \mathcal{E}\left\{\frac{q_{\mathrm{nrec}}^{N}[k]}{|q_{\mathrm{nrec}}^{N}[k]|}r_{\mathrm{DFE}}^*[k]\boldsymbol{b}[k-k_0-1]\right\}, \tag{33}$$

for X = F and X = B, respectively. As mentioned above, for finite N it is difficult to evaluate Eqs. (32) and (33). For $N \to \infty$, however, an analytical solution can be obtained. In the following, we will employ the relation

$$\lim_{N\to\infty}\frac{q_{\mathrm{nrec}}^{N}[k]}{N} = \mathcal{E}\{r_{\mathrm{DFE}}[k]y_{\mathrm{DFE}}^*[k]|\Theta\} \triangleq \mathrm{e}^{j\phi}|\mathcal{E}\{r_{\mathrm{DFE}}[k]y_{\mathrm{DFE}}^*[k]|\Theta\}|, \tag{34}$$

which holds under steady–state conditions, i.e., $\boldsymbol{f}_{\mathrm{F}}[k]=\boldsymbol{f}_{\mathrm{F}}$, $\boldsymbol{f}_{\mathrm{B}}[k]=\boldsymbol{f}_{\mathrm{B}}$, since all stochastic processes considered here are assumed to be ergodic. Note, that the phase term $\mathrm{e}^{j\phi}$ depends on the channel and the FF filter impulse response and can be obtained directly from

$$\mathrm{e}^{j\phi}=\lim_{N\to\infty}\frac{q^{N}_{\mathrm{nrec}}[k]}{|q^{N}_{\mathrm{nrec}}[k]|}. \tag{35}$$

Using Eqs. (9), (13), (14), (35), and the definitions

$$\begin{aligned}
\boldsymbol{\Phi}_{rr} &\triangleq \mathcal{E}\{\boldsymbol{r}[k]\boldsymbol{r}^{H}[k]\}, && (36)\\
\boldsymbol{\Phi}_{rb} &\triangleq \mathcal{E}\{\boldsymbol{r}[k]\boldsymbol{b}^{H}[k-k_0-1]|\Theta\}, && (37)\\
\boldsymbol{\varphi}_{rb} &\triangleq \mathcal{E}\{\boldsymbol{r}[k]b^{*}[k-k_0]|\Theta\}=\mathrm{e}^{j\Theta}[h_{k_0}\,h_{k_0-1}\ \ldots\ h_{k_0-L_{\mathrm{F}}+1}]^{T}, && (38)
\end{aligned}$$

the following solutions for $\boldsymbol{f}_{\mathrm{F}}[k]$ and $\boldsymbol{f}_{\mathrm{B}}[k]$ may be obtained from Eqs. (32), (33) by straightforward calculations[6]:

$$\begin{aligned}
\boldsymbol{f}_{\mathrm{F}}[k] &= \boldsymbol{f}_{\mathrm{F}}=\mathrm{e}^{-j\phi}(\boldsymbol{\Phi}_{rr}-\boldsymbol{\Phi}_{rb}\boldsymbol{\Phi}^{H}_{rb})^{-1}\boldsymbol{\varphi}_{rb}, && (39)\\
\boldsymbol{f}_{\mathrm{B}}[k] &= \boldsymbol{f}_{\mathrm{B}}=\boldsymbol{\Phi}^{H}_{rb}(\boldsymbol{\Phi}_{rr}-\boldsymbol{\Phi}_{rb}\boldsymbol{\Phi}^{H}_{rb})^{-1}\boldsymbol{\varphi}_{rb}. && (40)
\end{aligned}$$

For derivation of these results, also $\boldsymbol{\Phi}_{bb}\triangleq\mathcal{E}\{\boldsymbol{b}[k-k_0-1]\boldsymbol{b}^{H}[k-k_0-1]\}=\boldsymbol{I}_{(L_{\mathrm{B}}-1)\times(L_{\mathrm{B}}-1)}$ ($\boldsymbol{I}_{Y\times Y}$ is the $Y\times Y$ identity matrix) has to be taken into account, which is valid because $b[\cdot]$ is an independent, identically distributed (i.i.d.) sequence. In order to find a relation between the filters defined in Eqs. (39), (40) and the filters of a coherent MMSE–DFE receiver, we define

$$\begin{aligned}
\boldsymbol{r}_{\mathrm{coh}}[k] &\triangleq [r_{\mathrm{coh}}[k]\ r_{\mathrm{coh}}[k-1]\ \ldots\ r_{\mathrm{coh}}[k-L_{\mathrm{F}}+1]]^{T}=\mathrm{e}^{-j\Theta}\boldsymbol{r}[k], && (41)\\
\boldsymbol{\Phi}^{\mathrm{coh}}_{rr} &\triangleq \mathcal{E}\{\boldsymbol{r}_{\mathrm{coh}}[k]\boldsymbol{r}^{H}_{\mathrm{coh}}[k]\}=\boldsymbol{\Phi}_{rr}, && (42)\\
\boldsymbol{\Phi}^{\mathrm{coh}}_{rb} &\triangleq \mathcal{E}\{\boldsymbol{r}_{\mathrm{coh}}[k]\boldsymbol{b}^{H}[k-k_0-1]\}=\mathrm{e}^{-j\Theta}\boldsymbol{\Phi}_{rb}, && (43)\\
\boldsymbol{\varphi}^{\mathrm{coh}}_{rb} &\triangleq \mathcal{E}\{\boldsymbol{r}_{\mathrm{coh}}[k]b^{*}[k-k_0]\}=\mathrm{e}^{-j\Theta}\boldsymbol{\varphi}_{rb}. && (44)
\end{aligned}$$

The optimum FF and FB filters of a coherent MMSE–DFE receiver may be expressed as [23, 32]

$$\begin{aligned}
\boldsymbol{f}^{\mathrm{coh}}_{\mathrm{F}}[k] &= (\boldsymbol{\Phi}^{\mathrm{coh}}_{rr}-\boldsymbol{\Phi}^{\mathrm{coh}}_{rb}(\boldsymbol{\Phi}^{\mathrm{coh}}_{rb})^{H})^{-1}\boldsymbol{\varphi}^{\mathrm{coh}}_{rb}\triangleq\boldsymbol{f}^{\mathrm{coh}}_{\mathrm{F}}, && (45)\\
\boldsymbol{f}^{\mathrm{coh}}_{\mathrm{B}}[k] &= (\boldsymbol{\Phi}^{\mathrm{coh}}_{rb})^{H}(\boldsymbol{\Phi}^{\mathrm{coh}}_{rr}-\boldsymbol{\Phi}^{\mathrm{coh}}_{rb}(\boldsymbol{\Phi}^{\mathrm{coh}}_{rb})^{H})^{-1}\boldsymbol{\varphi}^{\mathrm{coh}}_{rb}\triangleq\boldsymbol{f}^{\mathrm{coh}}_{\mathrm{B}}. && (46)
\end{aligned}$$

Using Eqs. (39)–(46) it is straightforward to show that the relations

$$\begin{aligned}
\boldsymbol{f}_{\mathrm{F}} &= \mathrm{e}^{j\phi_0}\boldsymbol{f}^{\mathrm{coh}}_{\mathrm{F}}, && (47)\\
\boldsymbol{f}_{\mathrm{B}} &= \boldsymbol{f}^{\mathrm{coh}}_{\mathrm{B}}, && (48)
\end{aligned}$$

[6] In Eqs. (34), (37) and (38) the expected value is conditioned on Θ in order to emphasize that Θ is treated as a constant for expectation. This is not necessary in Eqs. (32) and (33) since in this case the argument does not depend on Θ.

with

$$\phi_0 \triangleq \Theta - \phi, \tag{49}$$

are valid, i.e., the only difference between the filters for coherent MMSE–DFE and NDFE for $N \rightarrow \infty$ is the multiplicative term $\mathrm{e}^{j\phi_0}$ in the FF filter coefficients. Note, that since the underlying cost function is 'noncoherent' (cf. Eq. (26)), $\boldsymbol{f}_\mathrm{F}$ is unique only up to a complex factor with magnitude one (cf. Eqs.(9), (22), (28), (32), and (33)), i.e., the phase ϕ_0 is arbitrary (but constant).

3.3.2 NDFE Decision Rule for $N \rightarrow \infty$

In Appendix A, it is shown that under steady–state conditions the NDFE decision rule for $N \rightarrow \infty$ is given by:

$$\hat{b}[k-k_0] = \underset{b[k-k_0]}{\operatorname{argmin}} \{|b[k-k_0] - d[k]|^2\}, \tag{50}$$

where $d[k]$ is defined as

$$d[k] = \mathrm{e}^{-j(\phi_0 - \Theta)} \boldsymbol{f}_\mathrm{F}^H \boldsymbol{r}[k] - \boldsymbol{f}_\mathrm{B}^H \hat{\boldsymbol{b}}[k-k_0-1]. \tag{51}$$

According to Eq. (50), $\hat{b}[k-k_0]$ is that $b[k-k_0]$ which has minimum Euclidean distance from the decision variable $d[k]$. Thus, like in the coherent case, the complex plane may be divided into M sectors corresponding to the M possible values of $b[k-k_0]$ and $\hat{b}[k-k_0]$ is determined uniquely by the sector into which $d[k]$ falls.

If Eqs. (41), (47), and (48) are used, Eq. (51) can be rewritten to

$$d[k] = (\boldsymbol{f}_\mathrm{F}^\mathrm{coh})^H \boldsymbol{r}_\mathrm{coh}[k] - (\boldsymbol{f}_\mathrm{B}^\mathrm{coh})^H \hat{\boldsymbol{b}}[k-k_0-1], \tag{52}$$

which is the decision variable of a coherent MMSE–DFE receiver [24]. This proves, that for $N \rightarrow \infty$, the proposed NDFE scheme approaches coherent MMSE–DFE. Note, that this is also true if the phase reference symbol is generated recursively (cf. Eq. (23)) with $\alpha \rightarrow 1$ since the nonrecursively and the recursively generated reference symbols are identical for the special cases $N \rightarrow \infty$ and $\alpha \rightarrow 1$.

4 Suboptimum NDFE and Linear Equalization

In this section, a suboptimum version of the proposed NDFE scheme and a NLE are derived. In addition, the relation of this NLE to a recently reported NLE [17] is investigated.

4.1 Suboptimum Noncoherent DFE

The decision rule for the proposed NDFE scheme according to Eq. (20) requires M metric calculations per symbol decision. Here, a suboptimum NDFE scheme is derived which demands only the calculation of one decision variable, like coherent DFE schemes.

$d[k]$ given in Eq. (51) might be used as decision variable for NDFE for $N \to \infty$. We propose to apply the rule according to Eqs. (50) and (51) also for finite N and use

$$e^{j\hat{\phi}_{\mathrm{nrec}}} = \frac{\hat{q}_{\mathrm{nrec}}^{N-1}[k-1]}{|\hat{q}_{\mathrm{nrec}}^{N-1}[k-1]|} \tag{53}$$

or

$$e^{j\hat{\phi}_{\mathrm{rec}}} = \frac{\hat{q}_{\mathrm{rec}}[k-1]}{|\hat{q}_{\mathrm{rec}}[k-1]|} \tag{54}$$

as estimate for $e^{j\phi} = e^{j(\phi_0 - \Theta)}$ in Eq. (51). Note, that under steady–state conditions

$$e^{j\phi} = \lim_{N\to\infty} e^{j\hat{\phi}_{\mathrm{nrec}}} = \lim_{\alpha\to 1} e^{j\hat{\phi}_{\mathrm{rec}}} \tag{55}$$

holds. Thus, in the limit $N \to \infty$ or $\alpha \to 1$, the suboptimum NDFE scheme also approaches coherent MMSE–DFE. For finite N and $\alpha < 1$, however, the original NDFE scheme offers a better performance as will be shown by computer simulations. If Eqs. (53) and (54) are applied in Eq. (51), the decision variables for nonrecursive and recursive suboptimum NDFE can be expressed as

$$d_{\mathrm{nrec}}[k] = \frac{(\hat{q}_{\mathrm{nrec}}^{N-1}[k-1])^*}{|\hat{q}_{\mathrm{nrec}}^{N-1}[k-1]|} \boldsymbol{f}_{\mathrm{F}}^H[k]\boldsymbol{r}[k] - \boldsymbol{f}_{\mathrm{B}}^H[k]\hat{\boldsymbol{b}}[k-k_0-1] \tag{56}$$

and

$$d_{\mathrm{rec}}[k] = \frac{(\hat{q}_{\mathrm{rec}}[k-1])^*}{|\hat{q}_{\mathrm{rec}}[k-1]|} \boldsymbol{f}_{\mathrm{F}}^H[k]\boldsymbol{r}[k] - \boldsymbol{f}_{\mathrm{B}}^H[k]\hat{\boldsymbol{b}}[k-k_0-1], \tag{57}$$

respectively. The corresponding structure for the nonrecursive case is shown in Fig. 3. It should be mentioned that for suboptimum NDFE an MMSE approach (cf. e.g. [17]) might be used for filter optimization. The variance of the error signal $e[k] = b[k-k_0] - d_{\mathrm{nrec}}[k]$ can be minimized and modified LMS and RLS algorithms (different from those presented in Section 5) may be derived.

4.2 Noncoherent Linear Equalization

An NLE can be obtained from the proposed optimum NDFE scheme for $L_{\mathrm{B}} = 1$, i.e., $f_{\mathrm{B},\nu}[k] = 0$, $\nu \geq 1$. In this case, the decision variable

$$d_{\mathrm{nrec}}^{\mathrm{LE}}[k] = r_{\mathrm{DFE}}[k](\hat{l}_{\mathrm{nrec}}^{N-1}[k-1])^*, \tag{58}$$

for direct estimation of $\hat{a}[k-k_0]$ (and not $\hat{b}[k-k_0]$) can be obtained from Eq. (20) by some straightforward manipulations. $\hat{l}_{\mathrm{nrec}}^{N-1}[k-1]$ is defined as

$$\hat{l}_{\mathrm{nrec}}^{N-1}[k-1] \triangleq \sum_{\nu=1}^{N-1} r_{\mathrm{DFE}}[k-\nu] \prod_{\mu=1}^{\nu-1} \hat{a}[k-k_0-\mu]. \tag{59}$$

For $N = 2$, $d^{\mathrm{LE}}_{\mathrm{nrec}}[k] = r_{\mathrm{DFE}}[k]r^*_{\mathrm{DFE}}[k-1]$ is obtained, i.e., $d^{\mathrm{LE}}_{\mathrm{nrec}}[k]$ is the output of a conventional differential detector [2]. For $N > 2$, $d^{\mathrm{LE}}_{\mathrm{nrec}}[k]$ is the output of a nonrecursive decision–feedback differential detector (DF–DD) [7, 9] with input $r_{\mathrm{DFE}}[\cdot]$.

If a recursively generated phase reference symbol $\hat{q}_{\mathrm{rec}}[k-1]$ is used in Eq. (20) the corresponding decision variable $d^{\mathrm{LE}}_{\mathrm{rec}}[k]$ results from Eq. (58) by replacing $\hat{l}^{N-1}_{\mathrm{nrec}}[k-1]$ with

$$\hat{l}_{\mathrm{rec}}[k-1] \stackrel{\triangle}{=} \alpha\hat{a}[k-k_0-1]\hat{l}_{\mathrm{rec}}[k-2] + r_{\mathrm{DFE}}[k-1]. \tag{60}$$

For $\alpha = 0$, $d^{\mathrm{LE}}_{\mathrm{rec}}[k]$ is the output of a conventional differential detector. On the other hand, for $0 < \alpha < 1$, the decision variable is the output of a recursive DF–DD [12, 13] with input $r_{\mathrm{DFE}}[\cdot]$.

The decision variables of the described nonrecursive and recursive NLE are – up to a constant – identical with those proposed in [17] (for the special cases $N = 2$, $\alpha = 0$, they are also equivalent to those of the NLE proposed in [4]). In addition, in both cases the same receiver structure, i.e., the combination of a linear filter and a DF–DD (cf. Fig. 1 of [17]), is obtained. Note, however, that the derivation of the receiver structure given in this paper is completely different from that provided in [17]. Moreover, the resulting equalizer filter is different from that described in [17] since the underlying cost functions (optimization criteria) are not identical. Thus, the filter adaptation which will be explained in Section 5, is different from the adaptive algorithms proposed in [17]. Moreover, the analytical solution for $N \to \infty$ ($\alpha \to 1$) for the filter coefficient vector given in Eq. (32) of [17] differs from that obtained from Eq. (39) of this paper for $L_{\mathrm{B}} = 1$. Hence, it has to be expected that the equalizer filter obtained here also differs from that of [17] for $N < \infty$ ($\alpha < 1$), where no analytical solution could be obtained. Despite these differences, our simulations showed that the performance of both schemes is almost identical. For the special case $N \to \infty$ ($\alpha \to 1$), it is shown in [17] that the resulting receiver approaches the performance of a coherent linear MMSE equalizer. On the other hand, the NLE proposed here is derived directly from NDFE which approaches the performance of coherent MMSE–DFE for $N \to \infty$ ($\alpha \to 1$) and a coherent linear MMSE equalizer may be viewed as a coherent MMSE decision–feedback equalizer with $L_{\mathrm{B}} = 1$ (no feedback filter). Thus, it can be concluded that the NLE proposed here also approaches the performance of a coherent linear MMSE equalizer for $N \to \infty$ ($\alpha \to 1$).

Finally, it is interesting to note that for an ISI–free AWGN channel, where no FF and FB filter is required, i.e., $r_{\mathrm{DFE}}[k] = r[k]$, the proposed NDFE scheme (cf. Eqs. (58)–(60)) simplifies to a DF–DD as proposed in [7, 9, 12, 13] for detection of MDPSK signals transmitted over frequency–nonselective channels.

5 Filter Adaptation

For coherent MMSE–DFE conventional LMS or RLS algorithms [3] might be used to find the optimum FF and FB filters recursively and in [4, 5, 17] modified LMS and RLS algorithms have been proposed for noncoherent equalizers. In the following, two novel adaptive algorithms are provided which minimize the (noncoherent) cost function defined in Section 3.2., recursively. Note, that conventional (coherent) adaptive algorithms are sensitive to frequency offset and should not be used in a noncoherent receiver.

5.1 Modified LMS Algorithm

If a gradient algorithm is to be employed for calculation of the FF filter, the recursive relation for updating the FF filter coefficient vector is given by

$$\boldsymbol{f}_{\mathrm{F}}[k+1] = \boldsymbol{f}_{\mathrm{F}}[k] - \delta \frac{\partial}{\partial \boldsymbol{f}_{\mathrm{F}}^{*}[k]} \lambda_{\mathrm{DFE}}[k], \tag{61}$$

where δ is the adaptation step size. Using Eq. (30), Eq. (61) can be rewritten to

$$\boldsymbol{f}_{\mathrm{F}}[k+1] = \boldsymbol{f}_{\mathrm{F}}[k] + \delta e_{\mathrm{F}}^{*}[k]\boldsymbol{r}[k], \tag{62}$$

with

$$e_{\mathrm{F}}[k] = \frac{q_{\mathrm{nrec}}^{N}[k]}{|q_{\mathrm{nrec}}^{N}[k]|} y_{\mathrm{DFE}}[k] - r_{\mathrm{DFE}}[k]. \tag{63}$$

Similarly, the FB filter coefficients can be updated according to

$$\boldsymbol{f}_{\mathrm{B}}[k+1] = \boldsymbol{f}_{\mathrm{B}}[k] + \delta e_{\mathrm{B}}^{*}[k]\boldsymbol{b}[k-k_0-1], \tag{64}$$

with

$$e_{\mathrm{B}}[k] = \frac{(q_{\mathrm{nrec}}^{N}[k])^{*}}{|q_{\mathrm{nrec}}^{N}[k]|} r_{\mathrm{DFE}}[k] - y_{\mathrm{DFE}}[k]. \tag{65}$$

It can be seen from Eqs. (9), (28), (62)–(65), that Θ has no influence on the FF and FB filter coefficients. For calculation of $e_{\mathrm{F}}[k]$ and $e_{\mathrm{B}}[k]$, knowledge of the transmitted symbol sequence $b[\cdot]$ is necessary. Therefore, a training sequence is required for adaptation of the FF and FB filter coefficients. If the filter coefficients have converged to the optimum solution, decision–feedback symbols may be used to track possible variations of the channel. Note, that the only differences between the derived modified LMS algorithm and the conventional LMS algorithm used for adaptation of the filters for coherent MMSE–DFE are the factors $q_{\mathrm{nrec}}^{N}[k]/|q_{\mathrm{nrec}}^{N}[k]|$ and $(q_{\mathrm{nrec}}^{N}[k])^{*}/|q_{\mathrm{nrec}}^{N}[k]|$ for calculation of $e_{\mathrm{F}}[k]$ and $e_{\mathrm{B}}[k]$, respectively.

Of course, it is also possible to derive modified LMS algorithms for the FF and FB filters if the recursively generated phase reference symbol $q_{\mathrm{rec}}[k]$ is used instead of $q_{\mathrm{nrec}}^{N}[k]$. The resulting algorithm is identical to that for the nonrecursive case, except that $q_{\mathrm{nrec}}^{N}[k]$ in Eqs. (63) and (65) has to be replaced by $q_{\mathrm{rec}}[k]$.

A stability analysis of the proposed modified LMS algorithm is very difficult and beyond the scope of this paper. Our simulations indicate that the adaptation constant δ has to fulfill similar conditions like for the conventional LMS algorithm [3]. For each adaptation process, we initialize the FF and FB filter coefficient vector with the zero vector. Thus, in the first iteration step $q_{\mathrm{nrec}}^N[0] = 0$ follows. Since $q_{\mathrm{nrec}}^N[0]/|q_{\mathrm{nrec}}^N[0]|$ is not defined in this case, we set $q_{\mathrm{nrec}}^N[0]/|q_{\mathrm{nrec}}^N[0]| = 1$ (cf. Eqs. (63), (65)). Note, that for $k > 0$, $q_{\mathrm{nrec}}^N[k] \neq 0$ holds with probability one in the presence of noise.

5.2 Modified RLS Algorithm

Besides the simple modified LMS algorithm, we present a modified RLS algorithm which provides a faster convergence. As shown in Appendix B, the FF filter coefficients may be calculated by using the following equations:

$$\begin{aligned}
\boldsymbol{k}_{\mathrm{F}}[k] &= \frac{\boldsymbol{P}_{\mathrm{F}}[k-1]\boldsymbol{r}[k]}{w + \boldsymbol{r}^H[k]\boldsymbol{P}_{\mathrm{F}}[k-1]\boldsymbol{r}[k]}, && (66)\\
\xi_{\mathrm{F}}[k] &= \frac{q_{\mathrm{nrec}}^N[k-1]}{|q_{\mathrm{nrec}}^N[k-1]|} y_{\mathrm{DFE}}[k] - \boldsymbol{f}_{\mathrm{F}}^H[k-1]\boldsymbol{r}[k], && (67)\\
\boldsymbol{f}_{\mathrm{F}}[k] &= \boldsymbol{f}_{\mathrm{F}}[k-1] + \boldsymbol{k}_{\mathrm{F}}[k]\xi_{\mathrm{F}}^*[k], && (68)\\
\boldsymbol{P}_{\mathrm{F}}[k] &= w^{-1}\boldsymbol{P}_{\mathrm{F}}[k-1] - w^{-1}\boldsymbol{k}_{\mathrm{F}}[k]\boldsymbol{r}^H[k]\boldsymbol{P}_{\mathrm{F}}[k-1], && (69)
\end{aligned}$$

where w, $0 < w \leq 1$, denotes the forgetting factor of the modified RLS algorithm. The modified RLS algorithm for the FB filter can be derived in a similar way as that for the FF filter in Appendix B. The resulting algorithm consists of the following equations:

$$\begin{aligned}
\boldsymbol{k}_{\mathrm{B}}[k] &= \frac{\boldsymbol{P}_{\mathrm{B}}[k-1]\boldsymbol{b}[k-k_0-1]}{w + \boldsymbol{b}^H[k-k_0-1]\boldsymbol{P}_{\mathrm{B}}[k-1]\boldsymbol{b}[k-k_0-1]}, && (70)\\
\xi_{\mathrm{B}}[k] &= \frac{(q_{\mathrm{nrec}}^N[k-1])^*}{|q_{\mathrm{nrec}}^N[k-1]|} r_{\mathrm{DFE}}[k] - b[k-k_0] - \boldsymbol{f}_{\mathrm{B}}^H[k-1]\boldsymbol{b}[k-k_0-1], && (71)\\
\boldsymbol{f}_{\mathrm{B}}[k] &= \boldsymbol{f}_{\mathrm{B}}[k-1] + \boldsymbol{k}_{\mathrm{B}}[k]\xi_{\mathrm{B}}^*[k], && (72)\\
\boldsymbol{P}_{\mathrm{B}}[k] &= w^{-1}\boldsymbol{P}_{\mathrm{B}}[k-1] - w^{-1}\boldsymbol{k}_{\mathrm{B}}[k]\boldsymbol{b}^H[k-k_0-1]\boldsymbol{P}_{\mathrm{B}}[k-1]. && (73)
\end{aligned}$$

The only difference between these modified RLS algorithms and the corresponding conventional RLS algorithms are the factors $q_{\mathrm{nrec}}^N[k-1]/|q_{\mathrm{nrec}}^N[k-1]|$ and $(q_{\mathrm{nrec}}^N[k-1])^*/|q_{\mathrm{nrec}}^N[k-1]|$ in Eqs. (67) and (71), respectively.

For initialization of $\boldsymbol{P}_{\mathrm{F}}[0]$ and $\boldsymbol{P}_{\mathrm{B}}[0]$ we propose

$$\boldsymbol{P}_{\mathrm{X}}[0] = \delta_{\mathrm{RLS}}^{-1}\boldsymbol{I}_{S_{\mathrm{X}}\times S_{\mathrm{X}}}, \qquad \mathrm{X} = \mathrm{F},\ \mathrm{B}. \tag{74}$$

δ_{RLS} is a small positive constant (typical value: 0.001). Furthermore, in our simulations, at time $k = 0$ all FF and FB filter coefficients are initialized with zero. Consequently, similar to the modified LMS algorithm, at $k = 1$, $(q^N_{\mathrm{nrec}}[0])^*/|q^N_{\mathrm{nrec}}[0]| = 1$ (cf. Eqs. (67), (71)) is employed.

Note, that the described modified RLS algorithms may also employ the recursively generated phase reference symbol $q_{\mathrm{rec}}[k-1]$ instead of $q^N_{\mathrm{nrec}}[k-1]$ (cf. Eqs. (67), (71)).

In principle, the values of N and α used for NDFE do not have to be the same as those used for the modified LMS and RLS algorithms. However, for simplicity here we always choose the same values for NDFE and the applied adaptive algorithms for filter adjustment.

6 Simulation Results

In this section, we evaluate the performance of the derived noncoherent equalization schemes and compare them with coherent equalizers. For this, Channel A ($h_0 = 0.304$, $h_1 = 0.903$, $h_2 = 0.304$, $L_h = 3$) and Channel B ($h_0 = 1/\sqrt{19}$, $h_1 = 2/\sqrt{19}$, $h_2 = 3/\sqrt{19}$, $h_3 = 2/\sqrt{19}$, $h_4 = 1/\sqrt{19}$, $L_h = 5$) specified in [4, 5] and [2], respectively, are used. Channel A has no spectral nulls and thus, may be equalized linearly. Channel B, however, has spectral nulls and DFE has to be employed. For all simulations, a QDPSK constellation ($M = 4$) is used.

First the convergence speed of the modified LMS and RLS algorithms is investigated for Channel A and NDFE with nonrecursively generated phase reference symbol. The FF and FB filter lengths are $L_{\mathrm{F}} = 4$ and $L_{\mathrm{B}} = 2$, respectively. The decision delay is $k_0 = 4$. Figs. 4a) and 4b) show the learning curves of the modified LMS and RLS algorithms, respectively. Note, that in both cases averaging is done over 500 adaptation processes and $10\log_{10}(E_b/N_0) = 12$ dB ($E_b = E_S/2$ is the mean received energy per bit) is valid. For the modified LMS algorithm ($\delta = 0.008$), $J'[k]$ is defined as $J'[k] \triangleq \mathcal{E}\{|e_{\mathrm{F}}[k]|^2\} = \mathcal{E}\{|e_{\mathrm{B}}[k]|^2\}$, whereas $J'[k] \triangleq \mathcal{E}\{|\xi_{\mathrm{F}}[k]|^2\} = \mathcal{E}\{|\xi_{\mathrm{B}}[k]|^2\}$ is used for the modified RLS algorithm[7]. It can be observed from Fig. 4a), that the influence of N on the convergence speed of the modified LMS algorithm is negligible. In addition, the modified LMS algorithm converges as fast as the conventional LMS algorithm with the same adaptation constant. The modified RLS algorithm ($w = 0.99$) provides a faster convergence than the modified LMS algorithm, however, it converges more slowly than the conventional RLS algorithm ($w = 0.99$) and causes a larger steady–state error for small N. The reason for this behaviour might be the approximation needed for derivation of the modified RLS algorithm (cf. Appendix B). The steady–state error of the modified LMS algorithm is almost independent of N[8].

For simplicity, in the following simulations, the modified LMS algorithm ($\delta = 0.001$) is used exclusively. After transmission of a training sequence, a transition to the decision–directed

[7]The definitions of $J'[k]$ for the modified LMS and RLS algorithms are similar to those proposed in [3] for the conventional LMS and RLS algorithms.

[8]Please note that for the modified LMS algorithm proposed for linear equalization in [17], both convergence speed and steady–state error depend on N (cf. Fig. 2 of [17]).

mode occurs. In order to illustrate the influence of the FF and FB filters, Fig. 5 shows h_ν, $0 \leq \nu \leq L_h - 1$, one possible solution for $f_{\mathrm{F},\nu}[k]$ (the phase of the FF filter coefficients is not unique), $0 \leq \nu \leq L_\mathrm{F} - 1$, the resulting $g_\nu[k]$, $0 \leq \nu \leq L_h + L_\mathrm{F} - 2$, and $f_{\mathrm{B},\nu}[k]$, $0 \leq \nu \leq L_\mathrm{B} - 1$, for Channel B under steady–state conditions, i.e., after full convergence. Here, $N = 3$, $L_\mathrm{F} = 5$, $L_\mathrm{B} = 5$, $k_0 = 4$, $10\log_{10}(E_b/N_0) = 15$ dB are valid. It can be observed, that the precursors $g_\nu[k]$, $0 \leq \nu \leq k_0 - 1$, are reduced by the FF filter, however, they are not forced to zero by the proposed filter optimization criterion. Because of the relation of the proposed scheme with coherent MMSE–DFE, this behaviour was to be expected at least for $N \gg 1$. The FB filter coefficients $f_{\mathrm{B},\nu}[k]$, $1 \leq \nu \leq L_\mathrm{B} - 1$, have the same magnitude as the postcursors $g_\nu[k]$, $k_0 + 1 \leq \nu \leq L_h + L_\mathrm{F} - 2$. Note that for coherent MMSE–DFE they also need to have the same phase.

For Fig. 6a) and 6b), NDFE with nonrecursively and recursively generated phase reference symbol, respectively, is used for equalization of Channel A. For the FF and FB filters, $L_\mathrm{F} = 4$, $L_\mathrm{B} = 2$, and $k_0 = 4$ are valid. It can be seen, that the BER for NDFE is always lower than that for suboptimum NDFE. As was predicted in Section 3.3, both NDFE and the suboptimum version approach coherent MMSE–DFE for $N \gg 1$ and $\alpha \to 1$. A comparison with Fig. 3 of [17] reveals that NDFE outperforms the NLE proposed there (which has almost the same performance as the NLE described in Section 4.2 of this paper) if N and α are equal in both cases.

In Fig. 7, BER vs. $10\log_{10}(E_b/N_0)$ is shown for Channel B when equalized with NDFE ($L_\mathrm{F} = 5$, $L_\mathrm{B} = 5$, $k_0 = 4$). Again, the nonrecursively and the recursively generated phase reference symbols are employed in Figs. 7a) and 7b), respectively. For small N and α, NDFE suffers a significant performance loss compared to coherent MMSE–DFE. However, as N and α increase, performance improves. For $N \gg 1$ and $\alpha \to 1$, both NDFE and suboptimum NDFE approach coherent MMSE–DFE. Note that a linear equalizer fails to equalize this channel since it has spectral nulls.

So far, we have assumed that the channel introduces an arbitrary but constant phase shift. Since practical receivers often have to cope with phase drifts, in Fig. 8 the performance of the proposed NDFE receiver ($L_\mathrm{F} = 4$, $L_\mathrm{B} = 2$, $k_0 = 4$) is evaluated for a constant frequency offset Δf, $10\log_{10}(E_b/N_0) = 10$ dB, and Channel A. Figs. 8a) and 8b) show BER vs. normalized frequency offset ΔfT for nonrecursively and recursively generated phase reference symbol, respectively. As N and α increase, the BER of NDFE for $\Delta fT = 0$ decreases, however, the robustness against frequency offset decreases, too. Thus, there is a trade–off between performance under pure AWGN conditions and robustness against frequency offset which is typical for noncoherent detection schemes [17, 20]. Note that coherent MMSE–DFE fails to equalize this channel even for very small frequency offsets since the conventional LMS algorithm cannot track the phase variations.

7 Conclusions

In this paper, a novel NDFE scheme is obtained from noncoherent MLSE. From this NDFE scheme a suboptimum NDFE scheme and an NLE are derived. Furthermore, the relation of the novel NLE to a previously proposed NLE is investigated. It has been shown analytically and by simulations, that the proposed NDFE (original and suboptimum version) can approach coherent MMSE–DFE, whereas the NLE can approach the performance of a coherent linear MMSE equalizer. A cost function for calculation of the optimum FF and FB filters is provided. Since, in general, the calculation of the FF and FB filters using this cost function is difficult, a modified LMS and a modified RLS algorithm are derived which find the optimum filter settings recursively. The performance of all derived noncoherent equalizers and novel adaptation algorithms is evaluated by simulations and compared with the performance of coherent equalizers and conventional adaptation algorithms. Finally, it is shown that for NDFE there is a trade–off between performance under pure AWGN conditions and sensitivity to frequency offset.

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Appendix A

For determination of the NDFE metric for $N \to \infty$ under steady–state conditions (i.e., $\boldsymbol{f}_{\mathrm{F}}[k] = \boldsymbol{f}_{\mathrm{F}}$, $\boldsymbol{f}_{\mathrm{B}}[k] = \boldsymbol{f}_{\mathrm{B}}$, $\forall\, k$), we assume that only correct symbols are fedback, i.e., $\hat{q}_{\mathrm{nrec}}^{N-1}[k-1] = q_{\mathrm{nrec}}^{N-1}[k-1]$, and apply Eq. (34) to Eqs. (20), (21). After some straightforward manipulations,

$$\begin{aligned}\hat{b}[k-k_0] &= \underset{b[k-k_0]}{\mathrm{argmin}} \Bigg\{ \left|b[k-k_0] + \boldsymbol{f}_{\mathrm{B}}^H \hat{\boldsymbol{b}}[k-k_0-1]\right|^2 - 2\Bigg| \mathrm{e}^{-j\phi} r_{\mathrm{DFE}}[k] \Big(b[k-k_0] \\ &\qquad + \boldsymbol{f}_{\mathrm{B}}^H \hat{\boldsymbol{b}}[k-k_0-1]\Big)^* + (N-1)|\mathcal{E}\{r_{\mathrm{DFE}}[k] y_{\mathrm{DFE}}^*[k] | \Theta\}| \Bigg| \Bigg\} \qquad (75)\end{aligned}$$

is obtained, where $N \to \infty$ is assumed. For the following, we make use of the relation

$$\lim_{x\to\infty} \{|x+z| - x\} = \lim_{x\to\infty} \left\{ x\sqrt{\left(1 + \frac{\mathrm{Re}\{z\}}{x}\right)^2 + \left(\frac{\mathrm{Im}\{z\}}{x}\right)^2} - x \right\} = \mathrm{Re}\{z\}, \qquad (76)$$

$x \in \mathbb{R}$, $z \in \mathbb{C}$ ($\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ denote the real and the imaginary part of a complex number, respectively). From Eq. (76), $|x+z| \to x + \mathrm{Re}\{z\}$ for $x \to \infty$, follows. Using this, assuming $|\mathcal{E}\{r_{\mathrm{DFE}}[k]y^*_{\mathrm{DFE}}[k]|\Theta\}| > 0$, which is certainly justified after the adaptive algorithms have converged, and omitting terms which do not influence the decision, for $N \to \infty$ Eq. (75) can be simplified to

$$\hat{b}[k-k_0] = \underset{b[k-k_0]}{\mathrm{argmin}} \left\{ \left| b[k-k_0] + \boldsymbol{f}_{\mathrm{B}}^H \hat{\boldsymbol{b}}[k-k_0-1] \right|^2 - 2\mathrm{Re}\left\{ \mathrm{e}^{-j\phi} r_{\mathrm{DFE}}[k] \left(b[k-k_0] + \boldsymbol{f}_{\mathrm{B}}^H \hat{\boldsymbol{b}}[k-k_0-1] \right)^* \right\} \right\}. \tag{77}$$

Now we make use of the fact that the decision rule remains unchanged if the term $|\mathrm{e}^{-j\phi} r_{\mathrm{DFE}}[k]|^2$ is added in Eq. (77). Applying $\phi_0 = \Theta - \phi$ (cf. Eq. (49)), this leads to the simple form

$$\hat{b}[k-k_0] = \underset{b[k-k_0]}{\mathrm{argmin}} \left\{ \left| b[k-k_0] + \boldsymbol{f}_{\mathrm{B}}^H \hat{\boldsymbol{b}}[k-k_0-1] - \mathrm{e}^{-j(\phi_0-\Theta)} r_{\mathrm{DFE}}[k] \right|^2 \right\}. \tag{78}$$

Appendix B

For derivation of the modified RLS algorithm for the FF filter coefficients we define the cost function

$$J_{\mathrm{F}}[k] \triangleq \sum_{\mu=1}^{k} w^{k-\mu} \lambda^{\mathrm{F}}_{\mathrm{DFE}}[k,\,\mu], \tag{79}$$

where w, $0 < w \leq 1$, is a forgetting factor and $\lambda^{\mathrm{F}}_{\mathrm{DFE}}[k,\,\mu]$ is given by

$$\lambda^{\mathrm{F}}_{\mathrm{DFE}}[k,\,\mu] \triangleq \sum_{\nu=0}^{N-1} |\boldsymbol{f}_{\mathrm{F}}^H[k-\nu]\boldsymbol{r}[\mu-\nu]|^2 + \sum_{\nu=0}^{N-1} |y_{\mathrm{DFE}}[\mu-\nu]|^2 - 2\left| \sum_{\nu=0}^{N-1} \boldsymbol{f}_{\mathrm{F}}^H[k-\nu]\boldsymbol{r}[\mu-\nu] y^*_{\mathrm{DFE}}[\mu-\nu] \right|, \tag{80}$$

cf. Eq. (15). In order to find the minimum of $J_{\mathrm{F}}[k]$ with the help of Eq. (29), we calculate

$$\frac{\partial}{\partial \boldsymbol{f}_{\mathrm{F}}^*[k]} J_{\mathrm{F}}[k] = \sum_{\mu=1}^{k} w^{k-\mu} \left(\boldsymbol{r}[\mu]\boldsymbol{r}^H[\mu]\boldsymbol{f}_{\mathrm{F}}[k] - \frac{\left(\sum_{\nu=0}^{N-1} \boldsymbol{f}_{\mathrm{F}}^H[k-\nu]\boldsymbol{r}[\mu-\nu]y^*_{\mathrm{DFE}}[\mu-\nu]\right)^*}{\left|\sum_{\nu=0}^{N-1} \boldsymbol{f}_{\mathrm{F}}^H[k-\nu]\boldsymbol{r}[\mu-\nu]y^*_{\mathrm{DFE}}[\mu-\nu]\right|} \boldsymbol{r}[\mu] y^*_{\mathrm{DFE}}[\mu] \right) \tag{81}$$

and set the result equal to zero. This leads to

$$\hat{\boldsymbol{\Phi}}_{rr}[k]\boldsymbol{f}_{\mathrm{F}}[k] = \hat{\boldsymbol{\varphi}}'_{ry}[k], \tag{82}$$

where the definitions

$$\hat{\boldsymbol{\Phi}}_{rr}[k] \triangleq \sum_{\mu=1}^{k} w^{k-\mu} \boldsymbol{r}[\mu]\boldsymbol{r}^H[\mu] = w\hat{\boldsymbol{\Phi}}_{rr}[k-1] + \boldsymbol{r}[k]\boldsymbol{r}^H[k], \tag{83}$$

$$\hat{\boldsymbol{\varphi}}'_{ry}[k] \triangleq \sum_{\mu=1}^{k} w^{k-\mu} \frac{\left(\sum_{\nu=0}^{N-1} \boldsymbol{f}_{\mathrm{F}}^H[k-\nu]\boldsymbol{r}[\mu-\nu]y^*_{\mathrm{DFE}}[\mu-\nu]\right)^*}{\left|\sum_{\nu=0}^{N-1} \boldsymbol{f}_{\mathrm{F}}^H[k-\nu]\boldsymbol{r}[\mu-\nu]y^*_{\mathrm{DFE}}[\mu-\nu]\right|} \boldsymbol{r}[\mu] y^*_{\mathrm{DFE}}[\mu], \tag{84}$$

are used. Unlike $\hat{\mathbf{\Phi}}_{rr}[k]$, $\hat{\boldsymbol{\varphi}}'_{ry}[k]$ cannot be calculated recursively. Thus, it is not possible to find a computationally efficient algorithm for exact minimization of $J_{\mathrm{F}}[k]$. In order to overcome this problem, we use the modified vector

$$\begin{aligned}\hat{\boldsymbol{\varphi}}_{ry}[k] &\triangleq \sum_{\mu=1}^{k} w^{k-\mu} \frac{\left(\sum_{\nu=1}^{N} \boldsymbol{f}_{\mathrm{F}}^{H}[\mu-\nu]\boldsymbol{r}[\mu-\nu]y_{\mathrm{DFE}}^{*}[\mu-\nu]\right)^{*}}{\left|\sum_{\nu=1}^{N} \boldsymbol{f}_{\mathrm{F}}^{H}[\mu-\nu]\boldsymbol{r}[\mu-\nu]y_{\mathrm{DFE}}^{*}[\mu-\nu]\right|}\boldsymbol{r}[\mu]y_{\mathrm{DFE}}^{*}[\mu] \\ &= w\hat{\boldsymbol{\varphi}}_{ry}[k-1] + \boldsymbol{r}[k]\left(\frac{q_{\mathrm{nrec}}^{N}[k-1]}{|q_{\mathrm{nrec}}^{N}[k-1]|}y_{\mathrm{DFE}}[k]\right)^{*} \qquad (85)\end{aligned}$$

instead of $\hat{\boldsymbol{\varphi}}'_{ry}[k]$ in Eq. (82). Although this measure may have a negative influence on the convergence speed, our simulations show that the resulting algorithm still converges considerably fast. Moreover, now $\boldsymbol{f}_{\mathrm{F}}[k]$ can be calculated recursively. A comparison of Eqs. (83) and (85) with the corresponding equations for the conventional RLS algorithm [3] shows, that the factor $q_{\mathrm{nrec}}^{N}[k-1]/|q_{\mathrm{nrec}}^{N}[k-1]|$ in Eq. (85) is the only difference. Hence, the FF filter coefficients may be calculated recursively using Eqs. (66)–(69).

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Figures:

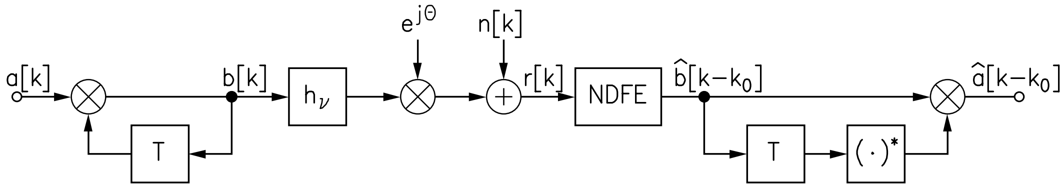

Figure 1: Block diagram of the discrete–time transmission model under consideration.

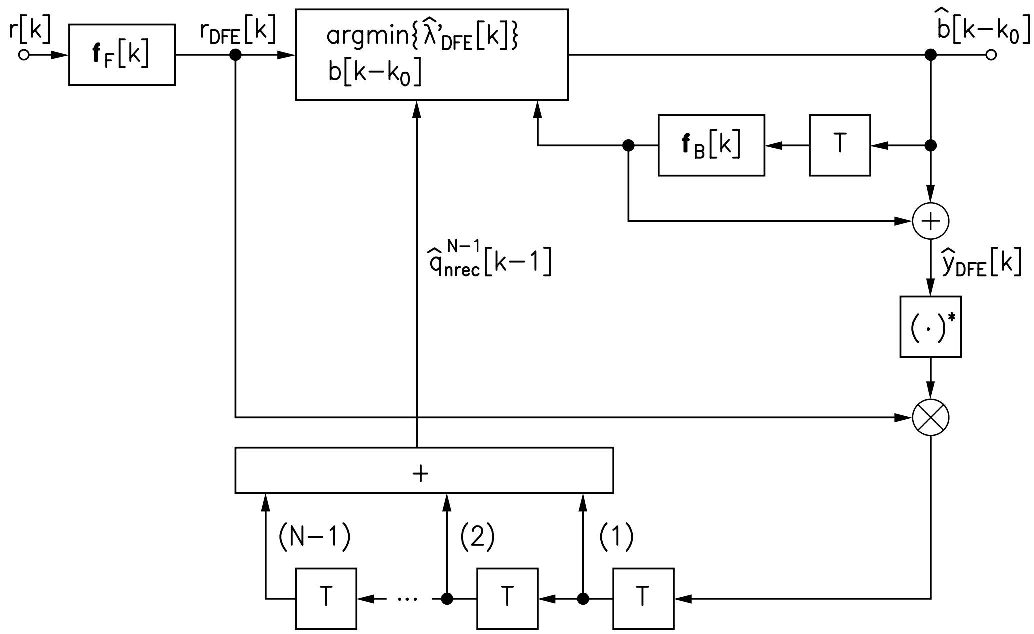

Figure 2: Receiver structure for the proposed NDFE scheme.

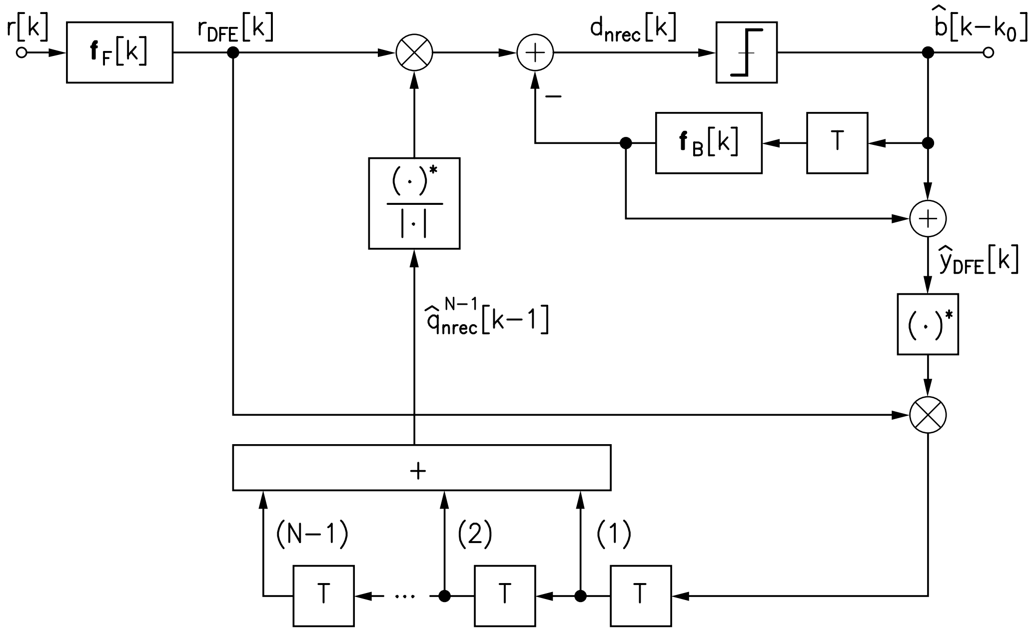

Figure 3: Receiver structure for suboptimum NDFE.

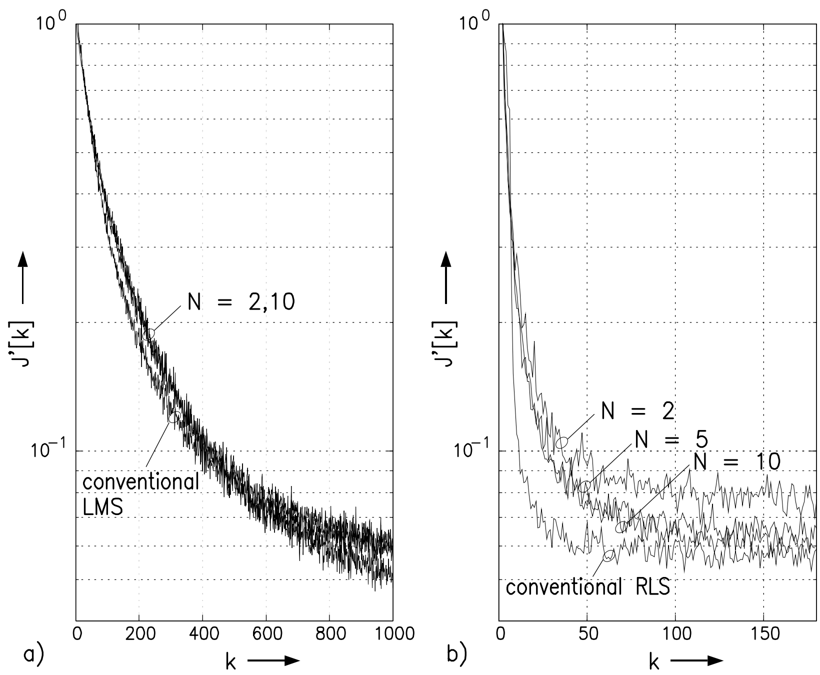

Figure 4: Learning curves for a) modified and conventional LMS algorithm; b) modified and conventional RLS algorithm.

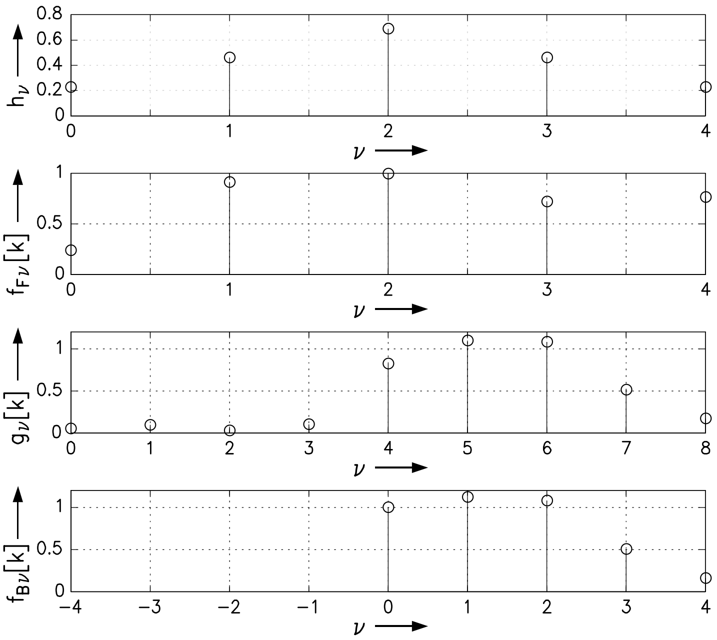

Figure 5: h_ν, $0 \le \nu \le L_h - 1$, $f_{\mathrm{F},\nu}[k]$, $0 \le \nu \le L_\mathrm{F} - 1$, $g_\nu[k]$, $0 \le \nu \le L_h + L_\mathrm{F} - 1$, and $f_{\mathrm{B},\nu}[k]$, $0 \le \nu \le L_\mathrm{B} - 1$, for Channel B and NDFE ($N = 3$).

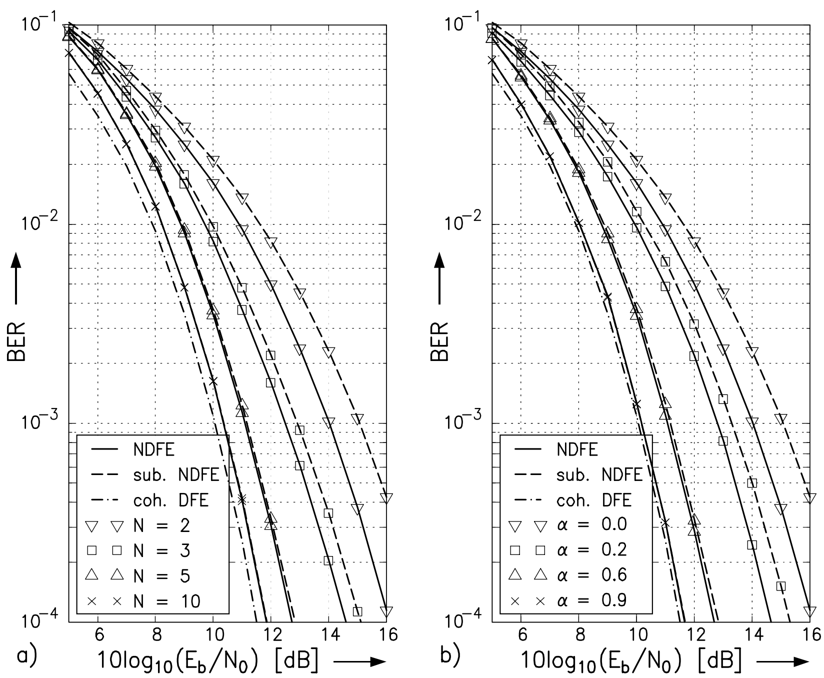

Figure 6: BER vs. $10\log_{10}(E_b/N_0)$ for NDFE and suboptimum NDFE with a) nonrecursively and b) recursively generated phase reference symbol. Channel A is used and for comparison the BER for coherent MMSE–DFE is also shown.

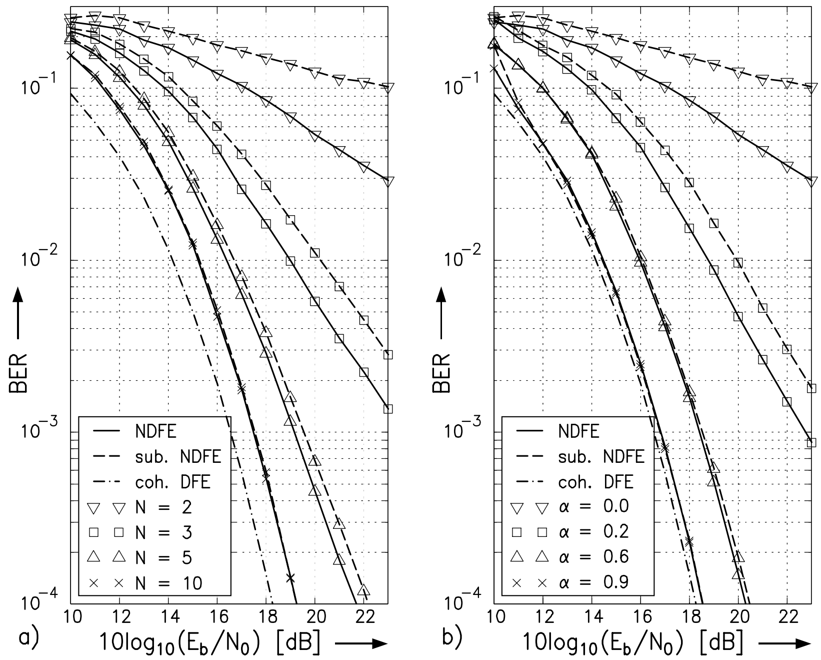

Figure 7: BER vs. $10\log_{10}(E_b/N_0)$ for NDFE and suboptimum NDFE with a) nonrecursively and b) recursively generated phase reference symbol. Channel B is used and for comparison the BER for coherent MMSE–DFE is also shown.

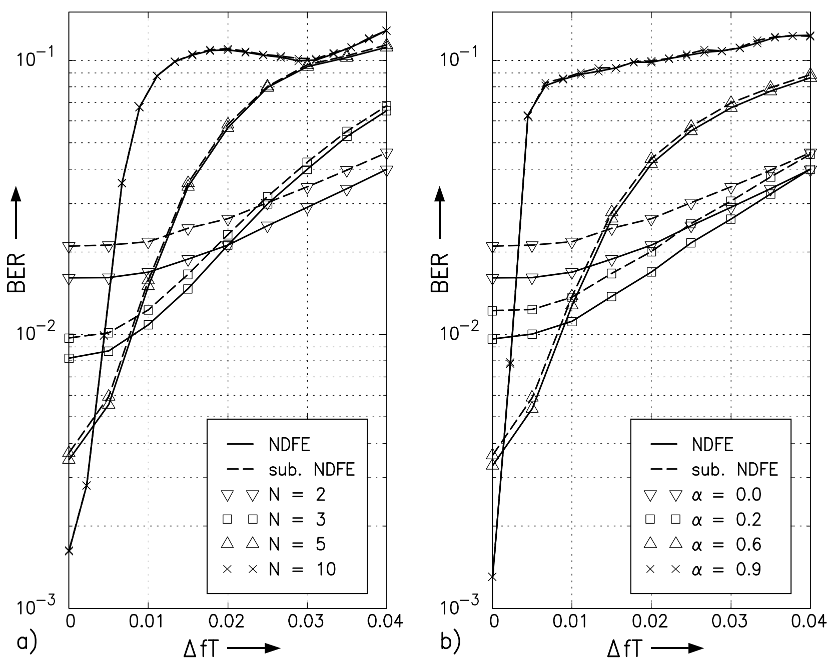

Figure 8: BER vs. ΔfT for NDFE and suboptimum NDFE with a) nonrecursively and b) recursively generated phase reference symbol.